GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3
AS/Advanced

SUMMER 2012

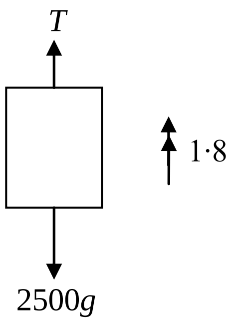
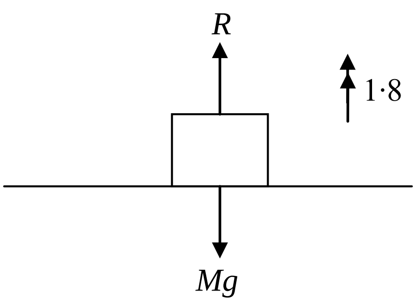
INTRODUCTION

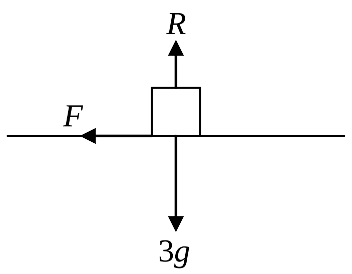
The marking schemes which follow were those used by WJEC for the Summer 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

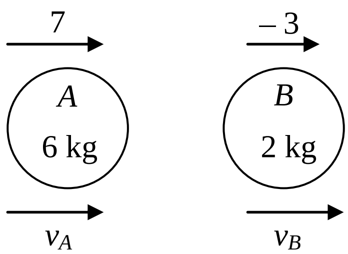
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

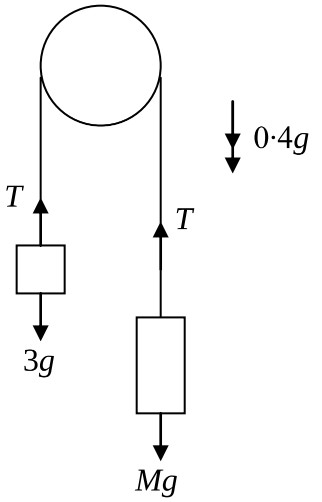
WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

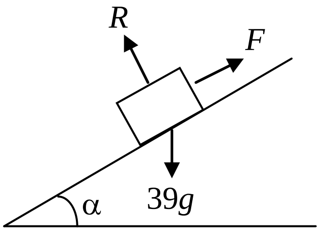
M1

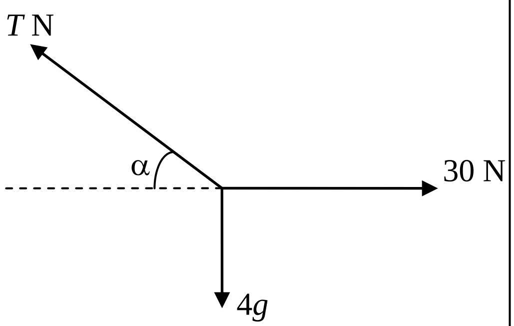
Q	Solution	Mark	Notes
1(a).			
	N2L dim correct equation attempted $T - 2500g = 2500 \times a$ $T = 2500(9.8 + 1.8)$ $T = \underline{29000\ (N)}$	M1 A1 A1	T, 2500g opposing Any form correct equ. cao
1(b)			
	N2L attempted $R - Mg = Ma$ $696 = M(9.8 + 1.8)$ $M = \underline{60\ (kg)}$	M1 A1 A1	R, Mg opposing, no extra forces Any form correct equ. cao

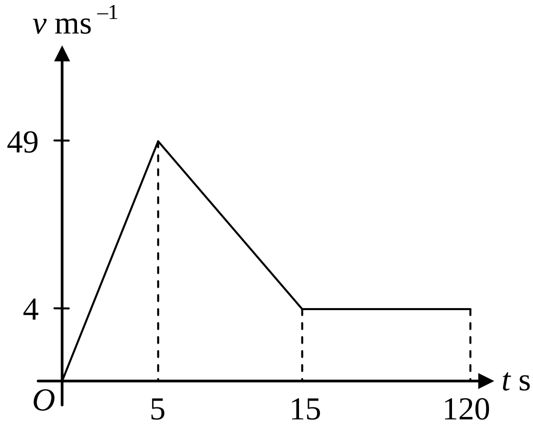
Q	Solution	Mark	Notes
2(a).			
	Resolve vertically R = 3g	B1	May be implied
	F = μR = $\frac{6}{49} \times 3 \times 9 \cdot 8$		
	F = <u>3.6 (N)</u>	B1	
	N2L F = ma	M1	used
	±3.6 = 3a		
	a = <u>-1.2 (ms^{-2})</u>	A1	needs to see -
2(b)	Using $v^2 = u^2 + 2as$ with u=9, v=0,a=(-)1.2	M1	allow sign errors, oe
	$0 = 9^2 + 2 \times (-1.2)$ s	A1	
	s = <u>33.75 (m)</u>	A1	allow -33.75

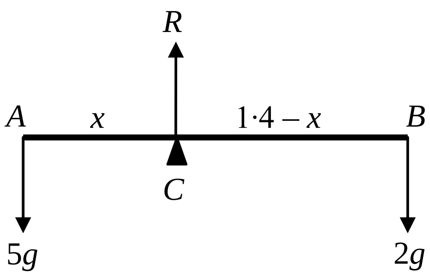
Q	Solution	Mark	Notes
3.			
3(a)	Conservation of momentum	M1	dim correct equation
	$6 \times 7 + 2 \times (-3) = 6v_A + 2v_B$	A1	
	$v_B = 2v_A$	m1	used
	$42 - 6 = 6v_A + 2 \times 2v_A$		
	$36 = 10v_A$		
	$v_A = 3.6$		
	$v_B = \underline{7.2\ (\text{ms}^{-1})}$	A1	
3(b)	Restitution equation	M1	attempted, ft c's vs, e on correct side. No more than one sign error.
	$7.2 - 3.6 = -e(-3 - 7)$	A1	
	$3.6 = 10e$		
	$e = \underline{0.36}$	A1	cao
3(c)	$I = 2 \times 7.2 - 2 \times (-3)$	M1	allow 6(7-3.6)
	$I = 14.4 + 6$		
	$I = \underline{20.4\ (\text{Ns})}$	A1	cao

