GCE AS/A level

983/01

MATHEMATICS S1

Statistics

P.M. THURSDAY, 16 June 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Cyril buys a bag containing 9 sweets of which 5 are red, 3 are green and 1 is yellow. He allows Gwyneth to choose 3 sweets at random from the bag.
Calculate the probability that she chooses

(a) 1 sweet of each colour, [3]

(b) no green sweets, [2]

(c) 3 sweets of the same colour. [3]

2. The random variable X has a Poisson distribution with mean 4. The random variable Y is defined by $Y = aX + b$, where a, b are positive constants.

(a) Given that the mean and variance of Y are both equal to 16, find the value of a and the value of b. [6]

(b) Bill states that, because the mean and variance of Y are equal, Y has a Poisson distribution. Give a reason why Bill's statement cannot be true. [1]

3. The events A and B are such that

$$P(A) = 0{\cdot}25,\ P(B) = 0{\cdot}4 \text{ and } P(A' \cap B') = 0{\cdot}45.$$

Determine whether

(a) A and B are mutually exclusive, [3]

(b) A and B are independent. [4]

4. Cars arrive at a petrol station in such a way that the number arriving during an interval of length t minutes has a Poisson distribution with mean $0{\cdot}2t$.

(a) Find the probability that

(i) exactly ten cars arrive between 9 a.m. and 10 a.m.,

(ii) more than five cars arrive between 11 a.m. and 11.30 a.m.. [6]

(b) The probability that no cars arrive during an interval of length t minutes is equal to $0{\cdot}03$. Without the use of tables, find the value of t. [4]

5. The probability distribution of the discrete random variable X is given by

$$P(X = x) = kx^2 \qquad \text{for } x = 1, 2, 3, 4,$$
$$P(X = x) = 0 \qquad \text{otherwise,}$$

where k is a constant.

(a) Show that $k = \frac{1}{30}$. [2]

(b) Calculate the mean and variance of X. [5]

(c) Two independent observations X_1, X_2 are taken from the distribution of X.
Calculate $P(X_1 + X_2 = 4)$. [4]

6. A box contains three coins. Two of these coins are fair and the third coin is double-headed so that when tossed a head is always obtained. One of these coins is selected at random and tossed three times.

(a) Find the probability that three heads are obtained. [4]

(b) Given that three heads are obtained, find the probability that the double-headed coin was selected. [3]

(c) The selected coin is tossed a fourth time. Find the probability that a head is obtained. [2]

7. *(a)* A series of trials is carried out, each resulting in either success or failure.
State **two** conditions that have to be satisfied in order for the total number of successes to be modelled by the binomial distribution. [2]

(b) Each time Ann shoots an arrow at a target, she hits it with probability 0·4.
She shoots 20 arrows at the target. Determine the probability that she hits it

(i) exactly 8 times,

(ii) between 6 and 10 times (both inclusive). [5]

(c) Each time she shoots an arrow, she hits the centre of the target with probability 0·04. She shoots 100 arrows at the target. Use a Poisson approximation to find the probability that she hits the centre of the target less than 5 times. [3]

8. *(a)* The continuous random variable X has probability density function f given by

$$f(x) = 12x^2(1 - x) \quad \text{for } 0 \leqslant x \leqslant 1,$$
$$f(x) = 0 \quad \text{otherwise.}$$

Calculate

(i) $E(X)$,

(ii) $E\left(\frac{1}{X}\right)$,

(iii) $P(0{\cdot}2 \leqslant X \leqslant 0{\cdot}5)$. [9]

(b) The continuous random variable Y takes values between 1 and 2 and its cumulative distribution function F is given, for $1 \leqslant y \leqslant 2$, by

$$F(y) = ay + by^2.$$

Find the values of the constants a and b. [4]