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MATHEMATICS S3

STATISTICS 3

A.M. MONDAY, 18 June 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Six cards are numbered 1, 2, 2, 3, 3, 3 respectively. Two of these cards are chosen at random without replacement. Let X denote the sum of the two numbers on the chosen cards.

(a) Show that

$$P(X = 3) = \frac{2}{15}.$$ [2]

(b) Find the sampling distribution of X. [6]

2. When Bill throws the discus, the distance thrown (in metres) can be assumed to be normally distributed with mean μ and variance σ^2. He throws the discus 10 times with the following results.

25·3 23·8 24·7 24·9 23·7 25·6 24·6 24·0 25·3 24·1

(a) Calculate unbiased estimates of μ and σ^2. [5]

(b) Calculate a 95% confidence interval for μ. [4]

3. Rachel visits a toy shop and buys a coin. The shopkeeper states that when the coin is tossed, 'heads' is more likely to occur than 'tails'. She tosses the coin 200 times and obtains 120 'heads'.

(a) Calculate an approximate 95% confidence interval for p, the probability that the coin will fall 'heads' when tossed. [4]

(b) State, with a reason, whether or not your result justifies the shopkeeper's statement. [1]

(c) Find, approximately, how many times the coin should be tossed to obtain a 95% confidence interval of width 0·05. [4]

4. The random variable X has a Poisson distribution with unknown mean μ. In order to estimate μ, 100 observations are made on X and it is found that $\sum x = 256$.

(a) Calculate an unbiased estimate of μ. Deduce an unbiased estimate of the variance of X. [2]

(b) Estimate the standard error of your estimate. [2]

(c) Determine an approximate 90% confidence interval for μ. [3]

(d) Explain briefly where your solution made use of the Central Limit Theorem. [1]

5. A psychologist puts forward the theory that, on average, girls complete jigsaws more quickly than boys. To investigate this theory, 100 girls and 120 boys were given identical jigsaws to complete. The times taken by the girls, x minutes, and the times taken by the boys, y minutes, to complete the jigsaws were recorded and the following calculations made.

$$\sum x = 4282,\ \sum x^2 = 185\,855,\ \sum y = 5208,\ \sum y^2 = 230\,347.$$

(a) State suitable hypotheses to test the psychologist's theory. [1]

(b) Calculate, approximately, the p-value of these results. Interpret its value in context. [11]

6. The table below shows the probability distribution of the discrete random variable X, where θ is an unknown parameter whose value lies between 0 and 0·25.

x	1	2	3
$P(X = x)$	3θ	$1 - 4\theta$	θ

To estimate the value of θ, n independent observations are made on X. Let X_1 denote the number of occurrences of the value 1 and X_2 the number of occurrences of the value 2. Consider the following two estimators for θ.

$$U_1 = \frac{X_1}{3n},\ U_2 = \frac{n - X_2}{4n}$$

(a) Show that both estimators are unbiased. [6]

(b) (i) Show that

$$\frac{\mathrm{Var}(U_1)}{\mathrm{Var}(U_2)} = \frac{4(1 - 3\theta)}{3(1 - 4\theta)}.$$

(ii) State, with a reason, which is the better estimator. [7]

7. The following table shows the temperature, y°C, of the water in a boiler at various times, x minutes, after switching on.

x	0	5	10	15	20	25	30
y	20	25	31	38	44	49	55

[You may assume that $\sum xy = 4760$, $\sum x^2 = 2275$]

(a) Assuming a linear relationship $y = \alpha + \beta x$, calculate least squares estimates a, b for α, β. Give your answers correct to four decimal places. [7]

(b) The boiler is being tested to determine whether or not the value of β is 1·2. Assuming that the values of x are exact whereas the values of y are subject to independent normally distributed errors with zero mean and standard deviation 0·25,

(i) state suitable hypotheses,

(ii) calculate the p-value of your value of b and state your conclusion. [9]