WJEC
CBAC

GCE AS/A level

981/01

MATHEMATICS M2
Mechanics 2

A.M. MONDAY, 13 June 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves along the x-axis and its velocity $v\,\text{ms}^{-1}$ at time t s is given by

$$v = 12\sin 3t - 8\cos 2t.$$

(a) Find an expression for the acceleration of the particle at time t s. [3]

(b) Given that at time $t = 0$ the particle is at the origin O, find an expression for the displacement of the particle from O at time t s. [5]

2. A particle of mass 0·5 kg is attached to one end of a light inextensible string of length 0·6 m. The other end of the string is fixed at a point O on a smooth horizontal surface. The particle moves on the surface in a circle with centre O, so that the string is taut and the angular velocity of the particle about O is 5 radians per second.

(a) Calculate the speed of the particle. [2]

(b) Find the tension in the string. [2]

3. A particle P, of mass 2 kg, is moving under the action of a force $\mathbf{F}$ N so that its velocity $\mathbf{v}\,\text{ms}^{-1}$ at time t s is given by

$$\mathbf{v} = 2\mathbf{i} + 6t\mathbf{j} + 4t^3\mathbf{k}.$$

(a) Find an expression for $\mathbf{F}$ at time t s. [3]

(b) Determine the value of $\mathbf{F}.\mathbf{v}$ when $t = 1$ and state the units of your answer. [4]

4. A car of mass 800 kg is travelling against a constant resistance to motion of 540 N.

(a) Find the power of the engine when the car is travelling on a level racing track at a constant speed of $60\,\text{ms}^{-1}$. [4]

(b) With the engine working at 32·4 kW and the resistance to motion unchanged, the car ascends a hill inclined at an angle α to the horizontal where $\sin\alpha = \frac{1}{16}$.

Find the acceleration of the car when its velocity is $15\,\text{ms}^{-1}$. [5]

5. A light elastic string, of natural length 1·6 m and modulus of elasticity 80 N, has one end attached to a fixed point A and the other end attached to a particle P, of mass 4 kg. Initially, P is held at a point 0·5 m vertically below the point A. The particle P is released from rest and allowed to fall.

(a) Calculate the tension in the string when the length of the string is 2 m. [2]

(b) Determine the speed of P when the length of the string is 2 m. [8]

6. A stone is thrown from the top of a vertical cliff, 100 m above sea level. The initial velocity of the stone is $6{\cdot}5\,\text{ms}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$.

(a) Find the time taken for the stone to reach the sea. Give your answer correct to two decimal places. [5]

(b) Calculate the horizontal distance from the bottom of the cliff to the point where the stone hits the sea. [2]

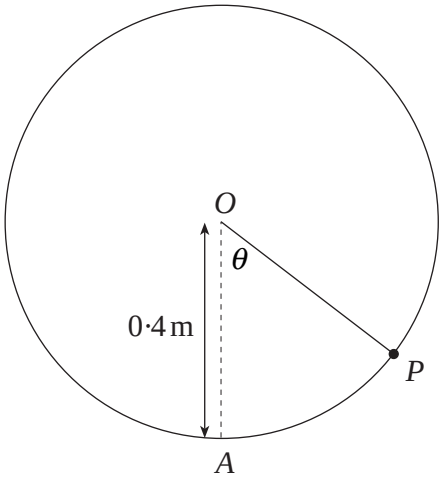
(c) Calculate the magnitude and direction of the velocity with which the stone hits the sea. [7]

7. At time t, the position vectors relative to a fixed origin O, of two particles A and B are given by $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k})$ and $\mathbf{OB} = 5\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} + t(3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$.

(a) Find the speed of particle A. [3]

(b) Show that the distance AB at time t is given by $AB^2 = 5t^2 - 30t + 211$. Determine the time at which the particles A and B are closest together. [7]

8. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 0·4 m to a fixed point O. Initially, P is projected from the point A, which is vertically below O, with a horizontal speed of $4\,\text{ms}^{-1}$.



(a) The speed of P when OP makes an angle θ with OA is $v\,\text{ms}^{-1}$.
Show that $v^2 = 8{\cdot}16 + 7{\cdot}84\cos\theta$. [4]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [3]

(d) Would your conclusion to *(c)* be different if the string was replaced by a light rigid rod? Justify your answer. [2]