GCE AS/A level

974/01

MATHEMATICS C2
Pure Mathematics

A.M. THURSDAY, 15 May 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1.   Use the Trapezium Rule with **four** ordinates to find an approximate value for the integral

$$\int_0^{0.6} \left(1 + x^2\right)^{\frac{3}{2}} \, dx.$$

Show your working and give your answer correct to three decimal places.　　　　　　[4]

2.   *(a)*   Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\sin\theta = 3\cos\theta.$$
　　　　　　[3]

(b)   Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\cos 3x = 0.9.$$
　　　　　　[4]

(c)   Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sin^2\theta - 4\cos^2\theta = 8\sin\theta.$$
　　　　　　[5]

3.   The diagram below shows the triangle ABC with $AB = x$ cm, $AC = (x + 4)$ cm and $B\widehat{A}C = 150°$.
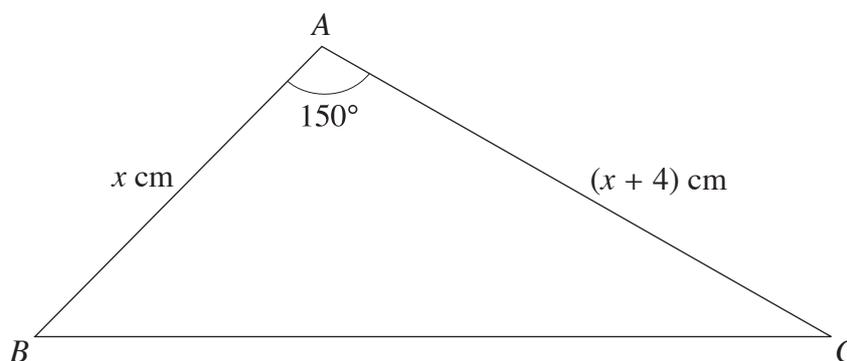
Given that the area of the triangle ABC is $15\,\text{cm}^2$,

(a)   find the value of x,　　　　　　[3]

(b)   find the length of BC correct to one decimal place.　　　　　　[2]

4. *(a)* An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] .$$

[3]

(b) The sum of the first ten terms of an arithmetic series is 320. The sum of the twelfth and sixteenth terms of the series is 166. Find the first term and the common difference of the series. [5]

5. A geometric series has first term a and common ratio r. The sum of the first two terms of the geometric series is 7·2. The sum to infinity of the series is 20. Given that r is positive, find the values of r and a. [6]

6. *(a)* Find $\displaystyle\int\left(5\sqrt{x} - \frac{4}{x^{\frac{2}{3}}}\right)\,dx$. [2]

(b)
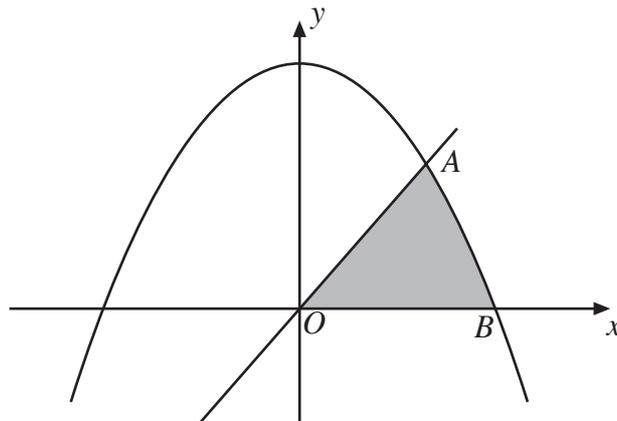
The diagram shows a sketch of the curve $y = 4 - x^2$ and the line $y = 3x$. The curve and the line intersect at the point A in the first quadrant and the curve intersects the positive x-axis at the point B.

(i) Showing your working, find the coordinates of A and the coordinates of B.

(ii) Find the area of the shaded region.

[12]

TURN OVER

7. *(a)* Given that $x > 0$, show that

$$\log_a x^n = n \log_a x .$$ [3]

(b) Solve the equation

$$\log_a(3x + 4) - \log_a x = 3 \log_a 2 .$$ [4]

(c) Solve the equation

$$4^{3y+2} = 7 ,$$

giving your answer correct to three decimal places. [3]

8. The circle C has centre A and radius r. The points $P(1, -4)$ and $Q(9, 10)$ are at either end of a diameter of C.
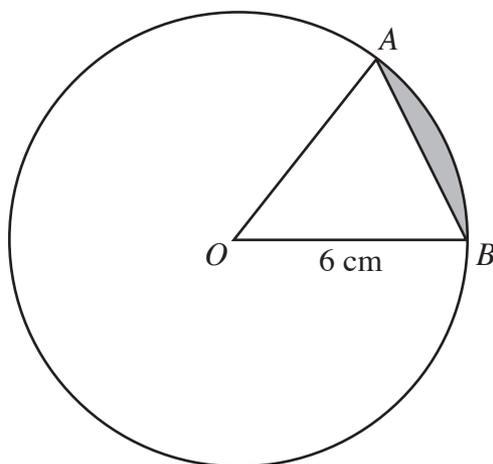
(a) (i) Write down the coordinates of A.

   (ii) Show that $r = \sqrt{65}$.

   (iii) Write down the equation of C. [4]

(b) Verify that the point $R(4, 11)$ lies on C. [2]

(c) Find $Q\hat{P}R$. [3]

9.

The diagram shows two points A and B on a circle with centre O and radius $6\,$cm. The length of the **arc** AB is $5{\cdot}4\,$cm.

(a) Show that the area of the **sector** AOB is $16{\cdot}2\,$cm^2. [4]

(b) Find the area of the shaded region, giving your answer correct to one decimal place. [3]