GCE AS/A level

978/01

MATHEMATICS FP2
Further Pure Mathematics

A.M. WEDNESDAY, 18 June 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. For each of the following functions state, with a reason, whether it is even, odd or neither even nor odd.

(a) $\dfrac{x}{x^2+1}$ [2]

(b) $e^x + 1$ [2]

2. The function f is defined by

$$f(x) = 1 + ax^3 \quad \text{for } x < 2,$$
$$f(x) = bx^2 - 3 \quad \text{for } x \geqslant 2.$$

Given that both f and its derivative f' are continuous at $x = 2$, find the values of the constants a and b. [6]

3. *(a)* Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{3}} \frac{x\,dx}{(9+x^4)},$$

giving your answer in the form $\dfrac{\pi}{k}$, where k is an integer. [5]

(b) Evaluate the integral

$$\int_0^1 \frac{dx}{\sqrt{25-9x^2}}.$$ [4]

4. Consider the equation

$$2\sin\theta + 3\cos\theta = 1.$$

(a) Putting $t = \tan\left(\dfrac{\theta}{2}\right)$, show that

$$2t^2 - 2t - 1 = 0.$$ [3]

(b) Hence find the general solution, in radians, of the above trigonometric equation. [5]

5. *(a)* Show that the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ is

$$y + px = ap(2 + p^2).$$ [4]

(b) This normal meets the x-axis at Q and the mid-point of PQ is R.

(i) Find the coordinates of R.

(ii) The locus of R as p varies is a parabola. Find the equation of this parabola and the coordinates of its focus. [8]

6. *(a)* Given that

$$z = \cos\theta + \mathrm{i}\sin\theta,$$

show that

$$z^n - z^{-n} = 2\mathrm{i}\sin n\theta. \qquad [3]$$

(b) Expand $(z - z^{-1})^3$ and hence show that

$$\sin^3\theta = a\sin 3\theta + b\sin\theta$$

where the values of the constants a and b are to be determined. [5]

7. The function f is defined by

$$f(x) = \frac{5 - 3x}{(x-1)(x-3)}.$$

(a) Express $f(x)$ in partial fractions. [3]

(b) Obtain an expression for $f'(x)$ and hence show that there are no stationary points on the graph of f. [3]

(c) Sketch the graph of f. State

(i) the coordinates of all the points of intersection of the graph and the coordinate axes,

(ii) the equations of all the asymptotes. [7]

(d) Find $f^{-1}(A)$ where A is the interval $(0, 1)$. [5]

8. *(a)* Find the modulus and argument of the complex number $8\mathrm{i}$. [2]

(b) Hence find the three cube roots of $8\mathrm{i}$, giving your answers in the form $x + \mathrm{i}y$. [8]