GCE AS/A level

977/01

MATHEMATICS FP1

Further Pure Mathematics

A.M. THURSDAY, 12 June 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$S_n = \sum_{r=1}^{n} r^2\left(r+1\right),$$

obtain an expression for S_n in terms of n, giving your answer as a product of linear factors. [6]

2. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$ [6]

(b) **Hence** solve the equations

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 5 \end{bmatrix}.$$ [2]

3. Given that

$$z = \left(2 - \mathrm{i}\right)^2 + \frac{\left(7 - 4\mathrm{i}\right)}{\left(2 + \mathrm{i}\right)} - 8,$$

(a) express z in the form $x + \mathrm{i}y$, [5]

(b) find the modulus and argument of z. [3]

4. *(a)* Use reduction to echelon form to find the value of k for which the following equations are consistent.

$$\begin{aligned} 2x + y + 3z &= 5 \\ x - 2y + 2z &= 6 \\ 4x + 7y + 5z &= k \end{aligned}$$ [5]

(b) For this value of k, find the general solution to these equations. [3]

5. Use mathematical induction to show that $7^n + 5$ is divisible by 6 for all positive integers n. [7]

6. *(a)* The roots of the cubic equation

$$ax^3 + bx^2 + cx + d = 0$$

are the first three terms of a geometric series with common ratio 2. Show that

$$4bc - 49ad = 0.$$ [7]

(b) Given that

$$8x^3 - 42x^2 + 63x - 27 = 0$$

is such an equation, find its three roots. [3]

7. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [3]

(b) Find the coordinates of the fixed point of T. [4]

(c) Find the equation of the image under T of the line $y = 2x - 1$. [5]

8. The function f is defined on the domain $\left(0, \frac{\pi}{2}\right)$ by

$$f(x) = x^{\cos x}.$$

(a) Obtain an expression for $f'(x)$ in terms of x. [4]

(b) The x-coordinate of the maximum point on the graph of f is denoted by α.

(i) Show that

$$\alpha \ln\alpha \tan\alpha = 1.$$

(ii) Show that α lies between 1·27 and 1·28. [4]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{z+1}.$$

(a) By first writing

$$z + 1 = \frac{1}{w},$$

show that

$$x + 1 = \frac{u}{u^2 + v^2}$$

and find an expression for y in terms of u and v. [4]

(b) The point P moves along the circle $(x + 1)^2 + y^2 = 4$. Find the equation of the locus of Q. [4]