GCE AS/A level

0973/01

MATHEMATICS – C1
Pure Mathematics

P.M. MONDAY, 13 May 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A, B, C have coordinates (8, 4), (6, –5), (3, 7), respectively. The line through A perpendicular to the line BC intersects BC at the point D.

(a) (i) Find the gradient of BC.

(ii) Show that the equation of BC is

$$4x + y - 19 = 0.$$

(iii) Find the equation of AD. [7]

(b) Show that the coordinates of D are (4, 3). [2]

(c) Find the length of BD. [2]

(d) The line AD is extended to E so that D is the mid-point of AE. Find the coordinates of E. [2]

2. Simplify

(a) $\dfrac{2 + 5\sqrt{7}}{4 + \sqrt{7}}$, [4]

(b) $\sqrt{360} - \sqrt{2} \times \left(\sqrt{5}\right)^3 - \dfrac{\sqrt{30} \times \sqrt{8}}{\sqrt{6}}$. [4]

3. The curve C has equation $y = 2x^2 - 10x + 7$.

(a) The point P has coordinates (3, –5) and lies on C. Find the equation of the **normal** to C at P. [5]

(b) The point Q lies on C and is such that the **tangent** to C at Q is parallel to the x-axis. Find the x-coordinate of Q. [2]

4. *(a)* Express $2x^2 - 16x - 8$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, find the least value of $x^2 - 8x - 4$ and the corresponding value of x. [2]

5. *(a)* Using the binomial theorem, write down and simplify the first three terms in the expansion of $(1 + 2x)^7$ in ascending powers of x. [3]

(b) Use your answer to part *(a)* to find the first three terms in the expansion of $(1 - 4x)(1 + 2x)^7$ in ascending powers of x. [3]

6. *(a)* (i) Assuming that the quadratic equation

$$(k + 1)x^2 + (4k + 1)x + (k - 5) = 0$$

has **two equal** roots, show that

$$4k^2 + 8k + 7 = 0.$$

(ii) Hence show that there are **no real** values of k such that the quadratic equation

$$(k + 1)x^2 + (4k + 1)x + (k - 5) = 0$$

has two equal roots. [6]

(b) Find the range of values of x satisfying the inequality

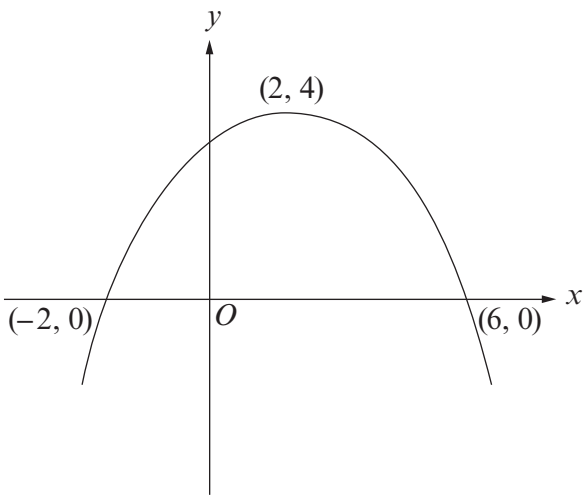
$$4x^2 - 9x - 9 \geqslant 0.$$ [3]

7. *(a)* Given that $y = 5x^2 + 8x - 11$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $6x^{\frac{2}{3}} + \frac{5}{x^2} - 4$ with respect to x. [2]

8. Solve the equation $8x^3 - 2x^2 - 7x + 3 = 0$. [6]

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$ and $(6, 0)$ and has a maximum point at $(2, 4)$.



Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

(a) $y = f(x + 5)$ [3]

(b) $y = f(-2x)$ [3]

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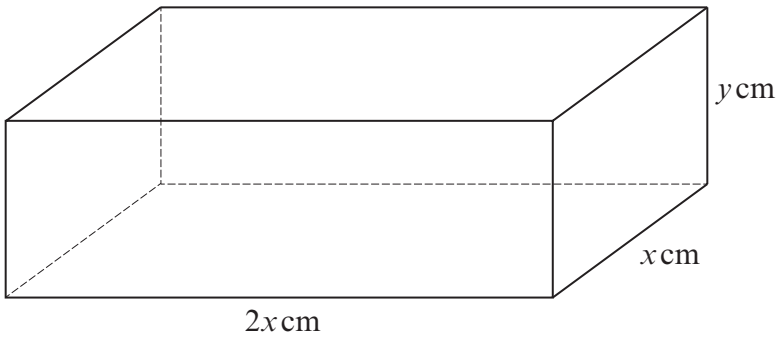
10. The diagram shows a **closed** box in the form of a cuboid. The length of the box is $2x$ cm, its width is x cm and its height is y cm.



The total surface area of the box is 108 cm^2.

(a) (i) Write down an equation involving x and y and hence show that

$$xy = 18 - \frac{2}{3}x^2 .$$

(ii) Hence show that the volume V cm^3 of the box is given by

$$V = 36x - \frac{4}{3}x^3 .$$

[3]

(b) Find the maximum value of V, showing that the value you have found is a maximum value. [5]

