WJEC CBAC

GCE AS/A level

977/01

MATHEMATICS FP1

Further Pure Mathematics

A.M. WEDNESDAY, 22 June 2011

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S11-0977-01)

1. Differentiate $\dfrac{1}{x^3}$ from first principles. [6]

2. Find an expression for the sum of the first n terms of the series whose rth term is $r(2r - 1)$. Simplify your answer as far as possible. [5]

3. Given that the complex number z and its complex conjugate $\bar{z}$ satisfy the equation

$$2\bar{z} + \mathrm{i}z = (1 + 2\mathrm{i})(2 - 3\mathrm{i}),$$

find z in the form $x + \mathrm{i}y$. [7]

4. *(a)* Show that the following matrix is singular.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & -1 & 7 \end{bmatrix}$$ [2]

(b) Given that the following system of equations is consistent,

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & -1 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ \lambda \end{bmatrix}$$

(i) find the value of λ,

(ii) find the general solution. [7]

5. Consider the polynomial equation

$$x^4 - 2x^3 - 2x^2 + 6x + 5 = 0.$$

Given that one of the roots of this equation is $2 + \mathrm{i}$, determine all the other roots of the equation. [7]

6. Use mathematical induction to prove that $6^n + 4$ is divisible by 10 for all positive integers n. [7]

7. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 1)$ followed by a reflection in the line $x + y = 0$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ [5]

(b) Find the equation of the image under T of the line $y = 2x - 1$. [5]

8. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(a) Evaluate $\mathbf{A}^2$ and show that

$$\mathbf{A}^2 = 5\mathbf{A} + 2\mathbf{I},$$

where $\mathbf{I}$ denotes the identity matrix. [4]

(b) Using the result in *(a)*, show that

$$\mathbf{A}^3 = \lambda\mathbf{A} + \mu\mathbf{I}$$

where λ, μ are constants to be determined. [3]

9. The roots of the following cubic equation are in geometric progression.

$$x^3 + fx^2 + gx + h = 0$$

Show that $g^3 = f^3h$. [7]

10. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{z^2}.$$

(a) Show that

$$u = \frac{x^2 - y^2}{\left(x^2 + y^2\right)^2}$$

and obtain an expression for v in terms of x and y. [3]

(b) The point P moves along the line L with equation $y = mx$.

(i) Show that the locus of Q is the line L' with equation of the form $v = m'u$ and find an expression for m' in terms of m.

(ii) Determine the values of m for which L and L' have the same gradient. [7]