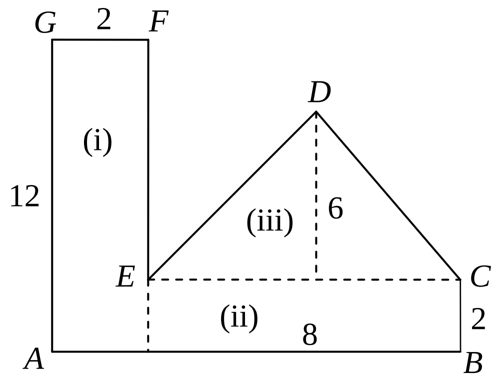
Q	Solution	Mark	Notes
4.	 		
	Apply N2L to B $Mg - T = Ma$	M1 A1	dim correct equation
	Apply N2L to A $T - 3g = 3a$	M1 A1	dim correct equation
	Adding	m1	correct method. dep on both M's
	$Mg - 3g = 0.4g(M + 3)$ $M - 3 = 0.4M + 1.2$ $0.6M = 4.2$ $M = \underline{7}$	A1	cao
	$T = 3 \times 9.8 + 3 \times 0.4 \times 9.8$ $T = \underline{41.16\ (N)}$	A1	cao
	<u>Alternative solution</u> Apply N2L to A $T - 3g = 3a$	M1 A1	dim. correct equation
	$T = 3(9.8 + 0.4 \times 9.8)$ $T = \underline{41.16\ (N)}$	A1	cao
	Apply N2L to B $Mg - T = Ma$ $9.8M - 0.4 \times 9.8M = 41.16$ $5.88M = 41.16$ $M = \underline{7}$	M1 A1 m1 A1	dim correct equation cao

Q	Solution	Mark	Notes
5.			
5(a)	Resolve perp to plane	M1	allow sin or cos
	R = 39gcosα		
	$R = 39 \times 9 \cdot 8 \times \frac{12}{13} = 352.8$ N		
	F = μR		
	F = 0.3 × 352.8	m1	
	F = 105.84 N	A1	si
	N2L down slope	M1	dim correct equation, -F
	39gsinα - F = 39a	A1	
	$39 \times 9 \cdot 8 \times \frac{5}{13} - 105 \cdot 84 = 39a$		
	a = 1.0554		
	a = <u>1.06 (ms^{-2})</u>	A1	
5(b)	N2L up slope	M1	dim correct equation, all forces, sin/cos, -F
	T – 39gsinα - F = 39a	A1	
	T = 147 + 105.84 + 39 × 0.4		
	T = <u>268.44 (N)</u>	A1	cao

Q	Solution	Mark	Notes
6.			
	Resolve vertically	M1	
	$T\sin\alpha = 4g$	A1	
	Resolve horizontally	M1	
	$T\cos\alpha = 30$	A1	
	Dividing		
	$\tan\alpha = \dfrac{4 \times 9 \cdot 8}{30}$	m1	dep on both M's
	$\alpha = \underline{52.5(7)°}$	A1	cao
	$T^2 = (4 \times 9.8)^2 + (30)^2$	m1	
	$T = \underline{49.36 \text{ (N)}}$	A1	cao

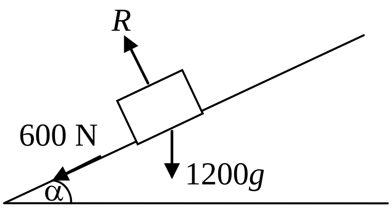
Q	Solution	Mark	Notes
7(a)	Using $v = u + at$ with $u=0$, $a=(\pm)9.8$, $t=5$	M1	
	$v = 0 + 9.8 \times 5$	A1	
	$v = \underline{49\ (ms^{-1})}$	A1	accept -49
7(b)			
		B1	units, labels and correct shape starting (0,0)
		B1	(0, 0) to (5, v)
		B1	(5, v) to (15, 4)
		B1	(15, 4) to (120, 4)
7(c)	Distance = Area under graph	M1	oe
	Distance $= 0.5 \times 5 \times 49 + 0.5(4 + 49) \times 10 + 105 \times 4$	B1	any one correct area, ft graph
	Distance = 122.5 + 265 + 420		
	Distance = $\underline{807.5\ (m)}$	A1	ft graph

Q	Solution	Mark	Notes
8.			
8(a)	Resolve vertically	M1	
	R = 5g + 2g		
	R = <u>7g (N)</u>	A1	
8(b)	Moments about C	M1	dim correct equation, no extra forces
	5gx = 2g(1.4 – x)	A1	rhs correct
		A1	lhs correct
	5x = 2.8 – 2x		
	7x = 2.8		
	x = 0.4	A1	cao
	AC = <u>0.4 (m)</u>		
	<u>Alternative solution</u>		
	Moments about A	M1	dim correct equation
	7gx = 2g × 1.4	A1	rhs correct
		A1	lhs correct
	x = <u>0.4 (m)</u>	A1	cao
		SC1	No marks at all, one correct moment, sc1.

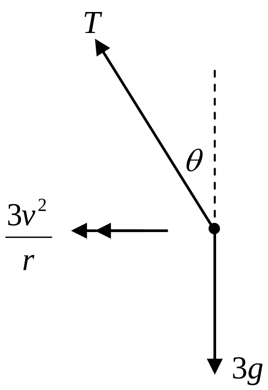
Q	Solution	Mark	Notes
9.			
9(a)	Area / from AG / from AB		
	(i) 24 1 6	B1	correct distances
	(ii) 12 5 1	B1	correct distances
	(iii) 18 5 4	B1	correct distances
	Lamina 54 x y	B1	areas all correct
	Moments about AG	M1	
	$54x = 24 \times 1 + 12 \times 5 + 18 \times 5$	A1	ft table if 2 or more B marks for distances gained.
	$x = \frac{29}{9} = 3.22$	A1	cao
	Moments about AB	M1	
	$54y = 24 \times 6 + 12 \times 1 + 18 \times 4$	A1	ft table
	$y = \frac{38}{9} = 4.22$	A1	cao
9(b)	$\theta = \tan^{-1}\left(\frac{x}{12 - y}\right)$	M1	correct triangle
	$= \tan^{-1}\left(\frac{29}{12\times9 - 38}\right)$	A1	correct equation, ft x, y
	$\theta = \underline{22.5°}$	A1	ft x and y

