# GCE AS/A level

0982/01

# MATHEMATICS M3
## Mechanics 3

P.M. THURSDAY, 21 June 2012

1½ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take $g$ as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** A car of mass 600 kg starts from rest and moves along a straight horizontal road. At time $t$ s, the force acting on the car has magnitude $\frac{27000}{(t+3)^2}$ N acting in the direction of motion. Resistance to motion may be ignored.

*(a)* Find an expression for $v$ ms$^{-1}$, the velocity of the car at time $t$ s. Hence show that the speed of the car has a limiting value as $t$ increases and find this limiting value. [7]

*(b)* Calculate the distance travelled by the car in the first 6 s of motion. Give your answer correct to two decimal places. [5]

**2.** The points $O$, $A$ and $B$ lie, in that order, on a straight line with $OA = 0{\cdot}6$ m and $OB = 0{\cdot}8$ m. A particle $P$ performs Simple Harmonic Motion along the line with centre $O$. The speed of $P$ at $A$ is $0{\cdot}3\sqrt{3}$ ms$^{-1}$ and its speed at $B$ is $0{\cdot}2\sqrt{5}$ ms$^{-1}$.

*(a)* Show that the amplitude of the motion is 1·2 m and that the period is $4\pi$ s. [7]

*(b)* Determine the magnitude of the acceleration of $P$ at $A$. [2]

*(c)* Calculate the time taken for $P$ to move directly from $A$ to $B$. Give your answer correct to 3 significant figures. [4]

*(d)* Given that $P$ is at $O$ at time $t = 0$, find the distance of $P$ from $O$ when $t = \frac{2\pi}{3}$. [2]

*(e)* Given that $P$ is at $O$ when $t = 0$, find the speed of $P$ when $t = \frac{2\pi}{3}$. [3]

**3.** Find the solution of the second order differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 6t + 5$$

such that $x = 3$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$ when $t = 0$. [12]

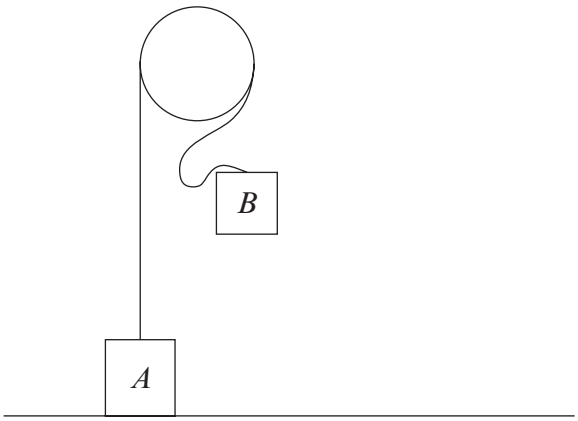
**4.** A particle $P$, of mass 0·5 kg, moves along the positive $x$-axis away from the origin $O$. At time $t$ s, the displacement of $P$ from $O$ is $x$ m and its speed is $v$ ms$^{-1}$. The particle is moving under the action of a force of magnitude $\frac{4}{2x+1}$ N acting in the direction of motion. As $P$ passes point $A$, where $OA = 3$ m, its speed is 4 ms$^{-1}$.

*(a)* Find an expression for $v^2$ in terms of $x$, and hence calculate the speed of $P$ when it is 10 m from $O$. [8]

*(b)* Find the distance of $P$ from $O$ when its speed is 6 ms$^{-1}$. [3]

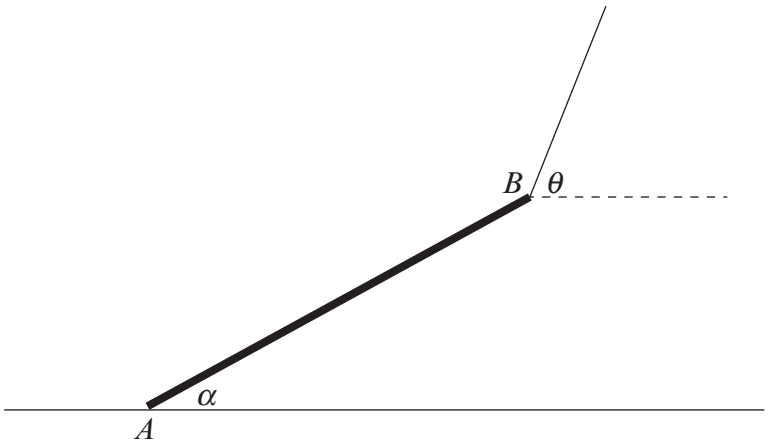
 

**5.** A particle $A$, of mass 5 kg, rests on a horizontal surface. It is attached to one end of a light inextensible string which passes over a smooth light fixed pulley. The other end of the string is attached to another particle $B$ of mass 2 kg. Initially, the particles are held at rest with the string just taut. Particle $B$ is then raised vertically and released from rest. After dropping for 0·5 s, the string becomes taut.




Find the speed with which particle $A$ begins to rise and the impulsive tension in the string. [8]

**6.** The diagram shows a straight uniform beam $AB$ of weight 2100 N and length 2 m resting in equilibrium with its end $A$ on rough horizontal ground. A light cable, which is attached to the other end $B$, is holding the beam with the end $B$ off the ground so that the beam makes an angle $\alpha$ with the ground, where $\tan\alpha = \frac{5}{12}$. The cable makes an angle $\theta$ with the horizontal.




The coefficient of friction between the end $A$ and the ground is $\frac{3}{4}$. Given that the end $A$ of the beam is about to slip,

*(a)* find the normal reaction of the ground on the beam at $A$, [6]

*(b)* calculate the tension in the cable and the value of the angle $\theta$. [8]