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MATHEMATICS S2

STATISTICS 2

P.M. MONDAY, 11 June 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The weights, in kg, of babies born during a particular week in a hospital are given below.

3·25 3·38 3·04 3·59 3·42 3·13 3·38 3·09 3·15

(a) Assuming that this is a random sample from a normal distribution with mean μ and standard deviation 0·15, calculate a 90% confidence interval for μ. [5]

(b) How many observations would be required to halve the width of this 90% confidence interval? [2]

2. The independent random variables X and Y are Poisson distributed with means 2 and 3 respectively.

(a) (i) Show that $E(X^2) = 6$ and evaluate $E(Y^2)$.

(ii) Deduce the value of $E(X^2Y^2)$. [5]

(b) The random variable U is defined by

$$U = XY.$$

Determine the standard deviation of U. [4]

3. The weights of apples may be assumed to be normally distributed with mean 75 grams and standard deviation 5 grams.

(a) (i) Find the probability that a randomly chosen apple weighs less than 80 grams.

(ii) Find the upper quartile of the weights of apples. [6]

(b) The weights of plums may be assumed to be normally distributed with mean 56 grams and standard deviation 4 grams. Calculate the probability that the combined weight of 3 plums exceeds the combined weight of 2 apples. [6]

4. A circle has radius R cm, where R is a continuous random variable that is uniformly distributed on the interval [0, 5].

(a) State the probability density function of R. [1]

(b) Find the expected area of the circle. [4]

(c) Find the probability that the area of the circle is greater than 25 cm^2, giving your answer correct to three decimal places. [4]

5. Jim is a darts player. When he throws a dart, he claims to be able to hit the 'bull' with probability 0·75. His friends believe that the probability is less than this.

(a) State suitable hypotheses to test Jim's claim. [1]

(b) They decide to set up a trial in which Jim throws 20 darts and they define X to be the number of darts hitting the 'bull'.

(i) Taking the critical region to be $X \leqslant k$, find the value of k for which the significance level is nearest to 10%.

(ii) The actual value of the probability of Jim hitting the 'bull' is 0·5. With the value of k found in (i), find the probability of reaching an incorrect conclusion. [7]

6. A plumber knows that the number of emergency calls received per day follows a Poisson distribution with mean $\mu = 2$.

(a) Calculate the probability that, in a 7-day period, he receives

(i) exactly 10 calls,

(ii) more than 12 calls. [5]

(b) Wishing to increase the value of μ, he increases his advertising budget.

(i) State suitable hypotheses for investigating whether or not this achieves the desired result.

(ii) In the first 7-day period after increasing the budget, he receives 20 emergency calls. Calculate and interpret the p-value of this result.

(iii) In the next 100-day period, he receives 230 emergency calls. Calculate an approximate p-value of this result and interpret it. [12]

7. A scientist wishes to determine whether or not there is a difference in the acidity levels of two different liquids. He therefore makes five independent measurements of the acidity level of each liquid with the following results.

Liquid 1	6·31	6·38	6·33	6·34	6·35
Liquid 2	6·28	6·31	6·29	6·35	6·30

You may assume that these are random samples from normal distributions with common standard deviation 0·025.

(a) (i) State suitable hypotheses.

(ii) Calculate the p-value of the above measurements and interpret your value in context. [10]

(b) Find a 95% confidence interval for the difference in the acidity levels of the two liquids. [3]