M2

Q	Solution	Mark	Notes
1.	s = $\int_0^{\frac{\pi}{6}} 4\cos 2t\, dt$	M1	limits not required
	s = [2sin2t]	A1	correct integration
	s = $2\sin\frac{\pi}{3}$ - 0		
	s = $\sqrt{3}$ = <u>1.732</u>	A1	cao
2(a)	N2L T = 7.5g	B1	
	Hooke's Law T = $\frac{245x}{5/3}$ (= 147x)	M1	
	7.5 × 9.8 = 147x		
	x = <u>0.5</u>	A1	cao
2(b)	Elastic Energy = $\frac{1}{2} \times \frac{x^2}{l}$	M1	used
	EE = $\frac{1}{2} \times \frac{245 \times 0 \cdot 5^2}{5/3}$		
	EE = <u>18.375 (J)</u>	A1	ft c's x value
3(a).	<u>v</u> = $\frac{dr}{dt}$	M1	used
	<u>v</u> = (1 + 4t)<u>i</u> + (3t – 2)<u>j</u>	A1	
	we required <u>v</u>.(-<u>i</u> + 2<u>j</u>) = 0	M1	
	-(1 + 4t) + 2(3t – 2) = 0	m1	
	-1 -4t + 6t -4 = 0		
	2t = 5		
	t = <u>2.5</u>	A1	cao
3(b)	<u>a</u> = $\frac{dv}{dt}$	M1	used
	<u>a</u> = 4<u>i</u> + 3<u>j</u>	A1	ft c's **v** provided constant
	<u>a</u> is independent of t and constant.		
	$\lvert a \rvert$ = $\sqrt{4^2 + 3^2}$ = <u>5</u>	A1	ft constant **a**=x**i**+y**j**

Q	Solution	Mark	Notes
4.			
4(a)	$T = \frac{P}{v} = \frac{75\times1000}{25}$	M1	
	$T = 3000$ N		
	N2L up plane	M1	dim correct, all forces
	$T - 1200g\sin\alpha - 600 = 1200a$		A2 -1 each error
	$1200a = 3000 - 1200 \times 9.8 \times 0.1 - 600$		
	$a = \underline{1.02\ (\text{ms}^{-2})}$	A1	cao
4(b)	$T = \frac{90\times1000}{v}$	M1	
	N2L up plane	M1	dim correct, all forces
	$T - 1200g\sin\alpha - 600 = 1200a$		
	$a = 0$	m1	si
	$\frac{90000}{v} = 1776$		
	$v = \underline{50.7\ (\text{ms}^{-1})}$	A1	cao
5.	KE at A $= 0.5 \times 0.1 \times v^2$	B1	
	PE at A $= 0.1 \times 9.8 \times 0.5$	M1	
	PE at B $= 0.1 \times 9.8 \times 1.4$	A1	both or difference
	WD against resistance $= 6 \times 1.2$	B1	
	Work-energy principle	M1	all terms included
	$0.05v^2 = 7.2 + 0.1 \times 9.8 \times 0.9$	A1	correct equation
	$v^2 = 161.64$		
	$v = \underline{12.7\ (\text{ms}^{-1})}$	A1	cao

Q	Solution	Mark	Notes
6(a).	$u_H = V\cos\alpha \ (= 0.8V)$ $u_V = V\sin\alpha \ (= 0.6V)$	M1 A1	attempt to resolve both answers correct
6(b)	Consider horizontal motion $0.8V \times T = 12$ $VT = 15$	 M1 A1	 correctly obtained
6(c)	Consider vertical motion $s = ut + 0.5at^2$ with $s=(\pm)5.4$, $u=0.6V$, $t=T$ $a=(\pm)9.8$ $-5.4 = 0.6VT - 4.9T^2$ $-5.4 = 0.6 \times 15 - 4.9T^2$ $4.9T^2 = 14.4$ $T = \frac{12}{7}$ $\frac{12}{7}V = 15$ $V = \underline{8.75}$	 M1 A1 A1 A1	 cao cao
6(d)	Using $v = u + at$ with $u=5.25$, $a=(\pm)9.8$, $t=\frac{12}{7}$ $v = 5.25 - 9.8 \times \frac{12}{7}$ $v = -11.55$ $u_H = 0.8 \times 8.75 = 7$ Speed $= \sqrt{11.55^2 + 7^2}$ Speed $= \underline{13.5\ (\text{ms}^{-1})}$	 M1 A1 B1 M1 A1	 si, cao

Q	Solution	Mark	Notes
7.			
7(a)	Resolve vertically	M1	
	$T\cos\theta = mg$	A1	
	$\theta = \cos^{-1}\left(\frac{3\times9\cdot8}{88\cdot2}\right)$		
	$\theta = \underline{70.5°}$	A1	cao
7(b)	N2L towards centre	M1	attempted
	$T\sin\theta = ma$	A1	
	$a = r\omega^2$	m1	used
	$r = \frac{T\sin\theta}{m\omega^2}$		
	length of string $= l$		
	$l\sin\theta = r$	m1	
	$l = \frac{r}{\sin\theta}$		
	$l = \frac{T}{m\omega^2} = \frac{88\cdot2}{3\times2\cdot8^2}$		
	$l = \underline{3.75\text{ (m)}}$	A1	cao
	<u>Alternative Solution</u>		
	N2l towards centre	M1	attempted
	$T\sin\theta = ma$	A1	
	$a = r\omega^2$	m1	used
	$88.2\sin\theta = 3\times r\times 2.8^2$		
	$r = 3.53553$ m		
	$AP = \frac{r}{\sin\theta}$	m1	
	$AP = \underline{3.75\text{ (m)}}$	A1	cao

Q	Solution	Mark	Notes
8(a)	$\underline{v} = \frac{1}{3}[(14\underline{i} - 5\underline{j}) - (8\underline{i} + 7\underline{j})]$	M1	
	$\underline{v} = \frac{1}{3}(6\underline{i} - 12\underline{j})$		
	$\underline{v} = (2\underline{i} - 4\underline{j})$	A1	
8(b)	$\underline{r}_S = (8\underline{i} + 7\underline{j}) + (2\underline{i} - 4\underline{j})t$	M1 A1	
	$\underline{r}_S = (8 + 2t)\underline{i} + (7 - 4t)\underline{j}$		
8(c)	$\underline{r}_B = (x\underline{i} + y\underline{j})(t - 10)$	M1 A1	
	$\underline{r}_B = x(t - 10)\underline{i} + y(t-10)\underline{j}$		
	At $t = 50$ $\quad \underline{r}_S = \underline{r}_B$	M1	
	$8 + 2t = x(t - 10)$	m1	
	$40x = 108$		
	$x = \underline{2.7}$	A1	cao
	$7 - 4 \times 50 = 40y$		
	$y = \underline{-4.825}$	A1	cao
	<u>Alternative solution</u>		
	At $t = 50$		
	$\underline{r}_S = 108\underline{i} - 193\underline{j}$		
	$\underline{r}_B = 40x\underline{i} + 40y\underline{j}$	M1 A1	
	$\underline{r}_S = \underline{r}_B$	M1	si
	$40x = 108$	m1	
	$x = \underline{2.7}$	A1	cao
	$40y = -193$		
	$y = \underline{-4.825}$	A1	cao

