General Certificate of Education
Advanced Subsidiary/Advanced

974/01

MATHEMATICS C2
Pure Mathematics

P.M. WEDNESDAY, 9 January 2008
($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^1 \frac{1}{\sqrt{2+x^3}}\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$12\sin^2\theta - 5\cos\theta = 9.$$ [6]

(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\sin(3x + 15°) = 0{\cdot}5.$$ [4]

3. *(a)* An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$ [3]

(b) Find an expression, in terms of n, for the sum of the first n terms of the arithmetic series

$$1 + 3 + 5 + \dots .$$

Simplify your answer. [2]

(c) The twentieth term of an arithmetic series is 98 and the sum of the first twenty terms of the series is 1010. Find the first term and the common difference of the series. [4]

4. A geometric series has first term a and common ratio r. The fifth term of the geometric series is 135 and the eighth term is 5.

(a) Show that $r = \frac{1}{3}$ and find the value a. [5]

(b) Find the sum to infinity of the series. [2]

5. In triangle ABC, $AB = 6$ cm, $BC = 13$ cm and $CA = 9$ cm.

(a) Find the value of $\cos B\hat{A}C$ as a fraction in its lowest terms. [3]

(b) Show that the area of triangle ABC is $4\sqrt{35}\,\text{cm}^2$. [3]

6. *(a)* Given that $x > 0$, $y > 0$, show that

$$\log_a \frac{x}{y} = \log_a x - \log_a y. \qquad [3]$$

(b) (i) Solve the equation

$$3^{2x-1} = 11,$$

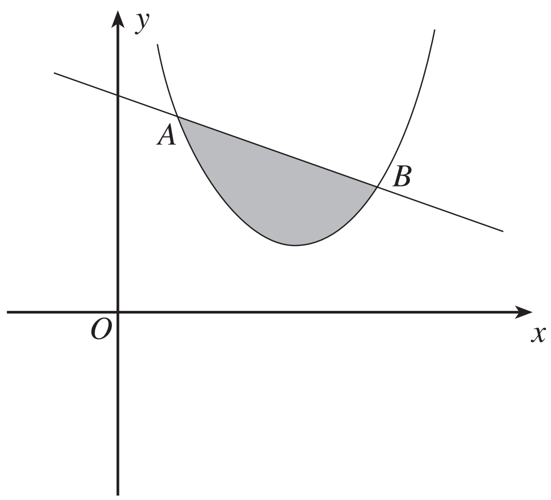
giving your answer correct to three decimal places.

(ii) Express $\frac{3}{2}\log_a 16 + \log_a 6 - 2\log_a 12$ as a single logarithm in its simplest form. [7]

7. *(a)* Find $\displaystyle\int\left(4x^{\frac{2}{3}} - \frac{7}{\sqrt{x}}\right)\mathrm{d}x.$

[2]

(b)



The diagram shows a sketch of the curve $y = x^2 - 6x + 11$ and the line $y = -x + 7$. The curve and the line intersect at the points A and B.

(i) Showing your working, find the coordinates of A and B.

(ii) Find the area of the shaded region. [11]

8. The circle C has centre A and equation

$$x^2 + y^2 - 4x + 6y - 12 = 0.$$

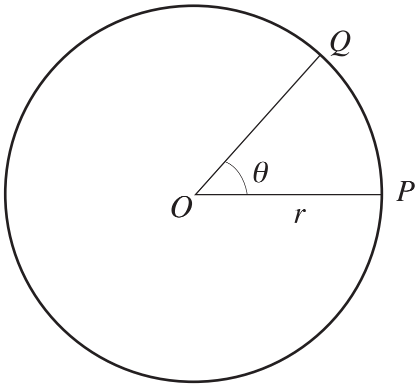
(a) Find the coordinates of A and the radius of C. [3]

(b) The point P has coordinates (5, 1) and lies on C. Find the equation of the tangent to C at P. [4]

(c) The line L has equation $y = x + 3$. Show that L and C do not intersect. [4]

TURN OVER.

9.



The diagram shows two points P and Q on a circle with centre O. The radius of the circle is r cm and $P\hat{O}Q = \theta$ radians. The length of the arc PQ is 6 cm and the area of the sector POQ is 22·5 cm^2. Find the values of r and θ. [5]