ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_{1\cdot6}^{2} \frac{1}{9-x^3}\,dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. (a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sin\theta + 12\cos^2\theta = 6.$$ [6]

(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\cos(2x - 35°) = 0\cdot891.$$ [3]

(c) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\sin\phi + \cos\phi = 0.$$ [3]

3. The diagram below shows a sketch of the triangle ABC with $\sin A = \dfrac{3}{5}$, $\sin B = \dfrac{5}{13}$, $\sin C = \dfrac{56}{65}$, $AC = x\,cm$ and $BC = y\,cm$.
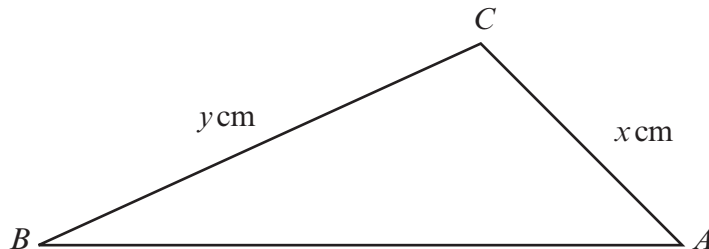
(a) Show that $y = 1\cdot56x$. [2]

(b) Given that the area of triangle ABC is $4\cdot2\,cm^2$, find the value of x and the value of y. [5]

4. (a) The sum of the first fifteen terms of an arithmetic series is 780. The sum of the second, fourth and tenth terms of the series is 100. Find the first term and the common difference of the series. [5]

(b) The pth term of another arithmetic series is 1023. The $(p + 4)$th term of this series is 1059. Find the $(p + 7)$th term of the series. [3]

5. *(a)* A geometric series has first term *a* and common ratio *r*. Prove that the sum of the first *n* terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$ [3]

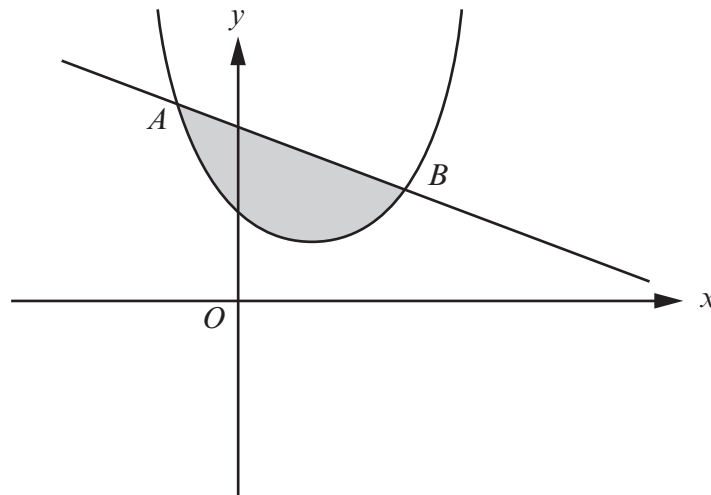*(b)* The sum to infinity of a geometric series is equal to four times the first term of the series.

(i) Find the value of the common ratio of the series.

(ii) Given that the sum of the first two terms of the series is 35, find the sum of the first nine terms of the series. Give your answer correct to the nearest whole number.

[6]

6. *(a)* Find $\displaystyle\int \left(\sqrt[3]{x} - \frac{2}{x^{\frac{3}{4}}} \right) dx.$ [2]

(b)

The diagram shows a sketch of the curve $y = x^2 - 4x + 6$ and the line $y = -x + 10$. The curve and the line intersect at the points *A* and *B*.

(i) Showing your working, find the coordinates of *A* and *B*.

(ii) Find the area of the shaded region.

[11]

TURN OVER

7. *(a)* Given that $x > 0$, $y > 0$, show that

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y.$$

[3]

(b) Express

$$\frac{1}{2}\log_a x^8 - \log_a 4x + 3\log_a \frac{2}{x}$$

as a single logarithm in its simplest form. [4]

8. The circle C_1 has centre A and equation
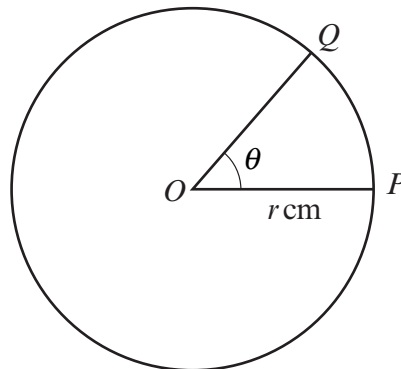
$$x^2 + y^2 - 4x + 2y - 20 = 0.$$

(a) Find the coordinates of A and the radius of C_1. [3]

(b) A second circle C_2 has centre $B(8, -9)$ and radius 15.

  (i) Show that C_1 and C_2 touch, justifying your answer.

  (ii) Given that the circles touch at the point $P(-1, 3)$, find the equation of the common tangent.

[7]

9.

The diagram shows two points P and Q on a circle with centre O. The radius of the circle is r cm and $P\hat{O}Q = \theta$ radians. The length of the arc PQ is $7 \cdot 6$ cm and the area of the sector POQ is $36 \cdot 1$ cm^2. Find the values of r and θ. [5]