Q	Solution	Mark	Notes
9(a).	Conservation of energy	M1	
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl(1 - \cos\theta)$	A1 A1	
	At max height, v=0, $\cos\theta = \frac{2}{3}$, l=1.2	m1	
	$\frac{1}{2}u^2 = 9.8 \times 1.2(1 - \frac{2}{3})$		
	$u^2 = 2 \times 9.8 \times 1.2 \times \frac{1}{3}$		
	$u = \underline{2.8\ (ms^{-1})}$	A1	cao
	$v^2 = u^2 - 2gl(1 - \cos\theta)$		
	$v^2 = 2.8^2 - 2 \times 9.8 \times 1.2(1 - \cos\theta)$		
	$\underline{v^2 = 23.52\cos\theta - 15.68}$	A1	cao
9(b)	N2L towards centre	M1	
	$T - mg\cos\theta = mv^2/l$	A1	
	$T = 3 \times 9.8\cos\theta + \frac{3}{1 \cdot 2}(23.52\cos\theta - 15.68)$	m1	
	$T = 29.4\cos\theta + 58.8\cos\theta - 39.2$		
	$T = \underline{88.2\cos\theta - 39.2}$	A1	cao
9(c)	Greatest value of T when $\cos\theta = 1$		
	$T = 88.2 - 39.2$		
	$T = \underline{49\ (N)}$	B1	
	Least value of T when $\cos\theta = \frac{2}{3}$		
	$T = 88.2 \times \frac{2}{3} - 39.2$		
	$T = \underline{19.6\ (N)}$	B1	

M3

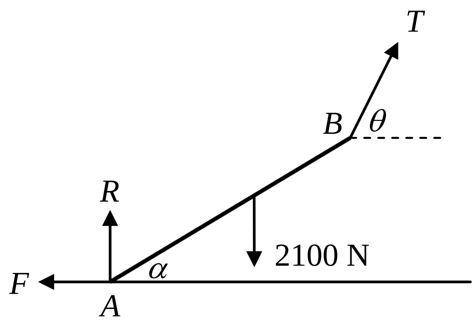
Q	Solution	Mark	Notes
1(a)	N2L $\frac{27000}{(t+3)^2} = 600a$	M1	+/-, no additional terms
	$\frac{45}{(t+3)^2} = \frac{dv}{dt}$	m1	use of dv/dt
	$v = -\frac{45}{(t+3)}$ (+ C)	A1	k/(t+3)
		A1	completely correct
	When t = 0, v = 0	m1	use of initial conditions
	C = 15		
	$v = 15 - \frac{45}{(t+3)}$	A1	
	As $t \to \infty$, $v \to 15$	A1	ft similar expression
1(b)	$v = \frac{dx}{dt} = 15 - \frac{45}{(t+3)}$	M1	
	x = 15t – 45 ln(t + 3) (+ C)	A1	ft similar expressions
	t = 0, x = 0 C = 45 ln3	A1	ft
	$x = 15t + 45\ln\left(\frac{3}{t+3}\right)$		
	When t = 6 $x = 90 + 45\ln\left(\frac{3}{9}\right)$	m1	
	x = 90 – 45 ln(3)		
	x = <u>40.56 (m)</u>	A1	cao

Q	Solution	Mark	Notes
2(a).	Using $v^2 = \omega^2(a^2 - x^2)$	M1	used
	$0.09 \times 3 = \omega^2(a^2 - 0.6^2)$	A1	
	$0.04 \times 5 = \omega^2(a^2 - 0.8^2)$	A1	
	$0.07 = 0.28\omega^2$	m1	
	$\omega = 0.5$		
	$0.2 = 0.25(a^2 - 0.64)$		
	$a = 1.2$	A1	
	Period $= \dfrac{2\pi}{\omega}$	M1	used
	Period $= \underline{4\pi}$	A1	
2(b)	$\ddot{x} = -\omega^2 x$	M1	used
	$\lvert\ddot{x}\rvert = 0.5^2 \times 0.6$		
	$\lvert\ddot{x}\rvert = \underline{0.15\ (\text{ms}^{-2})}$	A1	
2(c)	$x = 1.2\sin(0.5t)$	M1	used, accept cos
	At A, $0.6 = 1.2\sin(0.5t)$		
	$t = 2\sin^{-1}(0.5) = 1.0472$	A1	or 2.0944
	At B, $0.8 = 1.2\sin(0.5t)$		
	$t = 2\sin^{-1}(0.667) = 1.4595$	A1	or 1.6821
	Required $t = 1.4595 - 1.0472$		
	Required $t = \underline{0.412\ (\text{s})}$	A1	cao
2(d)	$x = a\sin(\omega t)$		
	$x = 1.2\sin(0.5t)$	M1	
	$x = 1.2\sin(0.5 \times 2\pi/3)$		
	$x = \underline{1.0392\ (\text{m})}$	A1	
2(e)	$v = a\omega\cos(\omega t)$	M1	oe
	$v = 1.2 \times 0.5\cos(0.5t)$		
	$v = 0.6\cos(0.5t)$	A1	
	When $t = 2\pi/3$, $v = 0.6\cos(0.5\times 2\pi/3)$		
	$v = 0.6\cos(\pi/3)$		
	$v = \underline{0.3\ (\text{ms}^{-1})}$	A1	cao

Q	Solution	Mark	Notes
3.	Auxiliary equation		
	$2m^2 + 5m + 2 = 0$	B1	cao
	$(2m + 1)(m + 2) = 0$		
	$m = -0.5, -2$	B1	cao
	CF is $x = Ae^{-0.5t} + Be^{-2t}$	B1	ft solutions for m
	For PI, try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$5a + 2(at + b) = 6t + 5$	A1	
	Comparing coefficients	m1	
	$2a = 6$		
	$a = 3$		
	$15 + 2b = 5$		
	$b = -5$	A1	both answers cao
	General solution is		
	$x = Ae^{-0.5t} + Be^{-2t} + 3t - 5$	B1	ft CF and PI
	When $t = 0$, $x = 3$	M1	use of conditions in GS
	$3 = A + B - 5$		
	$A + B = 8$		
	$\frac{dx}{dt} = -0.5Ae^{-0.5t} - 2Be^{-2t} + 3$	B1	ft similar expressions
	When $t = 0$, $\frac{dx}{dt} = 2$		
	$2 = -0.5A - 2B + 3$		
	$0.5A + 2B = 1$		
	$A + 4B = 2$		
	$A + B = 8$		
	$3B = -6$		
	$B = \underline{-2}$	A1	cao
	$A = \underline{10}$	A1	cao

