GCE AS/A level

0979/01

MATHEMATICS FP3

Further Pure Mathematics

P.M. MONDAY, 25 June 2012

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Show that

$$\int_0^1 x \sinh x \, dx = \frac{1}{e}. \qquad [6]$$

2. Consider the equation

$$\cosh^2 x = \sinh x + k$$

where k is a constant.

(a) Find the range of values of k for which the equation has no real solution. [4]

(b) Find the positive root of the equation when $k = 3$, giving your answer in the form $\ln(a + \sqrt{b})$, where a, b are positive integers. [3]

3. The Taylor series of $\tan^{-1}x$ about $x = 1$ is given by

$$\tan^{-1}x = p + q(x - 1) + r(x - 1)^2 + s(x - 1)^3 + \ldots$$

Find the values of the constants p, q, r, s. [7]

4. The curve C_1 has polar equation

$$r = 2\cos\theta - \sin\theta \quad (0 \leqslant \theta \leqslant \tfrac{\pi}{4}).$$

(a) Find the polar coordinates of the point on C_1 at which the tangent is parallel to the initial line. [6]

(b) The curve C_2 has polar equation

$$r = 1 + \sin\theta.$$

Find the polar coordinates of the point of intersection of C_1 and C_2. [7]

5. Use the substitution $t = \tan\left(\dfrac{x}{2}\right)$ to evaluate

$$\int_0^{\frac{\pi}{2}} \frac{1}{4\cos x + 3} \, dx.$$

Give your answer correct to three significant figures. [7]

6. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^{\frac{\pi}{2}} \theta^n \cos\theta \, d\theta.$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$$

[5]

(b) (i) Hence evaluate I_4, giving your answer correct to three significant figures.

(ii) Deduce the value of

$$\int_0^{\frac{\pi}{2}} \theta^5 \sin\theta \, d\theta.$$

[7]

7. The equation $x = 2\tanh x$ has a root α between 1·5 and 2.

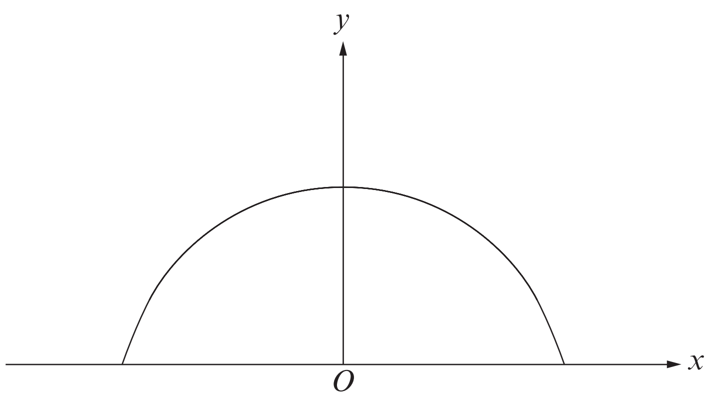
(a) Show that the Newton-Raphson iteration to find the value of α can be written in the form

$$x_{n+1} = \frac{\sinh 2x_n - 2x_n}{\cosh^2 x_n - 2}.$$

[5]

(b) Starting with $x_0 = 2$, write down the values of x_1 and x_2 given by your calculator. Show that rounding x_2 to three decimal places gives the value of α correct to three decimal places. [4]

8.



The diagram shows a sketch of the part of the curve $y = 2 - \cosh x$ which lies above the x-axis.

(a) Find the total length of the curve shown. [7]

(b) The region enclosed between the curve and the x-axis is rotated through 2π radians about the x-axis. Find the curved surface area of the solid generated, giving your answer correct to three significant figures. [7]