WJEC CBAC

GCE AS/A level

975/01

MATHEMATICS C3
PURE MATHEMATICS

A.M. FRIDAY, 23 May 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S08-975-01)

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_0^1 \sqrt{1+\mathrm{e}^x}\,\mathrm{d}x\,.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\tan 2\theta \equiv \frac{2\tan\theta}{1+\tan^2\theta}$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\sec^2\theta = 8 - \tan\theta.$$ [6]

3. Given that

$$x^2 + x\sin y + y^3 = \pi^3 + 1,$$

find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, \pi)$. [4]

4. Given that $x = \ln t$, $y = \mathrm{e}^{2t}$,

(a) show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2t\mathrm{e}^{2t}$, [4]

(b) find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t, simplifying your answer. [4]

5. *(a)* Show that $f(x) = \sin^{-1}x - 2x^{\frac{3}{2}} + 1$ has a stationary value when x satisfies

$$9x^3 - 9x + 1 = 0.$$ [4]

(b) Show that the equation

$$9x^3 - 9x + 1 = 0$$

has a root α between 0 and 0·2.

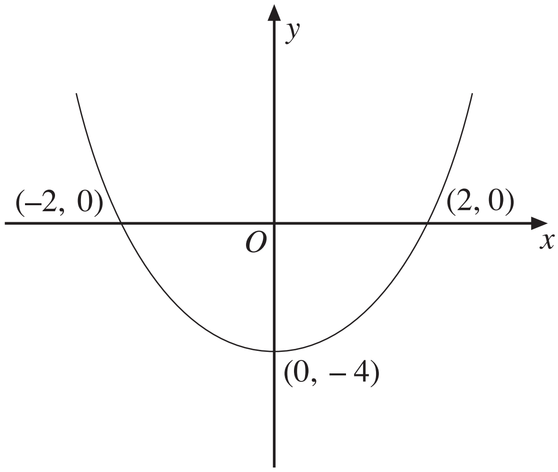
The recurrence relation

$$x_{n+1} = x_n^3 + \frac{1}{9}$$

with $x_0 = 0{\cdot}1$ can be used to find α. Find and record the values of x_1, x_2, x_3.

Write down the value of x_3 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

6. *(a)* The diagram shows the graph of $y = f(x)$. The graph has a stationary point at (0, –4) and it intersects the x-axis at the points (–2, 0) and (2, 0).



Sketch the graph of $y = 3f(x - 1)$, indicating the coordinates of the stationary point and of the points where the graph crosses the x-axis. [3]

(b) Solve $3|x| + 1 = 2 - |x|$. [2]

(c) Solve $|2x - 9| > 3$. [4]

7. *(a)* Find (i) $\int \sin 3x \, \mathrm{d}x$, (ii) $\int \frac{2}{3x+5} \, \mathrm{d}x$, (iii) $\int e^{3x+4} \mathrm{d}x$. [6]

(b) Evaluate $\int_0^1 \frac{1}{(2x+1)^4} \, \mathrm{d}x$. [4]

8. Differentiate *(a)* $\cot 2x$, *(b)* $x^2 \ln x$, *(c)* $\frac{x^2+1}{x^2-2}$,

simplifying your answers wherever possible. [2], [2], [3]

TURN OVER

9. The function f has domain $x \leqslant -1$ and is defined by

$$f(x) = (x + 1)^2 - 2.$$

(a) Find the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. State the domain and range of f^{-1}. [6]

10. The function f has domain $(-\infty, \infty)$ and is defined by

$$f(x) = 2e^x.$$

The function g has domain $[1, \infty)$ and is defined by

$$g(x) = 3 \ln x.$$

(a) Explain why $gf(-1)$ does not exist. [2]

(b) Find in its simplest form an expression for $fg(x)$. State the domain and range of fg. [5]