Q	Solution	Mark	Notes
4(a)	N2L F = ma	M1	used, no extra term
	$\frac{4}{2x+1} = 0\cdot5v\frac{dv}{dx}$	m1	use of vdv/dx
	$\int\frac{8}{2x+1}dx = \int v\,dv$	M1	separating variables
	$4\ln\lvert 2x+1\rvert = \frac{1}{2}v^2 + C$	A1	kln(2x+1)
		A1	all correct
	$v^2 = 8\ln\lvert 2x+1\rvert + C$		
	When $x = 3$, $v = 4$	m1	
	$16 = 8\ln7 + C$		
	$C = 16 - 8\ln7$	A1	ft kln(2x+1) + C
	$v^2 = 8\ln\left\lvert\frac{2x+1}{7}\right\rvert + 16$		
	When $x = 10$ $v^2 = 8\ln\left\lvert\frac{2\times10+1}{7}\right\rvert + 16$		
	$v^2 = 8\ln3 + 16$		
	$v = \underline{4.98\ (\text{ms}^{-1})}$	A1	cao
4(b)	$v = 6,\ 6^2 = 8\ln\left\lvert\frac{2x+1}{7}\right\rvert + 16$	M1	allow similar expressions
	$\ln\left\lvert\frac{2x+1}{7}\right\rvert = \frac{20}{8}$		
	$2x + 1 = 7e^{5/2}$	m1	correct inversion
	$x = 0.5[7e^{5/2} - 1]$		
	$x = \underline{42.1\ (\text{m})}$	A1	cao

Q	Solution	Mark	Notes
5.	Using $v = u + at$ with $u=0$, $a=(\pm)9.8$, $t=2.5$	M1	
	$v = 9.8 \times 0.5$		
	$v = 4.9 \text{ ms}^{-1}$	A1	
	Impulse = Change in momentum	M1	used
	For A $J = 5v$	B1	
	For B $J = 2 \times 4.9 - 2v$	A1	
	Solving $5v = 9.8 - 2v$	m1	
	$7v = 9.8$		
	$v = \underline{1.4 \text{ (ms}^{-1})}$	A1	cao
	$J = 5 \times 1.4$		
	$J = \underline{7 \text{ (Ns)}}$	A1	cao

Q	Solution	Mark	Notes
6.(a)			
	$F = \mu R = \frac{3}{4}R$	M1	
	Moments about B	M1	dim correct equation, 3 terms, perp distance
	$R \times 2\cos\alpha + F \times 2\sin\alpha = 2100 \times 1\cos\alpha$	A3	-1 each error
	$R \times 2 \times \frac{12}{13} + \frac{3}{4}R \times 2 \times \frac{5}{13} = 2100 \times \frac{12}{13}$		
	$24R + \frac{15}{2}R = 25200$		
	$R = \underline{800\ (N)}$	A1	cao
6(b)	Resolve vertically	M1	
	$T\sin\theta = 2100 - R$	A1	
	$T\sin\theta = 1300$		
	Resolve horizontally	M1	
	$T\cos\theta = F$	A1	
	$T\cos\theta = \frac{3}{4} \times 800$		
	$T\cos\theta = 600$		
	$T = \sqrt{1300^2 + 600^2}$	m1	oe
	$T = \underline{1432\ (N)}$	A1	cao
	$\theta = \tan^{-1}\left(\frac{1300}{600}\right)$	m1	oe
	$\theta = \underline{65.2°}$	A1	cao

S1

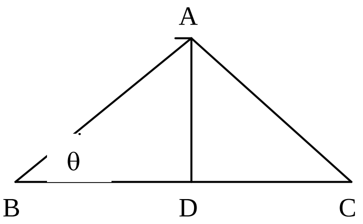
Ques	Solution	Mark	Notes
1(a)(i)	$P(A \cup B) = P(A) + P(B)$	**M1**	Award M1 for using formula
	$= 0.8$	**A1**	
(ii)	$P(A \cap B) = P(A)P(B) = 0.5 \times 0.3$	**B1**	
	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	**M1**	Award M1 for using formula
	$= 0.5 + 0.3 - 0.5 \times 0.3 = 0.65$	**A1**	
(b)	$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.1$	**B1**	
	$P(B\|A) = \dfrac{P(A \cap B)}{P(A)}$	**M1**	Award M1 for using formula
	$= 0.2$	**A1**	
2(a)	$E(X^2) = \text{Var}(X) + [E(X)]^2$	**M1**	Award M1 for using formula
	$= 66$	**A1**	
(b)	$E(Y) = 3E(X) + 4$	**M1**	Award M1 for using formula
	$= 28$	**A1**	
	$\text{Var}(Y) = 3^2 \text{Var}(X)$	**M1**	Award M1 for using formula
	$= 18$	**A1**	
3(a)	P(no white) $= \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7}$ or $\binom{4}{3} \div \binom{9}{3}$	**M1**	
	$= \frac{1}{21}$	**A1**	
(b)	P(2 white) $= \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \times 3$ or $\binom{5}{2} \times \binom{4}{1} \div \binom{9}{3}$	**M1**	M0 if 3 omitted.
	$= \frac{10}{21}$	**A1**	
(c)	EITHER		
	P(2 blue) $= \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} \times 3$ or $\binom{3}{2} \times \binom{6}{1} \div \binom{9}{3}$	**M1A1**	M0 if 3 omitted
	$= \left(\frac{3}{14}\right)$		
	P(2 the same) $= \frac{10}{21} + \frac{3}{14}$		
	$= \frac{29}{42}$ cao	**A1**	
	OR		
	P(2 the same) $= \frac{5}{9} \times \frac{4}{8} \times \frac{1}{7} \times 3 + \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times 3 + \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times 3 + \frac{3}{9} \times \frac{2}{8} \times \frac{5}{7} \times 3$	**M1A1**	M0 if 3 omitted Accept $\binom{5}{2} \times \binom{1}{1} \div \binom{9}{3} + \binom{5}{2} \times \binom{3}{1} \div \binom{9}{3} + \binom{3}{2} \times \binom{1}{1} \div \binom{9}{3} + \binom{3}{2} \times \binom{5}{1} \div \binom{9}{3}$
	$= \frac{29}{42}$ cao	**A1**	

