GCE AS/A level

978/01

MATHEMATICS FP2

Further Pure Mathematics

A.M. WEDNESDAY, 22 June 2011

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S11-0978-01)

1. Using the substitution $u = \sqrt{x}$, evaluate the integral

$$\int_1^4 \frac{1}{(9+x)\sqrt{x}}\,\mathrm{d}x.$$

Give your answer correct to four decimal places. [5]

2. Find the general solution to the equation

$$\cos\theta + \cos 3\theta + \cos 5\theta = 0.$$ [7]

3. The piecewise function f is defined by

$$f(x) = -x^2 + 6x - 7 \qquad (x \leqslant 2),$$

$$f(x) = x^2 - 2x + 4 \qquad (x > 2).$$

(a) Determine whether or not f is continuous for all values of x. [2]

(b) Determine whether or not f is a strictly increasing function. [4]

(c) The interval [1, 3] is denoted by A. Determine $f(A)$. [3]

4. Given that $z = -1 + \mathrm{i}$,

(a) find the modulus and argument of z, [3]

(b) find the three cube roots of z in the form $x + \mathrm{i}y$, giving x and y correct to three decimal places, [7]

(c) find the smallest positive integer n for which z^n is a positive real number. [2]

5. *(a)* Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

and find a similar expression for $z^n + \dfrac{1}{z^n}$. [5]

(b) Hence by expanding $\left(z - \dfrac{1}{z}\right)^4$, show that

$$\sin^4\theta = a\cos 4\theta + b\cos 2\theta + c,$$

where a, b, c are constants whose values should be determined. [5]

6. The ellipse E has equation

$$2x^2 + 3y^2 - 4x + 12y + 8 = 0.$$

Find

(a) the coordinates of the centre of E, [3]

(b) the eccentricity of E, [4]

(c) the coordinates of the foci of E, [2]

(d) the equations of the directrices of E. [2]

7. *(a)* Differentiate the following integral with respect to x.

$$\int_0^x \sin(e^t)\,dt$$ [1]

(b) By putting $u = x^2$ and using the chain rule, differentiate the following integral with respect to x.

$$\int_0^{x^2} \sin(e^t)\,dt$$ [2]

8. The function f is defined by

$$f(x) = \frac{(x+1)^2}{(x-1)(x-2)}\ .$$

(a) Prove that $f(x)$ can be written in the form

$$1 - \frac{4}{x-1} + \frac{9}{x-2}\ .$$

Hence find expressions for $f'(x)$ and $f''(x)$. [7]

(b) Find the coordinates of the stationary points on the graph of f and classify each point as a maximum or minimum. [6]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f. [3]