GCE AS/A level

0977/01

MATHEMATICS – FP1
Further Pure Mathematics

A.M. TUESDAY, 18 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$S_n = 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2,$$

obtain an expression for S_n in the form $an^3 - bn$, where a, b are positive rational numbers. [6]

2. The complex numbers u, v, w satisfy the equation

$$\frac{1}{w} = \frac{1}{u} + \frac{1}{v}.$$

(a) Given that $u = 1 - \mathrm{i}$ and $v = 1 + 2\mathrm{i}$, determine w in the form $x + \mathrm{i}y$. [6]

(b) Find the modulus and argument of w. [2]

3. The roots of the cubic equation $x^3 - 2x^2 + 2x + 1 = 0$ are denoted by α, β, γ.

(a) Show that

$$\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma} = -8.$$ [4]

(b) Find the cubic equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$. [7]

4. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x + 2, y + 1)$ followed by a reflection in the line $y = x$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Show that T has no fixed points. [3]

5. Using mathematical induction, prove that $7^n - 1$ is divisible by 6 for all positive integers n. [6]

6. Consider the system of equations **AX** = **B**, where

$$\mathbf{A} = \begin{bmatrix} 1 & \lambda & 3 \\ 2 & 1 & \lambda \\ 5 & 4 & 7 \end{bmatrix}; \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}.$$

(a) (i) Find the determinant of **A** in terms of the constant λ.

(ii) Show that **A** is singular when $\lambda = 2$ and determine the other value of λ for which **A** is singular. [4]

(b) Given that $\lambda = 2$,

(i) show that the equations are consistent,

(ii) determine the general solution of the equations. [7]

(c) Given that $\lambda = 1$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse of **A**,

(iii) hence solve the equations. [7]

7. The function *f* is defined by

$$f(x) = \frac{\sqrt{1 + \sin x}}{(1 + \tan x)^2}.$$

Using logarithmic differentiation, determine the value of $f'\left(\frac{\pi}{4}\right)$. Give your answer correct to three significant figures. [9]

8. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2.$$

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point P moves along the curve with equation $y^2 - 2x^2 = 1$. Find the equation of the locus of Q. [5]

