WJEC
CBAC

GCE AS/A level

982/01

MATHEMATICS M3
Mechanics 3

P.M. MONDAY, 16 June 2008

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Take g as 9·8 ms $^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A body, of mass 35 kg, is projected along a rough horizontal floor with an initial velocity of $28\,\text{ms}^{-1}$. The coefficient of friction between the body and the floor is $\frac{1}{7}$. In addition to friction, the body experiences a resistive force of $0{\cdot}7v$ N, where $v\,\text{ms}^{-1}$ is the velocity of the body at time t s.

(a) Show that v satisfies the equation $50\frac{\mathrm{d}v}{\mathrm{d}t} + v + 70 = 0$. [5]

(b) Find the time taken for the body to come to rest. [7]

2. *(a)* An experimental vehicle, of mass 2 kg, is designed such that, after crossing the starting line at a speed of $7\,\text{ms}^{-1}$, it moves in a straight horizontal line under a propulsive force of magnitude $0{\cdot}8x$ N and a resistive force of magnitude $1{\cdot}2v$ N, where x m is the distance from the starting line and $v\,\text{ms}^{-1}$ is the speed of the vehicle at time t s.

(i) Show that x satisfies the differential equation

$$5\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} - 2x = 0\ .$$ [3]

(ii) By solving the above equation, find an expression for x in terms of t. [6]

(iii) Show that x increases with t. [2]

(b) Find the general solution of the differential equation

$$5\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} - 2x = 20t - 70\,.$$ [5]

3. A particle is moving in a straight line with Simple Harmonic Motion with centre O. When the particle is 3 m from O its speed is $5\,\text{ms}^{-1}$ and when it is 4 m from O its speed is $3{\cdot}75\,\text{ms}^{-1}$.

(a) Show that the amplitude of the motion is 5 m and find the period of the motion. [8]

(b) Find, correct to two decimal places, the distance of the particle from O 2 s after the particle passes through O. [3]

(c) How long after passing through O is the speed of the particle two-fifths of its maximum speed? Give your answer correct to two decimal places. [5]

4. A particle is projected vertically upwards with initial speed $15\,\text{ms}^{-1}$ from a point O. In the subsequent motion,

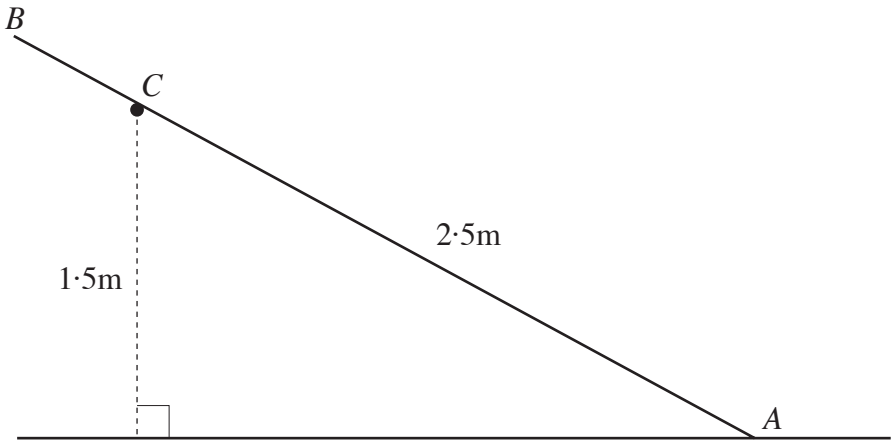
$$a + \frac{v^2}{90} + 10 = 0\ ,$$

where $a\,\text{ms}^{-2}$ is the acceleration and $v\,\text{ms}^{-1}$ is the speed when the height of the particle is x m above O.

(a) Find an expression for x in terms of v. [7]

(b) Find, correct to two decimal places, the greatest height of the particle above O. [2]

5. A small smooth peg C is fixed at a height of 1·5 m above the horizontal ground. A uniform rod AB, of mass 20 kg and length 3 m, rests on C with the end A on the rough ground as shown in the diagram.

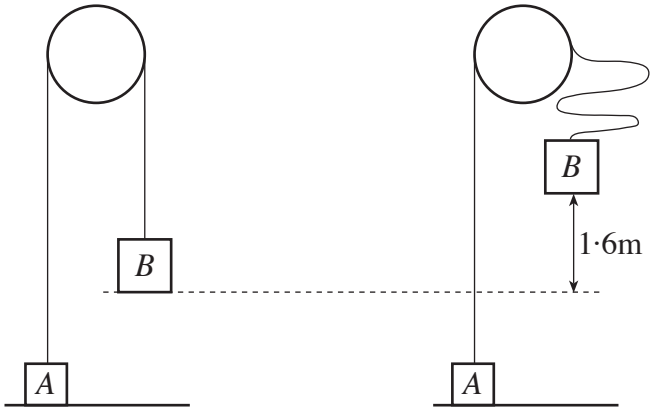


The rod AB is at rest in limiting equilibrium with $AC = 2{\cdot}5$ m.

(a) Calculate the magnitude of the reaction of C on the rod AB. [5]

(b) Find the coefficient of friction between the rod AB and the rough ground, giving your answer correct to two decimal places. [8]

6. A particle A, of mass 7 kg, rests on a horizontal table. It is attached to one end of a light inextensible string which passes over a smooth light pulley. The other end of the string is attached to another particle B, of mass 3 kg. Initially, the particles are held at rest with the string just taut. Particle B is raised vertically through a distance of 1·6 m and released from rest.



Find the speed with which particle A begins to rise, and the impulsive tension in the string. [9]