GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2012

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

C1

1. (*a*) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^4/_3$ (or equivalent) A1

(*b*) A correct method for finding C M1

$C(-1, 3)$ A1

(*c*) Use of $m_{AB} \times m_L = -1$ to find gradient of L M1

A correct method for finding the equation of L using candidate's coordinates for C and candidate's gradient for L. M1

Equation of L: $y - 3 = {}^3/_4\,[x - (-1)]$ (or equivalent)

(f.t. candidate's coordinates for C and candidate's gradient for AB) A1

Equation of L: $3x - 4y + 15 = 0$ (convincing, c.a.o.) A1

(*d*) (i) Substituting $x = 7$, $y = k$ in equation of L M1

$k = 9$ A1

(ii) A correct method for finding the length of $CA(DA)$ M1

$CA = 5$ (f.t. candidate's coordinates for C) A1

$DA = \sqrt{125}$ A1

(iii) $\text{Sin } ADC = \dfrac{CA}{DA} = \dfrac{5}{\sqrt{125}}$

(f.t. candidate's derived values for CA and DA) M1

$\text{Sin } ADC = \dfrac{CA}{DA} = \dfrac{1}{\sqrt{5}}$ (c.a.o.) A1

2. (*a*) $\dfrac{10}{7+2\sqrt{11}} = \dfrac{10(7-2\sqrt{11})}{(7+2\sqrt{11})(7-2\sqrt{11})}$ M1

Denominator: $49 - 44$ A1

$\dfrac{10}{7+2\sqrt{11}} = \dfrac{10(7-2\sqrt{11})}{5} = 2(7-2\sqrt{11}) = 14 - 4\sqrt{11}$ (c.a.o.) A1

Special case
If M1 not gained, allow B1 for correctly simplified denominator following multiplication of top and bottom by $7 + 2\sqrt{11}$

(*b*) $(4\sqrt{3})^2 = 48$ B1

$\sqrt{8} \times \sqrt{50} = 20$ B1

$\dfrac{5\sqrt{63}}{\sqrt{7}} = 15$ B1

$(4\sqrt{3})^2 - (\sqrt{8} \times \sqrt{50}) - \dfrac{5\sqrt{63}}{\sqrt{7}} = 13$ (c.a.o.) B1

3. (*a*) $\dfrac{dy}{dx} = 4x - 11$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Use of candidate's numerical value for $\dfrac{dy}{dx}$ as gradient of tangent at P m1

Equation of tangent at P: $y - (-1) = -3(x - 2)$ (or equivalent) (c.a.o.) A1

(*b*) Gradient of tangent at $Q = 9$ B1

An attempt to equate candidate's expression for $\dfrac{dy}{dx}$ and candidate's derived value for gradient of tangent at Q M1

$4x - 11 = 9 \Rightarrow x = 5$
(f.t. one error in candidate's expression for $\dfrac{dy}{dx}$) A1

4. $(1 - 2x)^6 = 1 - 12x + 60x^2 - 160x^3 + \ldots$ B1 B1 B1 B1
(– 1 for further incorrect simplification)

5. (*a*) $a = 3$ B1

$b = -2$ B1

$c = 17$ B1

(*b*) Stationary value = 17 (f.t. candidate's value for c) B1

This is a minimum B1

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b, c correct — M1

$b^2 - 4ac = (2k - 1)^2 - 4(k^2 - k + 2)$ — A1

$b^2 - 4ac = -7$ (c.a.o.) — A1

candidate's value for $b^2 - 4ac < 0$ ($\Rightarrow$ no real roots) — A1

(*b*) Finding critical values $x = -6$, $x = {}^2/_3$ — B1

A statement (mathematical or otherwise) to the effect that

$x < -6$ or ${}^2/_3 < x$ (or equivalent)

(f.t critical values ± 6, $\pm {}^2/_3$ only) — B2

Deduct 1 mark for each of the following errors

the use of $\le$ rather than $<$

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) + 5$ — B1

Subtracting y from above to find δy — M1

$\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$ — A1

Dividing by δx and letting $\delta x \to 0$ — M1

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 7$$ (c.a.o.) A1

(*b*) Required derivative $= {}^2/_3 \times {}^1/_4 \times x^{-3/4} + 12 \times (-3) \times x^{-4}$ — B1, B1

8. (*a*) Attempting to find $f(r) = 0$ for some value of r — M1

$f(2) = 0 \Rightarrow x - 2$ is a factor — A1

$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct — M1

$f(x) = (x - 2)(6x^2 - 7x - 3)$ — A1

$f(x) = (x - 2)(3x + 1)(2x - 3)$ (f.t. only $6x^2 + 7x - 3$ in above line) — A1

$x = 2, -{}^1/_3, {}^3/_2$ (f.t. for factors $3x \pm 1$, $2x \pm 3$) — A1

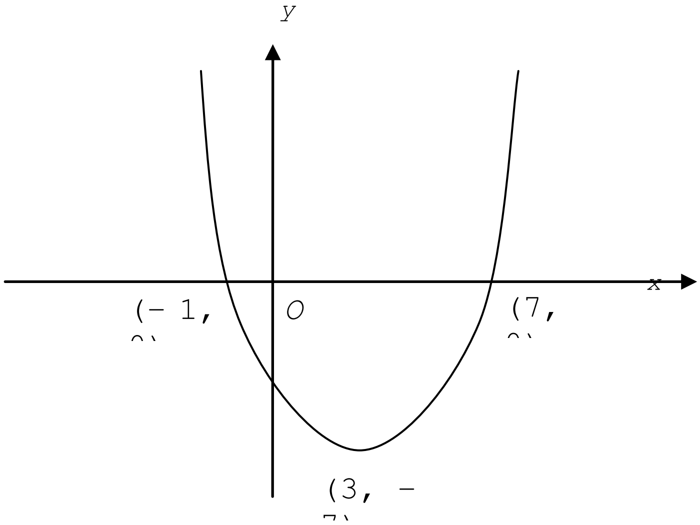
Special case

Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks

(*b*) Use of $g(a) = 11$ — M1

$a^3 - 53 = 11 \Rightarrow a = 4$ — A1

9. (*a*)

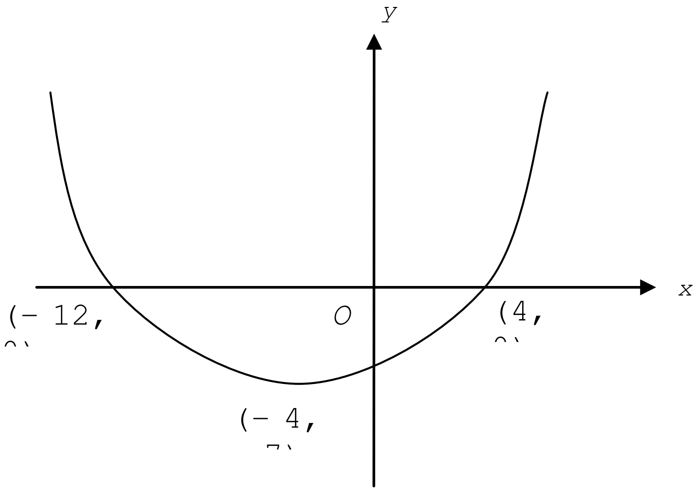


Concave up curve and y-coordinate of minimum $= -7$ B1

x-coordinate of minimum $= 3$ B1

Both points of intersection with x-axis B1

(*b*)