4(a)(i) **(ii)** **(b)**	$P(X = 4) = \binom{10}{4} \times 0.75^4 \times 0.25^6$ $= 0.0162$ Let Y denote the number of games won by Dave so that Y is B(10,0.25). si We require $P(Y \le 4)$ $= 0.9219$ The number of games lasting less than 1 hr, G, is B(45,0.08) ≈ Poi(3.6). si $P(G > 6) = 0.0733$	**M1** **A1** **M1** **m1** **A1** **B1** **M1A1**	Accept 0.9965 – 0.9803 or 0.0197 – 0.0035 Award M1A0 for use of adjacent row or column. FT their mean
5(a) **(b)**	$P(CB) = \frac{6}{10} \times \frac{8}{100} + \frac{4}{10} \times \frac{3}{100}$ $= 0.06$ $P(F\|CB) = \frac{12/1000}{0.06}$ $= 0.2$ cao	**M1A1** **A1** **B1B1** **B1**	M1 Use of Law of Total Prob (Accept tree diagram) FT denominator from (a) B1 num, B1 denom
6(a) **(b)** **(c)** **(d)**	$\frac{1}{6}$ $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ $\frac{1}{6}, \frac{25}{216}$ and $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \left(\frac{625}{7776}\right)$ Prob = $\frac{1/6}{1 - 25/36}$ $= \frac{6}{11}$	**B1** **M1A1** **M1A1** **M1** **A1**	 Award M1A1 if only 3^{rd} term given. FT their answer to (a)
7(a)(i) **(ii)** **(b)**	$P(X = 10) = \frac{e^{-12} \times 12^{10}}{10!}$ $= 0.105$ $P(X > 10) = 1 - 0.3472 = 0.6528$ Using tables, we see that $P(X \le 18) = 0.9626$ He needs to take 18 jars.	**M1** **A1** **M1A1** **M1** **A1**	Working must be shown. Accept 0.3472 – 0.2424 or 0.7576 – 0.6528 Award M1 for adjacent row/col Award M1A0 for 17 or 19

8(a) **(b)** **(c)(i)** **(ii)**	$0 \le \theta \le 0.3$ $E(X) = 2(0.3-\theta) + 3\times 2\theta + 4(0.7-\theta)$ $= 3.4$ E(X) is therefore independent of θ $\mathrm{E}(X^2) = 4(0.3-\theta) + 9\times 2\theta + 16(0.7-\theta)$. $= 12.4 - 2\theta$ Var(X) = $12.4 - 2\theta - 3.4^2$ $= 0.84 - 2\theta$ $0.84 - 2\theta = 0.8^2$ $\theta = 0.1$ cao Possibilities are 3,3; 4,2 si P(Sum = 6) = 0.2×0.6×2 + 0.2×0.2 = 0.28	**B1B1** **M1** **A1** **M1** **A1** **M1** **A1** **A1** **B1** **M1** **A1**	Accept use of <. Use of $\Sigma x p_x$ with θ. Need not be seen Must include θ FT their $E(X)$ if possible Award M1A0 if 2 is missing in 1^{st} term or present in 2^{nd} term FT their value of θ if sensible
9(a)(i) **(ii)** **(b)(i)** **(ii)** **(iii)**	$\mathrm{E}(X) = \frac{1}{10}\int_1^2 x(2x+3x^2)\mathrm{d}x$ $= \frac{1}{10}\left[\frac{2x^3}{3} + \frac{3x^4}{4}\right]_1^2$ $= 1.59$ $E(X^2) = \frac{1}{10}\int_1^2 x^2(2x+3x^2)\mathrm{d}x$ $= \frac{1}{10}\left[\frac{2x^4}{4} + \frac{3x^5}{5}\right]_1^2$ $= 2.61$ Var(X) = 2.61 – 1.59^2 = 0.08 $F(x) = \int_1^x \frac{1}{10}(2t+3t^2)\mathrm{d}t$ $= \frac{1}{10}\left[t^2 + t^3\right]_1^x$ $= \frac{1}{10}(x^2 + x^3 - 2)$ cao $\mathrm{P}(X \le 1.4) = F(1.4)$ $= 0.27$ The lower quartile is less than 1.4 since F(1.4) is more than 0.25.	**M1A1** **A1** **A1** **B1** **B1** **M1A1** **M1** **A1** **A1** **M1** **A1** **B1** **B1**	M1 for the integral of $xf(x)$, A1 for completely correct although limits may be left until 2^{nd} line. For evaluating the integral Integral and limits Correct evaluation of integral FT their $E(X)$ Limits may be left until 2^{nd} line FT their $F(x)$ if possible FT their answer to (a)(ii)

Ques	Solution	Mark	Notes
1(a)	$E(X^2) = \mathrm{Var}(X) + [E(X)]^2$ $= 27$ Similarly, $E(Y^2) = 39$	**M1** **A1** **A1**	Award M1 for **using** formula
(b)	E(U) = E(X)E(Y) = 30 $E(X^2Y^2) = E(X^2)E(Y^2) = 27 \times 39$ Var(U) = $E(X^2Y^2) - [E(XY)]^2$ = 27 × 39 – 30^2 = 153	**M1** **A1** **B1** **M1** **A1**	FT their $E(X^2), E(Y^2)$ but not their $E(X), E(Y)$ Award M1 for **using** formula
2(a)(i)	$z = \frac{4.5 - 4.4}{0.2} = 0.5$ P(X > 4.5) = 0.3085	**M1A1** **A1**	
(ii)	95th percentile = $\mu + 1.645\sigma$ = 4.73	**M1** **A1**	Award only for $\mu + z\sigma$
(b)(i)	E(2Y – X) = 0.8 Var(2Y – X) = 4Var(Y) + Var(X) = 0.13	**B1** **M1** **A1**	
(ii)	$z = \frac{0 - 0.8}{\sqrt{0.13}} = -2.22$ (Accept ±) We require P(2Y – X < 0) Prob = 0.0132	**M1A1** **M1** **A1**	FT their values from (b)(i)
(iii)	Let total weight = S E(S) = 2 × 4.4 + 3 × 2.6 = 16.6 Var(S) = 2 × 0.04 + 3 × 0.0225 = 0.1475 $z = \frac{16 - 16.6}{\sqrt{0.1475}} = -1.56$ Prob = 0.9406	**B1** **M1A1** **m1A1** **A1**	
3(a)	$\bar{x} = \frac{69.9}{75}$ (= 0.932) SE of $\bar{X} = \frac{0.1}{\sqrt{75}}$ (= 0.011547…) 90% conf limits are 0.932 ± 1.645 × 0.011547… giving [0.913, 0.951]	**B1** **B1** **M1A1** **A1**	M1 correct form, A1 correct z. SE must have $\sqrt{75}$ in denom for M1.
(b)	If the method for finding the confidence interval is repeated a large number of times, then 90% of the intervals obtained will contain μ (or equivalent)	**B1**	Award B0 for any solution which suggests that the calculated interval contains μ with a probability of 0.9

