GCE AS/A level

985/01

MATHEMATICS S3
STATISTICS 3

P.M. MONDAY, 16 June 2008

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A bag contains six £2 coins and four £1 coins. A random sample of three coins is selected from the bag without replacement.

(a) Find the sampling distribution of the total value of these three coins. [7]

(b) Verify that the expected value of this total is three times the mean value of the ten coins in the bag. [3]

2. David is a candidate in the local elections and he wishes to estimate the proportion, p, of the electorate supporting him. He questions a random sample of 1200 electors and he finds that 630 of these electors support him.

(a) Calculate

(i) an unbiased estimate of p,

(ii) the estimated standard error of this estimate. [3]

(b) Calculate an approximate 95% confidence interval for p. [3]

(c) David had claimed beforehand that he had the support of 60% of the electorate. State, with a reason, whether or not your confidence interval supports this claim. [2]

3. A farmer claims that the mean weight of apples grown in his orchard is 300 grams. His wife claims that the mean weight is less than this. To investigate these two claims, they agree to weigh a random sample of 12 apples. The results, in grams, are as follows.

286 294 305 277 289 312 301 284 281 315 289 295

You may assume that these weights form a random sample from a normal distribution.

(a) Calculate unbiased estimates of the mean and variance of the weights of apples grown in the orchard. [4]

(b) Stating your hypotheses clearly, test the claims using a significance level of

(i) 5%,

(ii) 10%. [9]

4. Ann cycles to school. She records the time taken, x minutes, on each of 75 days and she finds that

$$\sum x = 1815, \ \sum x^2 = 44\,213.$$

(a) Calculate an approximate 90% confidence interval for the mean time taken by Ann to cycle to school. [8]

(b) State, with a reason, whether or not your method required the assumption that the times are normally distributed. [1]

5. A zoologist wishes to determine whether or not the mean weights of two similar breeds of dog are equal. He obtains random samples of 60 dogs of each breed and he determines the weight, x kg, of each dog of breed A and the weight, y kg, of each dog of breed B. His results are summarised below.

$$\sum x = 1506,\ \sum x^2 = 38\,124,\ \sum y = 1530,\ \sum y^2 = 39\,327.$$

(a) State suitable hypotheses. [1]

(b) Calculate, approximately, the p-value of these results. Interpret its value in context. [11]

6. The continuous random variable X has probability density function f where

$$f(x) = 1 + \lambda x, \qquad \text{for } -\frac{1}{2} \leqslant x \leqslant \frac{1}{2},$$

$$f(x) = 0, \qquad \text{otherwise,}$$

and λ is an unknown parameter whose value lies between –2 and 2.

(a) Show that

$$P(X>0) = \frac{1}{2} + \frac{\lambda}{8}.$$ [2]

(b) In order to estimate the value of λ, n independent observations are made on X. Let Y denote the number of these observations which are positive.

(i) Show that

$$U = \frac{8Y}{n} - 4$$

is an unbiased estimator for λ.

(ii) Obtain an expression, in terms of n and λ, for the standard error of U. [9]

7. The length, y cm, of a wire is related to its temperature, x°C, by the equation $y = \alpha + \beta x$. The values of x can be controlled exactly whereas the measured values of y are subject to independent normally distributed errors with mean zero and standard deviation 0·15 cm. The following results were obtained for a particular wire.

Temperature (x°C)	20	30	40	50	60	70
Measured length (y cm)	82·3	83·9	85·3	86·8	88·6	90·1

[You are given that $\sum x = 270$, $\sum y = 517$, $\sum x^2 = 13\,900$, $\sum xy = 23\,538$]

(a) Calculate least squares estimates for α and β. [6]

(b) Calculate a 99% confidence inverval for the actual length of the wire when its temperature is 60°C. [6]