GCE AS/A level

976/01

MATHEMATICS C4
Pure Mathematics

A.M. THURSDAY, 12 June 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$f(x) = \frac{1}{x^2(2x-1)} \quad ,$$

(a) express $f(x)$ in partial fractions, [4]

(b) find $\int f(x)\mathrm{d}x$. [3]

2. Find the equation of the normal to the curve

$$x^2 + xy + 2y^2 = 8$$

at the point (–3, 1). [5]

3. *(a)* Express $3\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Find all values of x between 0° and 360° satisfying

$$3\cos x + 2\sin x = 1.$$ [3]

4. The region R is bounded by the curve $y = x + \frac{3}{\sqrt{x}}$, the x-axis and the lines $x = 1$, $x = 4$. Find the volume generated when R is rotated through four right-angles about the x-axis. [7]

5. The parametric equations of the curve C are $x = 4\sin t$, $y = \cos 2t$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, simplifying your answer as much as possible. [6]

(b) Show that the equation of the tangent to C at the point P with parameter p is

$$x\sin p + y = 1 + 2\sin^2 p.$$ [3]

6. *(a)* Find $\int (3x+1)\, e^{2x}\mathrm{d}x$. [4]

(b) Use the substitution $x = 3\sin\theta$ to show that

$$\int_{1\cdot 5}^{3} \sqrt{9-x^2}\,\mathrm{d}x = \int_a^b k\cos^2\theta\,\mathrm{d}\theta \quad ,$$

where the values of the constants a, b and k are to be found.

Hence evaluate $\int_{1\cdot 5}^{3} \sqrt{9-x^2}\,\mathrm{d}x$. [8]

7. A neglected large lawn contains a certain type of weed. The area of the lawn covered by the weed at time t years is $W\,\mathrm{m}^2$. The rate of increase of W is directly proportional to W.

(a) Write down a differential equation that is satisfied by W. [1]

(b) The area of the lawn covered by the weed initially is $0{\cdot}10\,\mathrm{m}^2$ and one year later the area covered is $2{\cdot}01\,\mathrm{m}^2$. Find an expression for W in terms of t. [6]

8. The position vectors of the points A and B are given by

$$\mathbf{a} = 4\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

(a) (i) Write down the vector $\mathbf{AB}$.

(ii) Find the vector equation of the line AB. [3]

The vector equation of the line L is

$$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k}).$$

(b) Given that the lines AB and L intersect, find the position vector of the point of intersection. [5]

(c) Find the angle between the line AB and the line L. [5]

9. Expand $\dfrac{1+3x}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^2. State the range of x for which the expansion is valid. [5]

10. Prove by contradiction the following proposition.

When x is real and positive,

$$x + \frac{49}{x} \geqslant 14 \ .$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$x + \frac{49}{x} < 14 \ .$$ [4]