4(a)	The total number of errors, X, is Poi(8) $P(X < 5) = 1 - 0.9004 = 0.0996$	**B1** **M1A1**	Award M1A0 for use of adjacent row/column
(b)(i) **(ii)**	$H_0 : \mu = 0.8; H_1 : \mu < 0.8$ Under H_0, number of errors is Poi(64) ≈ N(64,64). $z = \frac{60.5 - 64}{8}$ $= -0.4375$ p-value = 0.33 Insufficient evidence to reject H_0 /Accept H_0	**B1** **B1** **M1A1** **A1** **A1** **A1**	Award M1A0A1A1 for incorrect or no continuity correction No c/c gives $z = -0.5$, $p = 0.31$ Incorrect c/c gives $z = -0.5625$, $p = 0.29$ FT their p-value
5(a) **(b)**	$H_0 : \mu_D = \mu_F; H_1 : \mu_D \neq \mu_F$ $\bar{x}_D = \frac{890.4}{6} (= 148.4); \bar{x}_F = \frac{879}{6} (= 146.5)$ si SE of difference of means= $\sqrt{\frac{1.5^2}{6} + \frac{1.5^2}{6}}$ (0.866..) Test statistic = $\frac{148.4 - 146.5}{0.866..}$ = 2.19 Prob from tables = 0.01426 p-value = 0.02852 **Strong** evidence that there is a difference in mean distances for the two players. OR **Strong** evidence that David's mean is larger than Frank's mean.	**B1** **B1B1** **M1A1** **M1A1** **A1** **A1** **A1** **A1**	 FT arithmetic slip in evaluating means FT from previous line FT on their p-value

6(a)	 Drop a perpendicular from A to BC. X = 2BD = 2ABcosθ = 4cosθ	**M1** **A1**	Accept any valid method Must be convincing
(b)(i)	The probability density function of θ is $f(\theta) = \dfrac{2}{\pi}$ (for $0 < \theta < \pi/2$) si E(X) = $\displaystyle\int_0^{\pi/2} \frac{2}{\pi} \times 4\cos\theta \mathrm{d}\theta$ = $\dfrac{8}{\pi}\left[\sin\theta\right]_0^{\pi/2}$ = 8/π cao	**B1** **M1** **A1** **A1**	Limits not required, award M1 for $\int K \times 4\cos\theta \mathrm{d}\theta, K \neq 1$ Limits required here
(ii)	P($X \leq 3$) = P(cos$\theta \leq 0.75$) = P($\theta \geq 0.723$) = $\dfrac{\pi/2 - 0.723}{\pi/2}$ = 0.54	**M1** **A1** **M1** **A1**	An answer of 0.46 is given M1A0M1A0
7(a)	Let X denote the number of white flowers produced. If bag is Type B, X is B(120,0.7) ≈ N(84,25.2) P(label A) = P(X < 70) $z = \dfrac{69.5 - 84}{\sqrt{25.2}}$ = – 2.89 (Accept ±) Prob = 0.00193	**M1A1** **M1A1** **A1** **A1**	Award M1A0A1A1 for incorrect or no c/c. 70.5 → z = – 2.69, p = 0.00357 70 → z = – 2.79, p = 0.00264 69→ z = – 2.99, p = 0.00139 68.5 → z = – 3.09, p = 0.001
(b)	If bag is of Type A, X is B(120,0.5) ≈ N(60,30) P(label B) = P(X ≥ 70) $z = \dfrac{69.5 - 60}{\sqrt{30}}$ = 1.73 (Accept ±) Prob = 0.0418	**M1A1** **M1A1** **A1** **A1**	Award M1A0A1A1 for incorrect or no c/c. 70.5 → z = 1.92, p = 0.02743 70 → z = 1.83, p = 0.03362 69 → z = 1.64, p = 0.0505 68.5 → z = 1.55, p = 0.06057

S3

<table>
<tr><th>Ques</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>1(a)</td><td>The possibilities are
<table>
<tr><th>Numbers drawn</th><th>Mean</th><th>Median</th></tr>
<tr><td>1 2 3</td><td>2</td><td>2</td></tr>
<tr><td>1 2 4</td><td>7/3</td><td>2</td></tr>
<tr><td>1 2 5</td><td>8/3</td><td>2</td></tr>
<tr><td>1 3 4</td><td>8/3</td><td>3</td></tr>
<tr><td>1 3 5</td><td>3</td><td>3</td></tr>
<tr><td>1 4 5</td><td>10/3</td><td>4</td></tr>
<tr><td>2 3 4</td><td>3</td><td>3</td></tr>
<tr><td>2 3 5</td><td>10/3</td><td>3</td></tr>
<tr><td>2 4 5</td><td>11/3</td><td>4</td></tr>
<tr><td>3 4 5</td><td>4</td><td>4</td></tr>
</table></td><td>B1

B1

B1</td><td>B1 each column

Special case – B2 if one combination is missing.</td></tr>
<tr><td>(b)</td><td>The sampling distribution of the mean is
<table>
<tr><td>$\bar{x}$</td><td>2</td><td>7/3</td><td>8/3</td><td>3</td><td>10/3</td><td>11/3</td><td>4</td></tr>
<tr><td>Prob</td><td>1/10</td><td>1/10</td><td>2/10</td><td>2/10</td><td>2/10</td><td>1/10</td><td>1/10</td></tr>
</table>
The sampling distribution of the median is
<table>
<tr><td>Median</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Prob</td><td>3/10</td><td>4/10</td><td>3/10</td></tr>
</table></td><td>M1
A1

