GCE AS/A level

0979/01

MATHEMATICS – FP3
Further Pure Mathematics

P.M. MONDAY, 24 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S13-0979-01)

1. Determine the two positive roots of the equation

$$\cosh 2x - 7\cosh x + 7 = 0,$$

giving your answers correct to two decimal places. [6]

2. Jim wants to evaluate the real cube roots of several positive numbers but his calculator only performs the basic arithmetic operations add, subtract, multiply, divide.

(a) He therefore decides to determine $\sqrt[3]{a}$ by applying the Newton-Raphson method to the equation $x^3 - a = 0$ where $a > 0$.

(i) Show that this gives the iterative sequence

$$x_{n+1} = \frac{2x_n^3 + a}{3x_n^2}.$$

(ii) Taking $x_0 = 2$, use this method to find $\sqrt[3]{10}$ correct to four decimal places. [5]

(b) Huw suggests that an alternative method for determining $\sqrt[3]{a}$ could be to rearrange the equation $x^3 - a = 0$ in the form

$$x = \frac{a}{x^2}$$

and to define the iterative sequence

$$x_{n+1} = \frac{a}{x_n^2}.$$

Show, however, that this sequence diverges for all values of a. [4]

3. The function f is defined by

$$f(x) = \ln(2e^x - 1).$$

(a) Show that

$$f''(x) = \frac{-2e^x}{(2e^x - 1)^2}.$$

[3]

(b) Determine the Maclaurin series for $f(x)$ as far as the term in x^3. [6]

4. Determine the value of the integral

$$\int_1^2 \sqrt{(3 + 2x - x^2)}\, dx,$$

giving your answer correct to three significant figures. [10]

5. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^1 x^n \sinh x \, dx.$$

(a) Show that, for $n \geqslant 2$,

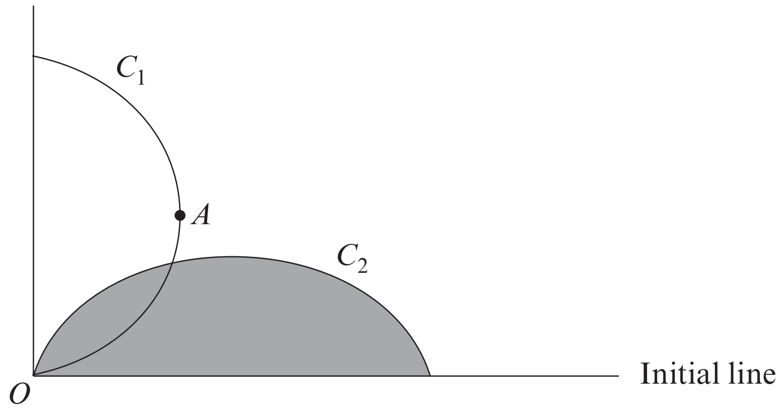
$$I_n = \cosh 1 - n \sinh 1 + n(n-1) I_{n-2}.$$ [5]

(b) Evaluate I_4, giving your answer in the form

$$a \cosh 1 + b \sinh 1 + c,$$

where a, b, c are integers. [5]

6.



The diagram shows sketches, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$, of the curve C_1 having polar equation $r = \sin^2\theta$ and the curve C_2 having polar equation $r = 1 - \sin\theta$.

(a) Find the polar coordinates of the point A on C_1 at which the tangent is perpendicular to the initial line. [8]

(b) Find the area of the shaded region enclosed between C_2 and the initial line. [6]

TURN OVER

7. *(a)* (i) Assuming the derivatives of $\cosh x$ and $\sinh x$, show that the derivatives of $\operatorname{cosech} x$ and $\coth x$ are respectively $-\operatorname{cosech} x \coth x$ and $-\operatorname{cosech}^2 x$.

(ii) Hence show that

$$\frac{\mathrm{d}}{\mathrm{d}x}[\ln(\operatorname{cosech} x + \coth x)] = -\operatorname{cosech} x.$$ [6]

(b) (i) Show that the length L of the arc joining the points (1, 0) and (e, 1) on the graph of $y = \ln x$ is given by

$$\int_1^{\mathrm{e}} \frac{\sqrt{1 + x^2}}{x}\,\mathrm{d}x.$$

(ii) Use the substitution $x = \sinh u$ to show that

$$L = \int_{\sinh^{-1} 1}^{\sinh^{-1} \mathrm{e}} (\operatorname{cosech} u + \sinh u)\,\mathrm{d}u.$$

(iii) Use the result in *(a)*(ii) to determine the value of L correct to three significant figures. [11]