



**GCE AS/A level**

982/01

**MATHEMATICS M3**

**Mechanics 3**

A.M. THURSDAY, 23 June 2011

1½ hours

#### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

#### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

#### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A vehicle  $P$ , of mass  $800\text{ kg}$ , on a straight horizontal road passes the point  $O$  with velocity  $5\text{ ms}^{-1}$ . At time  $t\text{ s}$  later its velocity is  $v\text{ ms}^{-1}$  and the vehicle is subject to a resistance given by  $(4000 + 1600v)\text{ N}$ .

(a) Show that  $v$  satisfies the differential equation

$$\frac{dv}{dt} = -(5 + 2v) . \quad [2]$$

(b) (i) Find the time when  $P$  is at rest.

(ii) Find an expression for  $v$  in terms of  $t$ . [9]

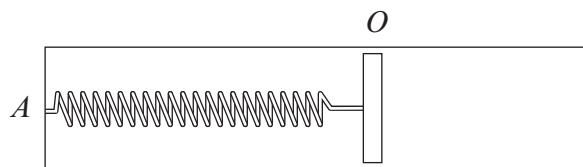
2. A particle, of mass  $8\text{ kg}$ , moves along the  $x$ -axis. At time  $t = 0$ , the particle is at  $O$  and its velocity is  $3\text{ ms}^{-1}$ . At time  $t\text{ s}$ , the velocity of the particle is  $v\text{ ms}^{-1}$  and it moves under the action of a propulsive force of magnitude  $4v\text{ N}$  and a resistive force of magnitude  $(4 - 16t)\text{ N}$ .

(a) Show that  $x$  satisfies the differential equation

$$2 \frac{d^2x}{dt^2} - \frac{dx}{dt} = 4t - 1 . \quad [3]$$

(b) Find an expression for  $x$  in terms of  $t$ . [12]

3. A piston of mass  $0.1\text{ kg}$  is free to slide inside a smooth cylinder whose axis is horizontal. One end of a light spring, of modulus of elasticity  $3.2\text{ N}$  and natural length  $0.5\text{ m}$ , is attached to the piston, and the other end is attached to a fixed point  $A$  along the line of the axis of the cylinder. Initially the piston is at rest at the point  $O$  and  $OA = 0.5\text{ m}$ .



The piston is given a velocity of  $0.8\text{ ms}^{-1}$  along the axis of the cylinder away from the point  $A$ .

(a) Show that the subsequent motion of the piston is Simple Harmonic.

State its centre and show that the period is  $\frac{\pi}{4}\text{ s}$ . [6]

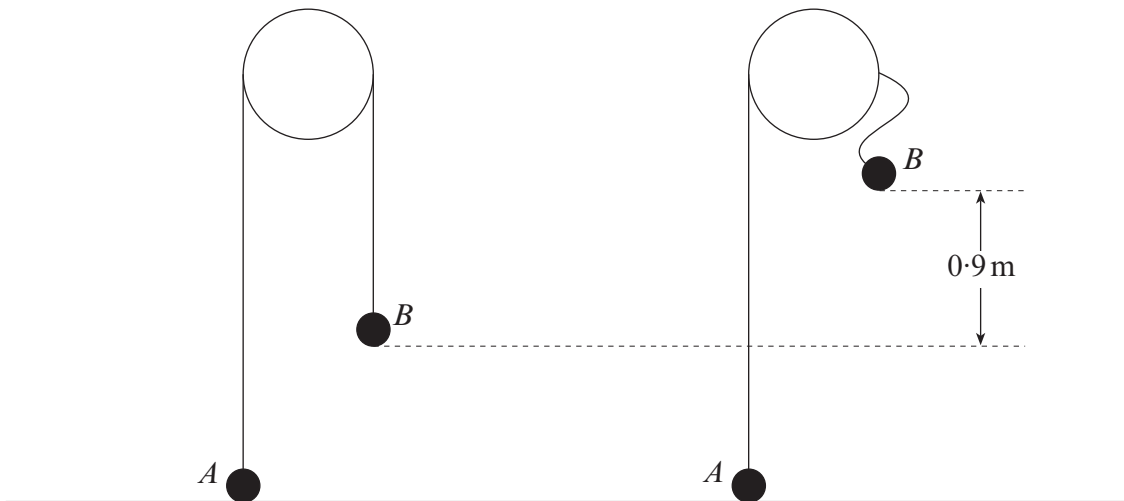
(b) Show that the amplitude of the motion is  $0.1\text{ m}$ . [2]

(c) Calculate the speed of the piston when it is  $0.08\text{ m}$  from  $O$ . [3]

(d) Find the maximum magnitude of the acceleration of the piston. [2]

(e) Calculate the time taken for the piston to reach a point  $0.05\text{ m}$  from  $O$  for the first time. [3]

4. A particle  $P$  moves along the  $x$ -axis. When the displacement of  $P$  from the origin  $O$  is  $x$  m, its acceleration is of magnitude  $\left(\frac{9}{2x^2}\right) \text{ms}^{-2}$  and is directed towards  $O$ .  
When  $x = \frac{3}{4}$ , the velocity of  $P$  is  $3 \text{ms}^{-1}$ . Find the speed of  $P$  when  $x = 2$  and the value of  $x$  when  $P$  comes to rest. [10]
5. The diagram shows two particles  $A$  and  $B$ , of masses  $4 \text{kg}$  and  $3 \text{kg}$  respectively, connected by a light inextensible string passing over a smooth light pulley fixed above a horizontal plane. Initially, the particle  $A$  is at rest on the plane and particle  $B$  hangs at a depth of  $1.0 \text{m}$  below the pulley.

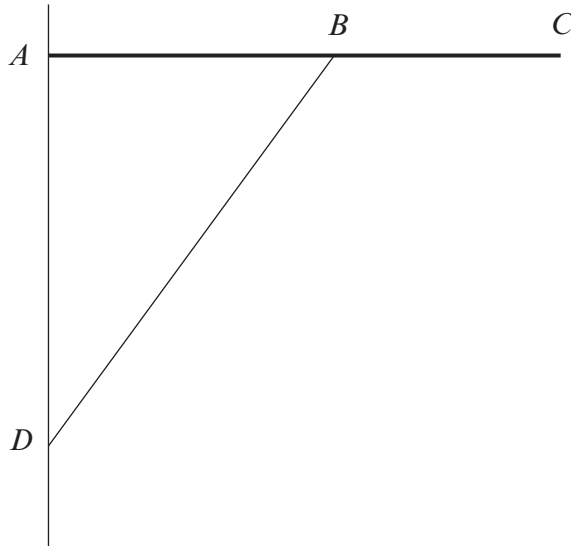


Particle  $B$  is then raised vertically a distance of  $0.9 \text{m}$  and released from rest from that position.

- (a) Calculate the speed of  $B$  immediately before the string tightens. [3]
- (b) Determine the speed with which  $A$  leaves the plane and the impulsive tension in the string immediately after the string tightens. [7]

**TURN OVER**

6. The diagram shows a uniform plank  $AC$ , of mass  $15\text{ kg}$  and length  $1.2\text{ m}$ , hinged to a vertical wall at  $A$ . The plank is supported in a horizontal position by a fixed light rod  $BD$ , where  $D$  is on the wall and  $B$  is the midpoint of  $AC$ . The length  $AD$  is  $0.8\text{ m}$ . A boy leans on the plank at  $C$  exerting a force of  $20\text{ N}$  vertically downwards.



- (a) Find the thrust in the rod  $BD$ . [5]
- (b) Calculate the magnitude and direction of the reaction at the hinge  $A$ . [8]