WJEC CBAC

General Certificate of Education
Advanced Subsidiary/Advanced

977/01

MATHEMATICS FP1
Further Pure Mathematics

P.M. TUESDAY, 22 January 2008
($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Solve the following equations by reduction to echelon form.

$$\begin{aligned} x + 3y + 2z &= 14 \\ 2x + y + z &= 7 \\ 3x + 2y - z &= 7 \end{aligned}$$ [7]

2. The roots of the equation

$$x^3 - x^2 + 3x + 5 = 0$$

are denoted by α, β, γ.

(a) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = -5.$$

Deduce that exactly one of the roots of the above cubic equation is real. [6]

(b) Given that one of the roots is $1 - 2\mathrm{i}$, find the other two roots. [3]

3. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & \lambda & 2 \end{bmatrix} .$$

(a) Find the value of λ for which **A** is singular. [3]

(b) Given that $\lambda = 4$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse of **A**. [6]

4. *(a)* Express

$$\frac{2}{\left(4x^2 - 1\right)}$$

in partial fractions. [4]

(b) Given that

$$S_n = \sum_{r=1}^{n} \frac{2}{\left(4r^2 - 1\right)} ,$$

obtain, in its simplest form, an expression for S_n in terms of n. [3]

5. The transformation T in the plane consists of a translation in which the point (x, y) is transformed to the point $(x + a, y + b)$, followed by a reflection in the line $y = x$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & b \\ 1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}.$$ [3]

(b) Given that $a + b = 0$,

(i) determine the set of fixed points of T,

(ii) describe, in words, the single transformation that is equivalent to T followed by T. [7]

6. *(a)* Find the modulus and argument of $(3 + 2\mathrm{i})^2$. [4]

(b) The complex numbers u, v and w are related by the equation

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{w}.$$

Given that $v = 2 + \mathrm{i}$ and $w = 1 - 2\mathrm{i}$, find u in the form $x + \mathrm{i}y$. [6]

7. Use mathematical induction to show that

$$\sum_{r=1}^{n} r \times 2^r = 2^{n+1}(n-1) + 2$$

for all positive integers n. [8]

8. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$|\, z - 1 \,| = \sqrt{2}\ |\, z - \mathrm{i} \,|,$$

show that the locus of P is a circle, and find its radius and the coordinates of its centre. [7]

9. Given that

$$f(x) = x^{-\sqrt{x}} \text{ for } x > 0,$$

(a) obtain an expression for $f'(x)$, [4]

(b) find the x-coordinate of the stationary point on the graph of f and determine whether this point is a maximum or a minimum. [4]