GCE AS/A level

979/01

MATHEMATICS FP3

Further Pure Mathematics

A.M. WEDNESDAY, 18 June 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S08-979-01)

1. *(a)* Draw sketches of suitable graphs to show that the equation

$$\cosh x = 1 + \sin x$$

has two roots, one of which is positive. [3]

(b) Use the Newton-Raphson method with a starting value $x_0 = 1{\cdot}5$ to find the value of the positive root correct to four decimal places. [6]

2. Use the substitution $x = 1 + \sinh\theta$ to evaluate the integral

$$\int_1^2 \sqrt{x^2 - 2x + 2}\ \mathrm{d}x.$$

Give your answer correct to two decimal places. [8]

3. The Taylor series of $f(x)$ about $x = a$ is

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + \ldots$$

(a) Find the first three terms of the Taylor series for $\dfrac{1}{\sqrt{x}}$ about $x = 1$. [4]

(b) Putting $x = \dfrac{8}{9}$, use your result to find a rational approximation for $\sqrt{2}$. [4]

4. *(a)* Using appropriate definitions in terms of exponential functions, show that

$$\operatorname{sech}^2 x \equiv 1 - \tanh^2 x.$$ [4]

(b) Solve the equation

$$5\operatorname{sech}^2 x = 11 - 13\tanh x$$

giving your answer as a natural logarithm. [8]

5. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_1^2 x(\ln x)^n \mathrm{d}x\ .$$

(a) Show that, for $n \geqslant 1$,

$$I_n = 2(\ln 2)^n - \frac{n}{2}I_{n-1}\ .$$ [5]

(b) Evaluate I_2, giving your answer correct to three decimal places. [5]

6. *(a)* The curve C has parametric equations

$$x = \cos^3\theta,\ y = \sin^3\theta,\quad 0 \leqslant \theta \leqslant \frac{\pi}{2}\ .$$

Show that

$$\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2} = \frac{3}{2}\sin 2\theta\ .$$ [5]

(b) (i) Find the arc length of C.

(ii) The curve C is rotated through 360° about the x-axis. Show that the curved surface area of the solid of revolution generated is given by

$$6\pi\int_0^{\frac{\pi}{2}} \sin^4\theta\cos\theta\,\mathrm{d}\theta.$$

Hence find this curved surface area. [9]

TURN OVER FOR QUESTION 7

7.

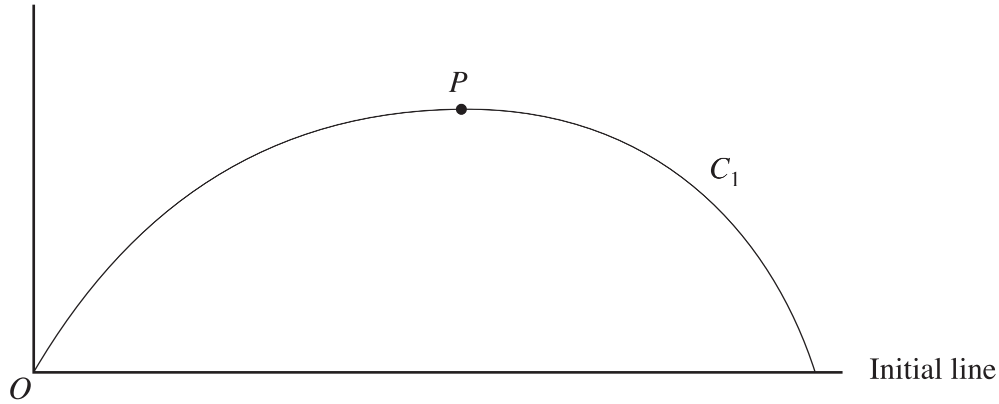

Figure 1

Figure 1 above shows a sketch of the curve C_1 with polar equation

$$r = 1 - \theta, \quad 0 \leqslant \theta \leqslant 1.$$

(a) (i) Given that P is the point on C_1 at which the tangent to C_1 is parallel to the initial line, show that the θ coordinate of P satisfies the equation

$$\theta + \tan\theta = 1.$$

(ii) Show that the area of the region enclosed by C_1 and the initial line is $\frac{1}{6}$. [6]

(b)

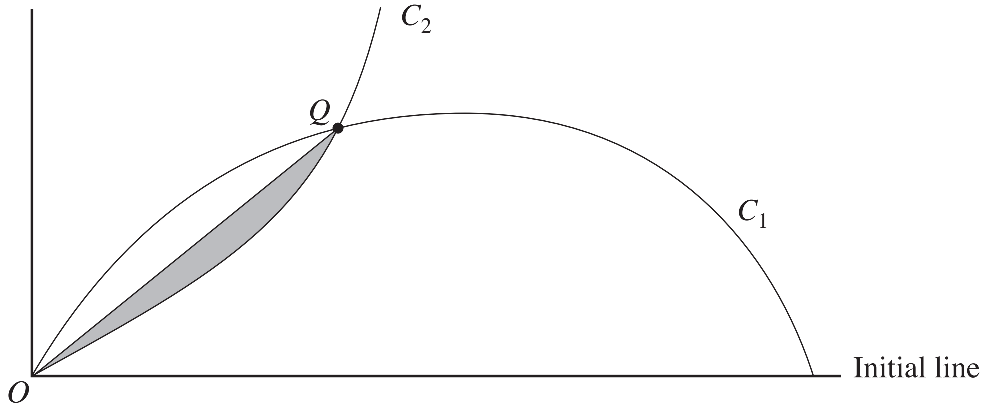

Figure 2

Figure 2 above shows a sketch of the curve C_1 and part of the curve C_2 with polar equation

$$r = 2\theta^2, \quad 0 \leqslant \theta \leqslant 1.$$

(i) Find the polar coordinates of Q, the point of intersection of C_1 and C_2.

(ii) Find the area of the region, shaded in Figure 2, enclosed by C_2 and the straight line OQ. [8]