GCE AS/A level

0984/01

MATHEMATICS S2
Statistics 2

P.M. FRIDAY, 22 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- a statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variables X and Y are independent, X with mean 5 and variance 2 and Y with mean 6 and variance 3.

(a) Determine the values of $E(X^2)$ and $E(Y^2)$. [3]

(b) Given that $U = XY$, find the mean and variance of U. [5]

2. The weights X kg of male birds of a certain species are normally distributed with mean 4·4 kg and standard deviation 0·2 kg.

(a) (i) Find the probability that the weight of a randomly selected male bird exceeds 4·5 kg.

(ii) Determine the 95th percentile of X. [5]

(b) The weights Y kg of female birds of the same species are normally distributed with mean 2·6 kg and standard deviation 0·15 kg.

(i) Find the mean and variance of $2Y - X$.

(ii) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird.

(iii) Two male birds and three female birds are placed on a weighing machine whose maximum permissible weight is 16 kg. Find the probability that the maximum weight is exceeded. [13]

3. The lifetime, X thousand hours, of a certain type of electric light bulb may be assumed to be normally distributed with unknown mean μ and standard deviation 0·1. The lifetimes of a random sample of 75 of these bulbs were measured and it was found that $\sum x = 69{\cdot}9$.

(a) Find a 90% confidence interval for μ. [5]

(b) Give an interpretation of this confidence interval. [1]

4. *(a)* When Jack types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean 0·8. He types a 10-page document. Determine the probability that the total number of errors is less than 5. [3]

(b) When Mary types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean μ. Mary claims that the value of μ is less than 0·8 but Jack claims that μ is equal to 0·8.

(i) State suitable hypotheses for testing these claims.

(ii) Mary types an 80-page document and makes 60 errors. Find the approximate p-value of this result and state your conclusion. [7]

5. David and Frank are golfers and they wish to determine whether or not there is a difference between the mean distances that they can hit a golf ball. They decide that they should each hit six balls and measure the distances travelled in yards by these balls. The results are shown below.

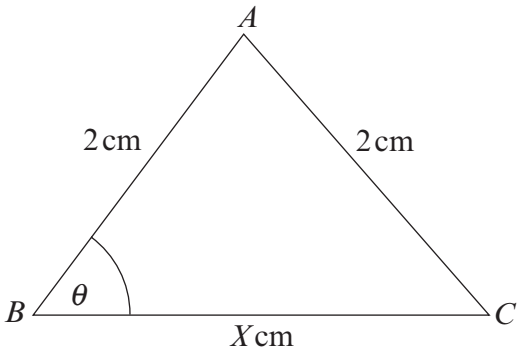
Distances hit by David	152·1	148·3	150·6	145·4	144·7	149·3
Distances hit by Frank	143·4	147·9	150·8	144·1	145·6	147·2

You may assume that these are random samples from normal populations with a common standard deviation of 1·5.

(a) State suitable hypotheses for testing whether or not there is a difference between the mean distances. [1]

(b) Determine the p-value of these results and state your conclusion in context. [10]

6.



The diagram shows an isosceles triangle ABC in which $AB = AC = 2\,\text{cm}$.

The angle $A\hat{B}C$, denoted by θ, is a random variable that is uniformly distributed on the interval $(0, \frac{\pi}{2})$. The length BC is denoted by $X\,\text{cm}$.

(a) Show that $X = 4\cos\theta$. [2]

(b) Evaluate

(i) $E(X)$,

(ii) $P(X \leqslant 3)$. [8]

TURN OVER

7. A garden centre sells large bags of wallflower seeds. Type A bags contain a mixture of seeds of which, on average, 50% will produce white flowers and 50% red flowers. Type B bags contain a mixture of seeds of which, on average, 70% will produce white flowers and 30% red flowers. The manager finds an unlabelled bag of these seeds and she wants to know if it is Type A or Type B.
She therefore plants 120 seeds and she decides to label the bag Type A if the number of these seeds producing white flowers is less than 70. You may assume that all the seeds germinate and produce flowers. Determine, approximately, the probability of

(a) labelling the bag as Type A when it is actually Type B, [6]

(b) labelling the bag as Type B when it is actually Type A. [6]