Concave up curve and y-coordinate of minimum $= -7$ B1

x-coordinate of minimum $= -4$ B1

Both points of intersection with x-axis B1

10. (*a*) $\frac{dy}{dx} = 3x^2 + 6x$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

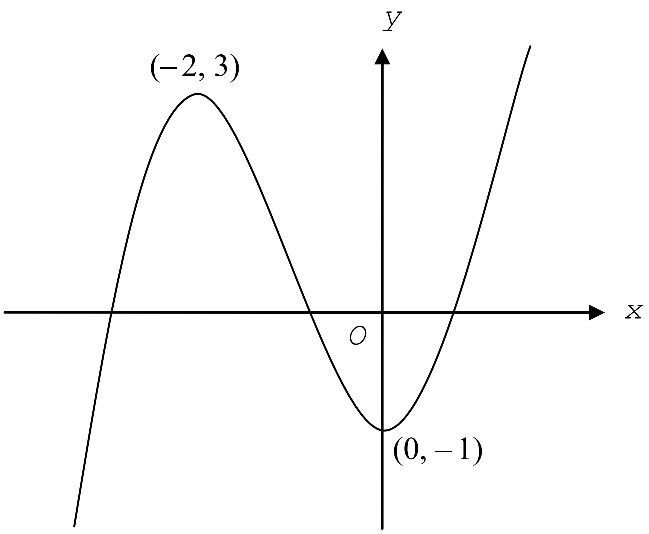
$x = 0, -2$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(0, -1)$ and $(-2, 3)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding
either $(0, -1)$ is a minimum point
or $(-2, 3)$ is a maximum point (f.t. candidate's derived values) M1

Correct conclusion for other point
(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points
(f.t. candidate's derived maximum and minimum points) A1

(*c*) One positive root (f.t. the number of times the candidate's curve crosses the positive x-axis) B1

C2

1.

1	0·5		
1·25	0·53935989		
1·5	0·603022689		
1·75	0·718421208	(5 values correct)	B2
2	1	(3 or 4 values correct)	B1

Correct formula with $h = 0·25$ M1

$I \approx \frac{0·25}{2} \times \{0·5 + 1 + 2(0·53935989 + 0·603022689 + 0·718421208)\}$

$I \approx 5·221607574 \div 8$

$I \approx 0·652700946$

$I \approx 0·6527$ (f.t. one slip) A1

Special case for candidates who put $h = 0·2$

1	0·5		
1·2	0·52999894		
1·4	0·573539334		
1·6	0·640184399		
1·8	0·753778361		
2	1	(all values correct)	B1

Correct formula with $h = 0·2$ M1

$I \approx \frac{0·2}{2} \times\{0·5 + 1 + 2(0·52999894 + 0·573539334 + 0·640184399 + 0·753778361)\}$

$I \approx 6·495002069 \div 10$

$I \approx 0·6495002069$

$I \approx 0·6495$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) $10\cos^2\theta + 3\cos\theta = 4(1 - \cos^2\theta) - 2$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$14\cos^2\theta + 3\cos\theta - 2 = 0 \Rightarrow (2\cos\theta + 1)(7\cos\theta - 2) = 0$

$\Rightarrow \cos\theta = \frac{2}{7}$, $\cos\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 73{\cdot}40°, 286{\cdot}60°$ B1

$\theta = 120°, 240°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks
$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $3x - 21° = -54°, 234°, 306°, 594$ (one value) B1

$x = 85°, 109°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) Use of $\frac{\sin\phi}{\cos\phi} = \tan\phi$ M1

$\tan\phi = 0{\cdot}2$ A1

$\phi = 11{\cdot}31°, 191{\cdot}31°$ (f.t $\tan\phi = a$) B1

3. (*a*) $11^2 = 5^2 + x^2 - 2 \times 5 \times x \times {}^2/_5$ (correct use of cos rule) M1

An attempt to collect terms, form and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + b)(x + d)$, with $b \times d$ = candidate's constant m1

$x^2 - 4x - 96 = 0 \Rightarrow x = 12$ (c.a.o.) A1

(*b*) $\frac{\sin XZY}{32} = \frac{\sin 19°}{15}$

(substituting the correct values in the correct places in the sin rule) M1

$XZY = 44°, 136°$ (at least one value) A1

Use of angle sum of a triangle = 180° M1

$YXZ = 117°, 25°$ (both values)

(f.t. candidate's values for XZY provided both M's awarded) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ n times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $a + 2d + a + 3d + a + 9d = 79$ B1

$a + 5d + a + 6d = 61$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = 3$, $d = 5$ (both values) (c.a.o.) A1

(*c*) $a = 15$, $d = -2$ B1

$S_n = \frac{n}{2}[2 \times 15 + (n - 1)(-2)]$ (f.t. candidate's d) M1

$S_n = n(16 - n)$ (c.a.o.) A1

5. (*a*) $a + ar = 72$ B1

$a + ar^2 = 120$ B1

An attempt to solve candidate's equations simultaneously by correctly eliminating a M1

$3r^2 - 5r - 2 = 0$ (convincing) A1

(*b*) An attempt to solve quadratic equation in r, either by using the quadratic formula or by getting the expression into the form $(ar + b)(cr + d)$, with $a \times c = 3$ and $b \times d = -2$ M1

$(3r + 1)(r - 2) = 0 \Rightarrow r = -{}^1/_3$ A1

$a \times (1 - {}^1/_3) = 72 \Rightarrow a = 108$ (f.t. candidate's derived value for r) B1

$S_\infty = \frac{108}{1 - (-{}^1/_3)}$ (correct use of formula for S_∞, f.t. candidate's derived values for r and a) M1

$S_\infty = 81$ (c.a.o.) A1

6. (*a*) $3 \times \dfrac{x^{3/2}}{3/2} - 2 \times \dfrac{x^{-2/3}}{-2/3} + c$ B1 B1

(−1 if no constant term present)

(*b*) (i) $36 - x^2 = 5x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = -36$ m1

$(x - 4)(x + 9) = 0 \Rightarrow A(4, 20)$ (c.a.o.) A1

$B(6, 0)$ B1

(ii) Area of triangle = 40 (f.t. candidate's coordinates for A) B1

Area under curve $= \displaystyle\int_4^6 (36 - x^2)\,dx$ (use of integration) M1

$\displaystyle\int 36\,dx = 36x$ **and** $\displaystyle\int x^2\,dx = \frac{x^3}{3}$ B1

Area under curve = [(216 – 216/3) – (144 – 64/3)]

(substitution of candidate's limits) m1

= 64/3

Use of candidate's, x_A, x_B as limits and trying to find total area by adding area of triangle and area under curve m1

Total area = 40 + 64/3 = 184/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) **Either:**

$(x/2 - 3) \log_{10} 9 = \log_{10} 6$

(taking logs on both sides and using the power law) M1

$x = \dfrac{2(\log_{10} 6 + 3 \log_{10} 9)}{\log_{10} 9}$ A1

$x = 7{\cdot}631$ (f.t. one slip, see below) A1

Or:

$x/2 - 3 = \log_9 6$ (rewriting as a log equation) M1

$x = 2(\log_9 6 + 3)$ A1

$x = 7{\cdot}631$ (f.t. one slip, see below) A1

Note: an answer of $x = -4{\cdot}369$ from $x = \dfrac{2(\log_{10} 6 - 3 \log_{10} 9)}{\log_{10} 9}$

earns M1 A0 A1

an answer of $x = 3{\cdot}815$ from $x = \dfrac{\log_{10} 6 + 3 \log_{10} 9}{\log_{10} 9}$

earns M1 A0 A1

an answer of $x = 1{\cdot}908$ from $x = \dfrac{(\log_{10} 6 + 3 \log_{10} 9)}{2\log_{10} 9}$

earns M1 A0 A1

an answer of $x = 4{\cdot}631$ from $x = \dfrac{2\log_{10} 6 + 3 \log_{10} 9}{\log_{10} 9}$

earns M1 A0 A1

Note: Answer only with no working earns 0 marks

(*c*) $\log_a (x - 2) + \log_a (4x + 1) = \log_a [(x - 2)(4x + 1)]$ (addition law) B1

$2 \log_a (2x - 3) = \log_a (2x - 3)^2$ (power law) B1

$(x - 2)(4x + 1) = (2x - 3)^2$ (removing logs) M1

$x = 2{\cdot}2$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $A(2, -3)$ B1

A correct method for finding the radius M1

Radius = $\sqrt{12}$ A1

(*b*) $AT^2 = 61$ (f.t. candidate's coordinates for A) B1

Use of $RT^2 = AT^2 - AR^2$ M1

$RT = 7$ (f.t. candidate's radius and coordinates for A) A1

9. Area of sector $POQ = {}^1/_2 \times r^2 \times 1\cdot12$ B1

Area of triangle $POQ = {}^1/_2 \times r^2 \times \sin(1\cdot12)$ B1

$10\cdot35 = {}^1/_2 \times r^2 \times 1\cdot12 - {}^1/_2 \times r^2 \times \sin(1\cdot12)$

(f.t. candidate's expressions for area of sector and area of triangle) M1

$r^2 = \frac{2 \times 10\cdot35}{(1\cdot12 - 0\cdot9)}$ (o.e.) (c.a.o.) A1

$r = 9\cdot7$ (f.t. one numerical slip) A1

C3

1. (*a*)

0	1		
0·25	1·064494459		
0·5	1·284025417		
0·75	1·755054657	(5 values correct)	B2
1	2·718281828	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$$I \approx \frac{0{\cdot}25}{3} \times \{1 + 2{\cdot}718281828 + 4(1{\cdot}064494459 + 1{\cdot}755054657) + 2(1{\cdot}284025417)\}$$

$I \approx 17{\cdot}56452913 \times 0{\cdot}25 \div 3$

$I \approx 1{\cdot}463710761$

$I \approx 1{\cdot}4637$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$$\int_0^1 e^{x^2+3}\,dx = e^3 \times \int_0^1 e^{x^2}\,dx$$ M1

$$\int_0^1 e^{x^2+3}\,dx = 29{\cdot}399$$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $\phi = 360° - \theta$ or $\phi = -\theta$ and noting that $\cos\theta = \cos\phi$ B1

$\sin\theta \neq \sin\phi$ (including correct evaluations) B1

(*b*) $13\tan^2\theta = 5(1 + \tan^2\theta) + 6\tan\theta.$

(correct use of $\sec^2\theta = 1 + \tan^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\tan\theta$, either by using the quadratic formula or by getting the expression into the form $(a\tan\theta + b)(c\tan\theta + d)$, with $a \times c$ = candidate's coefficient of $\tan^2\theta$ and $b \times d$ = candidate's constant m1

$8\tan^2\theta - 6\tan\theta - 5 = 0 \Rightarrow (4\tan\theta - 5)(2\tan\theta + 1) = 0$

$\Rightarrow \tan\theta = \frac{5}{4}, \tan\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 51{\cdot}34°, 231{\cdot}34°$ B1

$\theta = 153{\cdot}43°, 333{\cdot}43°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\tan\theta = +, -$, f.t. for 3 marks, $\tan\theta = -, -$, f.t. for 2 marks

$\tan\theta = +, +$, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(-3x - 2) = -3$ B1

$\frac{d}{dx}(-4x^2y) = -4x^2\frac{dy}{dx} - 8xy$ B1

$\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$ (c.a.o.) B1

(*b*) (i) Differentiating $\sin at$ and $\cos at$ with respect to t, at least one correct M1

candidate's x-derivative = $a\cos at$,

candidate's y-derivative = $-a\sin at$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\tan at$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left[\frac{dy}{dx}\right] = -a\sec^2 at$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dx}\right] \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = -\sec^3 at$ (c.a.o.) A1

4. $f(x) = \cos x - 5x + 2$

An attempt to check values or signs of $f(x)$ at $x = 0$, $x = \pi/4$ M1

$f(0) = 3 > 0$, $f(\pi/4) = -1{\cdot}22 < 0$

Change of sign $\Rightarrow f(x) = 0$ has root in $(0, \pi/4)$ A1

$x_0 = 0{\cdot}6$

$x_1 = 0{\cdot}565067123$ B1

$x_2 = 0{\cdot}568910532$

$x_3 = 0{\cdot}568497677$

$x_4 = 0{\cdot}568542145 = 0{\cdot}56854$ (x_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 0{\cdot}568535$, $x = 0{\cdot}568545$ M1

$f(0{\cdot}568535) = 1{\cdot}563 \times 10^{-5} > 0$, $f(0{\cdot}568545) = -3{\cdot}975 \times 10^{-5} < 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}56854$ correct to five decimal places A1

Note: 'change of sign' must appear at least once

5. (*a*) $\dfrac{dy}{dx} = \dfrac{a + bx}{7 + 2x - 3x^2}$ (including $a = 1$, $b = 0$) M1

$\dfrac{dy}{dx} = \dfrac{2 - 6x}{7 + 2x - 3x^2}$ A1

(*b*) $\dfrac{dy}{dx} = e^{\tan x} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\dfrac{dy}{dx} = e^{\tan x} \times \sec^2 x$ A1

(*c*) $\dfrac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$ ($f(x), g(x) \neq 1, 0$) M1

$\dfrac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$

(either $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$ or $g(x) = 10x$) A1

$\dfrac{dy}{dx} = 5x^2 \times \dfrac{1}{\sqrt{1 - x^2}} + 10x \times \sin^{-1} x$ A1

6. (*a*) (i) $\int 3e^{2 - x/4}\,dx = k \times 3e^{2 - x/4} + c$ $(k = 1, -{}^1/_4,\ 4, -\mathbf{4})$ M1

$\int 3e^{2 - x/4}\,dx = -4 \times 3e^{2 - x/4} + c$ A1

(ii) $\int \frac{9}{(2x-3)^6}\,dx = \frac{k \times 9 \times (2x-3)^{-5}}{-5} + c$ $(k = 1, 2, \mathbf{{}^1/_2})$ M1

$\int \frac{9}{(2x-3)^6}\,dx = \frac{9 \times (2x-3)^{-5}}{-5 \times 2} + c$ A1

(iii) $\int \frac{7}{3x+1}\,dx = k \times 7 \times \ln|3x+1| + c$ $(k = 1, 3, \mathbf{{}^1/_3})$ M1

$\int \frac{7}{3x+1}\,dx = {}^7/_3 \times \ln|3x+1| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \sin 2x\,dx = k \times \cos 2x$ $(k = -1, -2, {}^1/_2, -\mathbf{{}^1/_2})$ M1

$\int \sin 2x\,dx = -\frac{1}{2} \times \cos 2x$ A1

$k \times (\cos 2a - \cos 0) = {}^1/_4$

(f.t. candidate's value for k) M1

$\cos 2a = {}^1/_2$ (c.a.o.) A1

$a = \pi/6$ (f.t. $\cos 2a = b$ provided both M's are awarded) A1

7. (*a*) $9|x - 3| = 6$ B1

$x - 3 = \pm{}^2/_3$ (f.t. candidate's $a|x - 3| = b$, with at least one of a, b correct) B1

$x = {}^{11}/_3, {}^7/_3$ (f.t. candidate's $a|x - 3| = b$, with at least one of a, b correct) B1

(*b*) Trying to solve either $5x - 2 \le 3$ or $5x - 2 \ge -3$ M1

$5x - 2 \le 3 \Rightarrow x \le 1$

$5x - 2 \ge -3 \Rightarrow x \ge -{}^1/_5$ (both inequalities) A1

Required range: $-{}^1/_5 \le x \le 1$ (f.t. one slip) A1

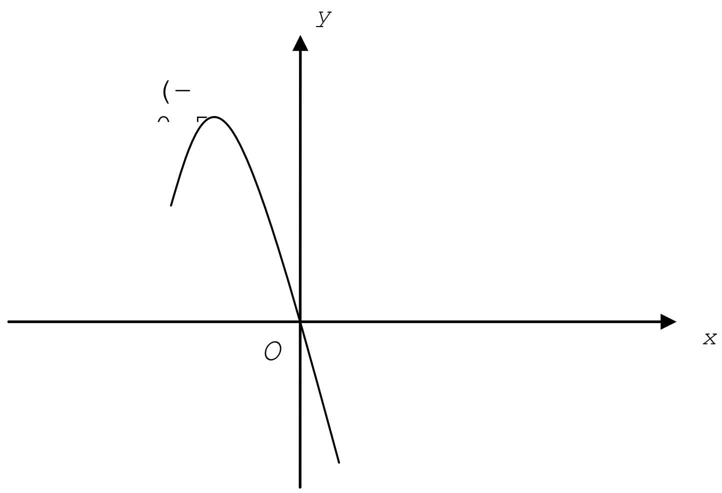
Alternative mark scheme

$(5x - 2)^2 \le 9$ (forming and trying to solve quadratic) M1

Critical points $x = -{}^1/_5$ and $x = 1$ A1

Required range: $-{}^1/_5 \le x \le 1$ (f.t. one slip) A1

8.



Concave down curve passing through the origin with maximum point in the second quadrant B1

x-coordinate of stationary point = – 0·5 B1

y-coordinate of stationary point = 8 B1

9. (*a*) (i) $f'(x) = \dfrac{(x^2 + 5) \times f(x) - (x^2 + 3) \times g(x)}{(x^2 + 5)^2}$ $(f(x), g(x) \neq 1)$ M1

$f'(x) = \dfrac{(x^2 + 5) \times 2x - (x^2 + 3) \times 2x}{(x^2 + 5)^2}$ A1

$f'(x) = \dfrac{4x}{(x^2 + 5)^2}$ (c.a.o.) A1

$f'(x) < 0$ since numerator is negative and denominator is positive B1

(ii) $R(f) = (^3/_5, 1)$ B1 B1

(*b*) (i) $x^2 = \dfrac{3 - 5y}{y - 1}$ (o.e.) (condone any incorrect signs) M1

$x = (\pm) \left(\dfrac{3 - 5y}{y - 1}\right)^{1/2}$ (f.t. at most one incorrect sign) A1

$x = -\left(\dfrac{3 - 5y}{y - 1}\right)^{1/2}$ (f.t. at most one incorrect sign) A1

$f^{-1}(x) = -\left(\dfrac{3 - 5x}{x - 1}\right)^{1/2}$ (c.a.o.) A1

(ii) $R(f^{-1}) = (-\infty, 0)$, $D(f^{-1}) = (^3/_5, 1)$,
(both intervals, f.t. candidate's $R(f)$) B1

10.	$gg(x) = (3(g(x))^2 + 7)^{1/2}$ or $gg(x) = g((3x^2 + 7)^{1/2})$			M1
	$gg(x) = (3(3x^2 + 7) + 7)^{1/2}$			A1
	An attempt to solve the equation by squaring both sides			M1
	$gg(x) = 8 \Rightarrow 9x^2 = 36$	(o.e.)	(c.a.o.)	A1
	$x = \pm 2$		(c.a.o.)	A1

C4

1. (*a*) $f(x) \equiv \dfrac{A}{(x+1)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$ (correct form) M1

$11 + x - x^2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 1,\ C = 3,\ B = -2$ (2 correct coefficients) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = -\dfrac{1}{(x+1)^2} + \dfrac{2}{(x-2)^2} - \dfrac{6}{(x-2)^3}$ (o.e.)

(f.t. candidate's values for *A*, *B*, *C*)

(at least one of the first two terms) B1

(third term) B1

$f'(0) = {}^1/_4$ (c.a.o.) B1

2. $3y^2\dfrac{dy}{dx} - 8x - 3x\dfrac{dy}{dx} - 3y = 0$ $\left[3y^2\dfrac{dy}{dx} - 8x\right]$ B1

$\left[-3x\dfrac{dy}{dx} - 3y\right]$ B1

Either $\dfrac{dy}{dx} = \dfrac{3y + 8x}{3y^2 - 3x}$ **or** $\dfrac{dy}{dx} = \dfrac{1}{3}$ (o.e.) (c.a.o.) B1

Equation of tangent: $y - (-3) = \dfrac{1}{3}(x-2)$ $\left[\text{f.t. candidate's value for } \dfrac{dy}{dx}\right]$ B1

3. (*a*) $4(1 - 2\sin^2\theta) = 1 - 2\sin\theta$. (correct use of $\cos 2\theta = 1 - 2\sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$8\sin^2\theta - 2\sin\theta - 3 = 0 \Rightarrow (4\sin\theta - 3)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{3}{4}, \quad \sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 48{\cdot}59°, 131{\cdot}41°$ B1

$\theta = 210°, 330°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) (i) $R = 17$ B1

Correctly expanding $\sin(x + \alpha)$ and using either $17\cos\alpha = 8$ **or** $17\sin\alpha = 15$ **or** $\tan\alpha = \frac{15}{8}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 61{\cdot}93°$ (c.a.o) A1

(ii) $\sin(x + \alpha) = \frac{11}{17}$ (f.t. candidate's value for R) B1

$x + 61{\cdot}93° = 40{\cdot}32°, 139{\cdot}68°, 400{\cdot}32°$,
(at least one value on R.H.S.,
f.t. candidate's values for α and R) B1

$x = 77{\cdot}75°, 338{\cdot}39°$ (c.a.o.) B1

(iii) Greatest possible value for k is 17 since greatest possible value for sin is 1 (f.t. candidate's value for R) E1

4.

$$\text{Volume} = \pi\int_3^4 \left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 dx$$ B1

$$\left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 = \left(x + 10 + \frac{25}{x}\right)$$ B1

$$\int\left(ax + b + \frac{c}{x}\right) dx = \frac{ax^2}{2} + bx + c\ln x, \text{ where } c \neq 0 \text{ and at least one of } a, b \neq 0$$ B1

Correct substitution of correct limits in candidate's integrated expression of form $\frac{ax^2}{2} + bx + c\ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$ M1

Volume = $65({\cdot}0059\ldots)$ (c.a.o.) A1

5. $\left(1 + \frac{x}{3}\right)^{-1/2} = 1 - \frac{x}{6} + \frac{x^2}{24}$ $\left(1 - \frac{x}{6}\right)$ B1

$\left(\frac{x^2}{24}\right)$ B1

$|x| < 3$ or $-3 < x < 3$ B1

$\left(\frac{16}{15}\right)^{-1/2} \approx 1 - \frac{1}{30} + \frac{1}{600}$ (f.t. candidate's coefficients) B1

$\sqrt{15} \approx \frac{581}{150}$ (c.a.o.) B1

6. (*a*) candidate's x-derivative = $2t$

candidate's y-derivative = 2 (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{1}{t}$ (o.e.) (c.a.o.) A1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ m1

Equation of normal at P: $y - 2p = -p(x - p^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y + px = p^3 + 2p$ (convincing) (c.a.o.) A1

(*b*) (i) Substituting $x = 9$, $y = 6$ in equation of normal M1

$p^3 - 7p - 6 = 0$ (convincing) A1

(ii) A correct method for solving $p^3 - 7p - 6 = 0$ M1

$p = -1$ A1

$p = -2$ A1

P is either $(1, -2)$ or $(4, -4)$ (c.a.o.) A1

7. (*a*) $u = x \Rightarrow du = dx$ (o.e.) B1

$dv = e^{-2x}\,dx \Rightarrow v = -\frac{1}{2}e^{-2x}$ (o.e.) B1

$\int x e^{-2x}\,dx = x \times -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx$ M1

$\int x e^{-2x}\,dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + c$ (c.a.o.) A1

(*b*) $\int \frac{1}{x(1+3\ln x)}\,dx = \int \frac{k}{u}\,du$ ($k = \frac{1}{3}$ or 3) M1

$\int \frac{a}{u}\,du = a\ln u$ B1

$\int_1^e \frac{1}{x(1+3\ln x)}\,dx = k[\ln u]_1^4$ or $k[\ln(1+3\ln x)]_1^e$ B1

$\int_1^e \frac{1}{x(1+3\ln x)}\,dx = 0{\cdot}4621$ (c.a.o.) A1

8. (*a*) $\frac{dV}{dt} = -kV^3$ (where $k > 0$) B1

(*b*) $\int \frac{dV}{V^3} = -\int k\,dt$ (o.e.) M1

$-\frac{V^{-2}}{2} = -kt + c$ A1

$c = -\frac{1}{7200}$ (c.a.o.) A1

$V^2 = \frac{3600}{7200kt+1} = \frac{3600}{at+1}$ (convincing)

where $a = 7200k$ A1

(*c*) Substituting $t = 2$ and $V = 50$ in expression for V^2 M1

$a = 0{\cdot}22$ A1

Substituting $V = 27$ in expression for V^2 with candidate's value for a M1

$t = 17{\cdot}9$ (c.a.o) A1

9. (*a*) An attempt to evaluate **a.b** M1

Correct evaluation of **a.b** and **a.b** $\neq 0 \Rightarrow$ **a** and **b** not perpendicular A1

(*b*) (i) **AB** = 2**i** + **j** + 2**k** B1

(ii) Use of **a** + λ**AB**, **a** + λ(**b** – **a**), **b** + λ**AB** or **b** + λ(**b** – **a**) to find vector equation of AB M1

r = 4**i** + **j** – 6**k** + λ (2**i** + **j** + 2**k**) (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $4 + 2\lambda = 2 - 2\mu$

$1 + \lambda = 6 + \mu$

$-6 + 2\lambda = p + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving the first two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = 2$, $\mu = -3$ (o.e.) (c.a.o.) A1

$p = 7$ from third equation

(f.t. candidates derived values for λ and μ) A1

10. $a^2 = 5b^2 \Rightarrow (5k)^2 = 5b^2 \Rightarrow b^2 = 5k^2$ B1

$\therefore$ 5 is a factor of b^2 and hence 5 is a factor of b B1

$\therefore$ a and b have a common factor, which is a contradiction to the original assumption B1

FP1

Ques	Solution	Mark	Notes
1	$S_n = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r$	**M1**	
	$= \dfrac{n^2(n+1)^2}{4} - \dfrac{n(n+1)}{2}$	**A1A1**	
	$= \dfrac{n(n+1)}{4}\left(n^2+n-2\right)$	**A1**	
	$= \dfrac{n(n-1)(n+1)(n+2)}{4}$	**A1**	
2(a)	$(1+2\mathrm{i})^2 = 1+4\mathrm{i}+4\mathrm{i}^2$	**M1**	Award for 3 reasonable terms.
	$= -3+4\mathrm{i}$	**A1**	
	$z = \dfrac{(-3+4\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})}$	**M1**	
	$= \dfrac{-6+8\mathrm{i}+3\mathrm{i}-4\mathrm{i}^2}{5}$	**A1A1**	A1 numerator, A1 denominator FT 1 arithmetic slip from line 2
	$= \dfrac{-2+11\mathrm{i}}{5}$ $(-0.4+2.2\mathrm{i})$ cao	**A1**	
(b)			
	$r = \sqrt{5}$ (2.24)	**B1**	FT on line above for r.
	$\tan^{-1}(-5.5) = -1.39$ (-79.6°) or	**B1**	FT on line above for this B1
	$\tan^{-1}(5.5) = 1.39$ (79.6°)		
	$\theta = 1.75$ (100.3°)	**B1**	FT only if in 2^{nd} or 3^{rd} quad
3(a)	$\alpha+\beta = -\dfrac{1}{2}, \alpha\beta = 1$	**B1**	
	$\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha} = \dfrac{\alpha^3+\beta^3}{\alpha\beta}$	**M1**	
	$= \dfrac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$	**M1A1**	
	$= \dfrac{(-1/2)^3-3\times1\times(-1/2)}{1}$	**A1**	
	$= \dfrac{11}{8}$		
(b)	Consider		
	$\dfrac{\alpha^2}{\beta}\times\dfrac{\beta^2}{\alpha} = \alpha\beta = 1$	**M1A1**	
	The required equation is		
	$x^2 - \dfrac{11}{8}x + 1 = 0$ $(8x^2-11x+8=0)$ cao	**B1**	

4(a)(i)	Cofactor matrix = $\begin{bmatrix} -13 & 9 & 1 \\ -18 & 13 & 1 \\ 14 & -10 & -1 \end{bmatrix}$	**M1** **A1**	Award M1 if at least 5 cofactors are correct.
	Adjugate matrix = $\begin{bmatrix} -13 & -18 & 14 \\ 9 & 13 & -10 \\ 1 & 1 & -1 \end{bmatrix}$	**A1**	No FT on cofactor matrix.
(ii)	Determinant = 3(7 – 20) +4(16 – 7) +2(5 – 4) = – 1	**M1** **A1**	
	Inverse matrix = $\begin{bmatrix} 13 & 18 & -14 \\ -9 & -13 & 10 \\ -1 & -1 & 1 \end{bmatrix}$	**A1**	FT the adjugate
(b)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 & 18 & -14 \\ -9 & -13 & 10 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}$	**M1**	
	$= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$	**A1**	FT their inverse matrix.
5(a)	By reduction to echelon form, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -2 & -10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ k-6 \end{bmatrix}$ It follows now that k – 6 = – 2 k = 4	**M1** **A1** **A1** **M1** **A1**	
(b)	Put $z = \alpha$. Then y = 1– 5α And x = 7α	**M1** **A1** **A1**	
6	Putting n = 1, the expression gives 3 which is divisible by 3 so the result is true for n = 1 Assume that the formula is true for n = k. ($k^3 + 2k$ is divisible by 3 or $k^3 + 2k = 3N$)). Consider (for n = k + 1) $(k+1)^3 + 2(k+1)$ $= k^3 + 3k^2 + 3k + 1 + 2k + 2$ $= 3N - 2k + 3k^2 + 3k + 1 + 2k + 2$ $= 3(N + k^2 + k + 1)$ (This is divisible by 3), therefore true for $n = k \Rightarrow$ true for n = k + 1 and since true for n = 1, the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	Award this M1 only if it is clearly stated that this is an assumption Do not award the second M1 if this is stated as an assumption but the three A1s may be awarded if either of the M1s is awarded

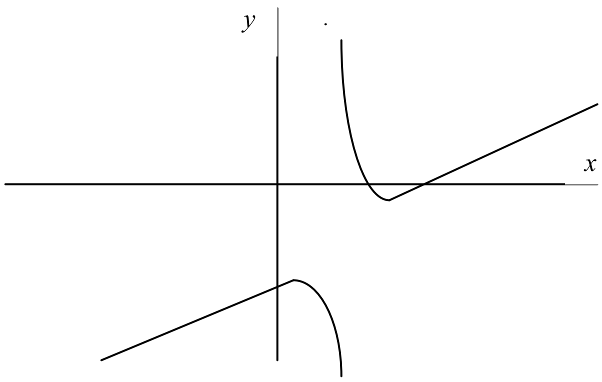
7(a)	Ref matrix in $y = x = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	Translation matrix $= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	Ref matrix in x-axis $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$	M1	
	$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	A1	
	$= \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$		
(b)	$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	M1	
	$y - 2 = x, \; -x - 2 = y$	A1	
	$x = -2, y = 0$ cao	A1A1	

8(a)	Taking logs, $\ln f(x) = x\ln x$ Differentiating, $\frac{f'(x)}{f(x)} = \ln x + 1$ $f'(x) = x^x(\ln x + 1)$	**B1** **B1B1** **B1**	B1 for LHS, B1 for RHS
(b)	At a stationary point, $f'(x) = 0$ $\ln x = -1$ $x = \frac{1}{\mathrm{e}}; y = \left(\frac{1}{\mathrm{e}}\right)^{1/\mathrm{e}}$ (0.368, 0.692)	**M1** **A1** **A1**	
(c)	Differentiating the expression in (a), $f''(x) = x^x(\ln x+1)(\ln x+1) + x^x \times \frac{1}{x}$ $= x^{x-1} + x^x(1+\ln x)^2$. $f''(1/\mathrm{e}) = 1.88$ Since this is positive it is a minimum.	**B1B1** **B1** **B1**	B1 each term Accept 'Since the first term is positive and the second term zero, it is a minimum' FT the final B1 on the line above
9(a)	$x + \mathrm{i}y = \frac{1}{u + \mathrm{i}v}$ $= \frac{u - \mathrm{i}v}{u^2 + v^2}$ $x = \frac{u}{u^2+v^2}$ $y = -\frac{v}{u^2+v^2}$	**M1** **A1** **A1**	
(b)(i)	We are given that $-\frac{v}{u^2+v^2} = \frac{mu}{u^2+v^2} + 1$ $-v = mu + u^2 + v^2$ $u^2 + v^2 + mu + v = 0$ (This is the equation of a circle).	**M1** **A1** **A1**	
(ii)	Completing the square or quoting the standard results, Radius $= \frac{1}{2}\sqrt{m^2+1}$ Centre $\left(-\frac{1}{2}m, -\frac{1}{2}\right)$	**M1** **A1** **A1**	FT on their circle equation
(iii)	$v = -\frac{1}{2}$	**A1**	Accept $y = -\frac{1}{2}$

FP2

Ques	Solution	Mark	Notes
1	Putting $x = 2$, $4a - 8 = 8 - 2b$ The two derivatives are $2ax$ and $3x^2 - b$ Putting $x = 2$, $4a = 12 - b$ Solving, $a = 2, b = 4$ cao	**M1A1** **M1** **A1** **A1**	
2	$u = e^x \Rightarrow du = e^x dx$, $[0,1] \to [1, e]$ $I = \int_1^e \frac{du/u}{u + 4/u}$ $= \int_1^e \frac{du}{u^2 + 4}$ $= \frac{1}{2}\left[\tan^{-1}(\frac{u}{2})\right]_1^e$ $= 0.236$	**B1** **B1** **M1** **A1** **A1** **A1**	
3	Put $t = \tan(x/2)$ $\frac{3 \times 2t}{1+t^2} = t$ $t(t^2 - 5) = 0$ $t = 0$ giving $x/2 = n\pi \to x = 2n\pi$ $(360n^\circ)$ $t = \sqrt{5}$ giving $x/2 = 1.15026.. + n\pi$ $\to x = 2.30 + 2n\pi$ $(360n^\circ + 132^\circ)$ $t = -\sqrt{5}$ giving $x/2 = -1.15026.. + n\pi$ $\to x = -2.30 + 2n\pi$ $(360n^\circ - 132^\circ)$	 **M1** **A1** **M1A1** **M1** **A1** **M1** **A1**	

4(a)	Let $\dfrac{3x^2-4x+1}{(x-2)(x^2+1)} \equiv \dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+1}$		
	$=\dfrac{A(x^2+1)+(Bx+C)(x-2)}{(x-2)(x^2+1)}$	**M1**	
	$x = 2$ gives $A = 1$	**A1**	
	Coeff of x^2 gives $A + B = 3$, $B = 2$	**A1**	
	Const term gives $A - 2C = 1$, $C = 0$	**A1**	
(b)	$\int_3^4 f(x)\mathrm{d}x = \int_3^4 \dfrac{1}{x-2}\mathrm{d}x + \int_3^4 \dfrac{2x}{x^2+1}\mathrm{d}x$	**M1**	
	$= [\ln(x-2)]_3^4 + [\ln(x^2+1)]_3^4$	**A1A1**	
	$= \ln 2 - \ln 1 + \ln 17 - \ln 10$	**A1**	
	$= \ln\left(\dfrac{34}{10}\right)$ or $\ln\left(\dfrac{17}{5}\right)$	**A1**	
5(a)	Consider $f(-x) = (-x)^2\sin(-x)$	**M1**	Accept a specific value for x.
	$= -x^2\sin x = -f(x)$	**A1**	
	f is therefore odd.	**A1**	
(b)	$\sin x$ is odd and x^n is even if n is even and odd if n is odd. si	**M1**	For a valid attempt. Accept a specific value for x.
	So g is even if n is odd	**A1**	
	and g is odd when n is even.	**A1**	

6(a)	Putting $x = 0$ gives $(0, -20/3)$ Putting $y = 0$, $\frac{2}{x-3} = 6 - x$ $(x-3)(6-x) = 2$ $x^2 - 9x + 20 = 0$ giving (4,0) ; (5,0) cao	**B1** **M1** **A1** **A1** **A1**	
(b)	Differentiating, $-\frac{2}{(x-3)^2} + 1 = 0$ $(x-3)^2 = 2$ $x = 3 + \sqrt{2}(4.41), y = 2\sqrt{2} - 3(-0.172)$ $x = 3 - \sqrt{2}(1.59), y = -3 - 2\sqrt{2})(-5.83)$	**M1** **A1** **A1** **A1** **A1**	Award A1A0 for the 2 x values only.
(c)	The asymptotes are $x = 3$ $y = x - 6$	**B1** **B1**	
(d)		**G1** **G1**	General shape of both branches. Correct shape including asymptotic behaviour.

7(a)(i)	Completing the square, $(y-1)^2 = 8x - 24$ The vertex is therefore (3,1)	**M1** **A1** **A1**	FT on 1 arithmetic slip
(ii)	In the usual notation, $a = 2$ si The focus is (5,1)	**B1** **B1**	
(iii)	The equation of the directrix is $x = 1$	**B1**	
(b)(i)	Substituting $y = mx$, $m^2x^2 - 2mx - 8x + 25 = 0$	**M1** **A1**	
(ii)	For coincident roots, $(2m+8)^2 = 100m^2$ $3m^2 - m - 2 = 0$ Solving using a valid method, $m = 1, -2/3$	**M1** **A1** **A1** **M1** **A1**	
8(a)	The result is true for $n = 1$ since it gives $(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos 1\theta + \mathrm{i}\sin 1\theta$ Let the result be true for $n = k$, ie $(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$ Consider $(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos\theta + \mathrm{i}\sin\theta)^k(\cos\theta + \mathrm{i}\sin\theta)$ $= (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$ $\cos k\theta\cos\theta - \sin k\theta\sin\theta$ $+\mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$ True for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$ the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	
(b)(i)	Consider $\left(w(\cos 2\pi/3 + \mathrm{i}\sin 2\pi/3)\right)^3$ $= w^3(\cos 2\pi + \mathrm{i}\sin 2\pi)$ $= z \times 1 = z$ Showing that $\left(w(\cos 2\pi/3 + \mathrm{i}\sin 2\pi/3)\right)$ is a cube root of z.	**M1** **A1** **A1**	
(ii)	The real cube root of -8 is -2. The other cube roots are $-2(\cos 2\pi/3 + \mathrm{i}\sin 2\pi/3) = 1 - \sqrt{3}\mathrm{i}$ $-2(\cos 4\pi/3 + \mathrm{i}\sin 4\pi/3) = 1 + \sqrt{3}\mathrm{i}$	**B1** **M1A1** **A1**	

FP3

Ques	Solution	Mark	Notes
1	$\int_0^1 x\sinh x\,\mathrm{d}x = [x\cosh x]_0^1 - \int_0^1 \cosh x\,\mathrm{d}x$	M1A1	
	$= \cosh 1 - [\sinh x]_0^1$	A1A1	
	$= \cosh 1 - \sinh 1$	A1	Do not accept an argument which evaluates this as 0.367879… and shows that this is also the numerical value of 1/e.
	$= \dfrac{e^1 + e^{-1}}{2} - \dfrac{e^1 - e^{-1}}{2}$	A1	
	$= \dfrac{1}{e}$		
2(a)	The equation can be rewritten as $\sinh^2 x - \sinh x + 1 - k = 0$	M1A1	
	The condition for no real roots is $1 - 4(1-k) = 4k - 3 < 0$	m1	
	$k < \dfrac{3}{4}$	A1	
(b)	$\sinh^2 x - \sinh x - 2 = 0$		
	$(\sinh x - 2)(\sinh x + 1) = 0$	M1	
	$\sinh x = 2$	A1	
	$x = \sinh^{-1} 2 = \ln(2 + \sqrt{5})$	A1	
3	Let $f(x) = \tan^{-1} x$		
	$p = f(1) = \dfrac{\pi}{4}$	B1	
	$f'(x) = \dfrac{1}{1+x^2}; q = f'(1) = \dfrac{1}{2}$	M1A1	
	$f''(x) = -\dfrac{2x}{(1+x^2)^2}; r = \dfrac{f''(1)}{2} = -\dfrac{1}{4}$	M1A1	
	$f'''(x) = \dfrac{-2(1+x^2)^2 + 2(1+x^2).4x^2}{(1+x^2)^4}; s = \dfrac{f'''(1)}{6} = \dfrac{1}{12}$	M1A1	

4(a)	Consider $y = r\sin\theta$ $= 2\sin\theta\cos\theta - \sin^2\theta$ $\frac{dy}{d\theta} = 2\cos^2\theta - 2\sin^2\theta - 2\sin\theta\cos\theta$ $= 2\cos 2\theta - \sin 2\theta$	M1 A1 A1	
	The tangent is parallel to the initial line where $2\cos 2\theta = \sin 2\theta$ $\tan 2\theta = 2$ $\theta = 0.554, r = 1.18$	M1 A1 A1	 Accept 31.7°
(b)	The curves intersect where $2\cos\theta - \sin\theta = 1 + \sin\theta$ $2\cos\theta - 2\sin\theta = 1$	M1 A1	
	EITHER Putting $t = \tan(\theta/2)$ $\frac{2(1-t^2)}{1+t^2} - \frac{4t}{1+t^2} = 1$ $3t^2 + 4t - 1 = 0$ $\tan(\theta/2) = \frac{-4+\sqrt{28}}{6}$ (0.21525..) $\theta = 0.424, r = 1.41$	 M1 A1 A1 A1 A1	 Accept 24.3°
	OR Putting $2\cos\theta - 2\sin\theta = r\cos(\theta + \alpha)$ $\alpha = \pi/4$ $r = 2\sqrt{2}$ $\cos(\theta + \pi/4) = \frac{1}{2\sqrt{2}}$ $\theta = 0.424, r = 1.41$	 M1 A1 A1 A1 A1	 Accept 24.3°

5	Putting $t=\tan(x/2)$ gives $\mathrm{d}x=\frac{2\mathrm{d}t}{1+t^2}$ $(0,\pi/2)\to(0,1)$ $I=\int_0^1 \frac{2\mathrm{d}t/(1+t^2)}{4(1-t^2)/(1+t^2)+3}$ $=2\int_0^1 \frac{\mathrm{d}t}{7-t^2}$ $=2\times\frac{1}{2\sqrt{7}}\left[\ln\left\|\frac{\sqrt{7}+t}{\sqrt{7}-t}\right\|\right]_0^1$ or $\frac{2}{\sqrt{7}}\left[\tanh^{-1}\left(\frac{t}{\sqrt{7}}\right)\right]_0^1$ $=\frac{1}{\sqrt{7}}\left(\ln\left(\frac{\sqrt{7}+1}{\sqrt{7}-1}\right)-\ln(1)\right)$ or $\frac{2}{\sqrt{7}}\left(\tanh^{-1}\left(\frac{1}{\sqrt{7}}\right)-\tanh^{-1}(0)\right)$ $=0.301$	B1 B1 M1 A1 A1 A1 A1	
6(a) **(b)(i)** **(b)(ii)**	$I_n=\left[\theta^n\sin\theta\right]_0^{\pi/2}-n\int_0^{\pi/2}\theta^{n-1}\sin\theta\mathrm{d}\theta$ $=\left(\frac{\pi}{2}\right)^n-n\int_0^{\pi/2}\theta^{n-1}\sin\theta\mathrm{d}\theta$ $=\left(\frac{\pi}{2}\right)^n+\left[n\theta^{n-1}\cos\theta\right]_0^{\pi/2}-n(n-1)I_{n-2}$ $=\left(\frac{\pi}{2}\right)^n-n(n-1)I_{n-2}$ $I_0=\int_0^{\pi/2}\cos\theta\mathrm{d}\theta=\left[\sin\theta\right]_0^{\pi/2}=1$ $I_4=\left(\frac{\pi}{2}\right)^4-12I_2$ $=\left(\frac{\pi}{2}\right)^4-12\left(\left(\frac{\pi}{2}\right)^2-2I_0\right)$ $=0.479$ $\int_0^{\pi/2}\theta^5\sin\theta\mathrm{d}\theta=-\left[\theta^5\cos\theta\right]_0^{\pi/2}+5\int_0^{\pi/2}\theta^4\cos\theta\mathrm{d}\theta$ $=5I_4=2.4$	M1A1 A1 M1A1 B1 M1 A1 A1 M1A1 A1	. FT their answer from (b)(i)

7(a)	The Newton-Raphson iteration is $x_{n+1} = x_n - \frac{(x_n - 2\tanh x_n)}{(1-2\text{sech}^2 x_n)}$	M1A1	
	$= \frac{x_n - 2x_n \text{sech}^2 x_n - x_n + 2\tanh x_n}{1-2\text{sech}^2 x_n}$	A1	
	$= \frac{-2x_n + 2\sinh x_n \cosh x_n}{\cosh^2 x_n - 2}$	A1	
	$= \frac{\sinh 2x_n - 2x_n}{\cosh^2 x_n - 2}$	A1	
(b)	$x_0 = 2$ $x_1 = 1.916216399$ $x_2 = 1.915008327$	B1	
	Rounding to three decimal places gives 1.915	B1	
	Let $f(x) = x - 2\tanh x$ $f(1.9155) = 4.1\times10^{-4}$	M1	
	$f(1.9145) = -4.2\times10^{-4}$ The change of sign shows $\alpha = 1.915$ correct 3dp	A1	The values are required
8(a)	The curve cuts the x-axis where $x = \cosh^{-1} 2 = \alpha$	B1	Seen or implied
	$\frac{dy}{dx} = -\sinh x$	B1	
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 x = \cosh^2 x$	B1	
	Arc length = $\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	M1	
	$= \int_{-\alpha}^{\alpha} \cosh x\, dx$	A1	
	$= [\sinh x]_{-\alpha}^{\alpha}$	A1	
	$= 2\sqrt{3}$ (3.46) cao	A1	
(b)	Curved surface area = $2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	M1	
	$= 2\pi \int_{-\alpha}^{\alpha} (2 - \cosh x)\cosh x\, dx$	A1	
	$= 4\pi \int_{-\alpha}^{\alpha} \cosh x\, dx - \pi \int_{-\alpha}^{\alpha} (\cosh 2x + 1)$	A1	
	$= \pi \left[4\sinh x - \frac{1}{2}\sinh 2x - x \right]_{-\alpha}^{\alpha}$	A2	Minus 1 each error
	$= 2\pi(4\sqrt{3} - 2\sqrt{3} - \cosh^{-1} 2)$	A1	
	$=13.5$	A1	

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