M1
A1</td><td>No FT from earlier work.</td></tr>
<tr><td>2(a)</td><td>UE of μ = 99.03
$\Sigma x^2 = 98088.11$
$\text{UE of } \sigma^2 = \frac{98088.11}{9} - \frac{990.3^2}{9\times 10}$
= 2.08 (2.0778…)</td><td>B1
B1

M1
A1</td><td>No working need be seen

Answer only no marks</td></tr>
<tr><td>(b)(i)</td><td>$H_0 : \mu = 100; H_1 : \mu \neq 100$</td><td>B1</td><td></td></tr>
<tr><td>(ii)</td><td>$t = \frac{99.03 - 100}{\sqrt{2.0778../10}}$
= – 2.13
DF = 9 si
Critical value = 2.262

Insufficient evidence to reject the manager’s claim or Accept the manager’s claim
Because 2.13 < 2.262 or equivalent using the term ‘acceptance region’ or by means of a diagram</td><td>M1

A1
B1
B1

B1

B1</td><td>M0 if treated as z

FT their critical value but not their p-value obtained from using the normal distribution</td></tr>
</table>

3(a)(i)	$\hat{p} = 0.45$	**B1**	
(ii)	ESE = $\sqrt{\dfrac{0.45 \times 0.55}{120}}$ = 0.0454.. si	**M1A1**	
	95% confidence limits are $0.45 \pm 1.96 \times 0.0454..$ giving [0.361,0.539]	**M1A1** **A1**	
(b)(i)	This time, $\hat{p} = \dfrac{0.455 + 0.581}{2}$ = 0.518	**M1A1**	
(ii)	Width of CI = $2 \times 1.645\sqrt{\dfrac{0.518 \times 0.482}{n}}$ = 0.581 – 0.455 = 0.126	**M1** **A1**	
	Solving, $n = \left(\dfrac{3.29}{0.126}\right)^2 \times 0.518 \times 0.482$	**M1**	Attempting to solve for *n*
	= 170	**A1**	
(iii)	x = 170 × 0.518 = 88	**B1**	FT their *n*
4(a)	$H_0 : \mu_A = \mu_B : H_1 : \mu_A \neq \mu_B$	**B1**	
(b)	$\bar{x} = 51.3; \bar{y} = 51.8$	**B1**	
	$s_x^2 = \dfrac{131659}{49} - \dfrac{2565^2}{49 \times 50} = 1.5204...$	**M1A1**	
	$s_y^2 = \dfrac{134232}{49} - \dfrac{2590^2}{49 \times 50} = 1.4285...$ [Accept division by 50 giving 1.49… and 1.4]	**A1**	
	SE = $\sqrt{\dfrac{1.5204..}{50} + \dfrac{1.4285}{50}}$	**M1**	
	= 0.2428.. (0.2404..)	**A1**	
	Test stat = $\dfrac{51.3 - 51.8}{0.2428..}$	**M1**	
	= 2.06 (2.08)	**A1**	
	p-value = 0.039 (0.038)	**A1**	
	Strong evidence for believing there is a difference in mean distances travelled (or that the Model A mean is less than the Model B mean).	**A1**	FT their p-value

5(a)	$\sum x = 15, \sum x^2 = 55, \sum y = 345.5, \sum xy = 1131.1$	**B2**	Minus 1 each error.
	$S_{xy} = 1131.1 - 15 \times 345.5/6 = 267.35$	**B1**	FT I error in sums.
	$S_{xx} = 55 - 15^2/6 = 17.5$	**B1**	
	$b = \dfrac{267.35}{17.5} = 15.3$	**M1** **A1**	
	$a = \dfrac{345.5 - 15 \times 15.277..}{6}$ = 19.4 (accept 19.3)	**M1** **A1**	
(b)	SE of $b = \dfrac{0.75}{\sqrt{17.5}}$ (0.179…)	**M1A1**	FT their values from (a)
	99% confidence limits for β are $15.277 \pm 2.576 \times 0.179..$	**M1A1**	
	giving [14.8,15.7]	**A1**	
6(a)	$\text{E}(X) = \int_0^a x \times \dfrac{2x}{a^2}\,\text{d}x$	**M1**	Limits not required in this line
	$= \left[\dfrac{2x^3}{3a^2}\right]_0^a$	**A1**	
	$= \dfrac{2a}{3}$	**A1**	
	$\text{E}(X^2) = \int_{-1}^{1} x^2 \times \dfrac{2x}{a^2}\,\text{d}x$	**M1**	Limits not required in this line
	$= \left[\dfrac{2x^4}{4a^2}\right]_0^a$	**A1**	
	$= \dfrac{a^2}{2}$	**A1**	
	$\text{Var}(X) = \dfrac{a^2}{2} - \dfrac{4a^2}{9}$	**A1**	
	$= \dfrac{a^2}{18}$		

(b)(i)	$\mathrm{E}(U) = c\mathrm{E}(\overline{X})$ (or $c\mathrm{E}(X)$) $= c \times \dfrac{2a}{3}$	**M1**	Penalise the omission of E once in the question
	$E(U) = a \Rightarrow c = \dfrac{3}{2}$	**A1**	
	$\mathrm{Var}(U) = \dfrac{9}{4}\mathrm{Var}(\overline{X})$	**M1**	
	$= \dfrac{9}{4} \times \dfrac{a^2}{18n}$	**A1**	
	$= \dfrac{a^2}{8n}$	**A1**	
(ii)	$\mathrm{E}(V) = d\mathrm{E}(Y) = d \times \dfrac{2na}{2n+1}$	**M1**	
	$E(V) = a \Rightarrow d = \dfrac{2n+1}{2n}$	**A1**	
	$\mathrm{Var}(V) = \left(\dfrac{2n+1}{2n}\right)^2 \mathrm{Var}(Y)$	**M1**	
	$= \left(\dfrac{2n+1}{2n}\right)^2 \times \left(\dfrac{na^2}{(n+1)(2n+1)^2}\right)$	**A1**	
	$= \dfrac{a^2}{4n(n+1)}$	**A1**	
(iii)	$\dfrac{\mathrm{Var}(U)}{\mathrm{Var}(V)} = \dfrac{a^2}{8n} \div \dfrac{a^2}{4n(n+1)}$	**B1**	
	$= \dfrac{n+1}{2}$		
	V is the better estimator	**B1**	
	Because (for $n > 1$) it has the smaller variance	**B1**	

WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk