WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

CYD-BWYLLGOR ADDYSG CYMRU
Tystysgrif Addysg Gyffredinol
Uwch Gyfrannol/Uwch

979/01

MATHEMATICS FP3

Further Pure Mathematics

A.M. FRIDAY, 22 June 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use the substitution $x = 2\sinh\theta - 1$ to evaluate the integral

$$\int_0^1 \frac{dx}{\sqrt{x^2 + 2x + 5}}.$$ [8]

2. The function f is defined by

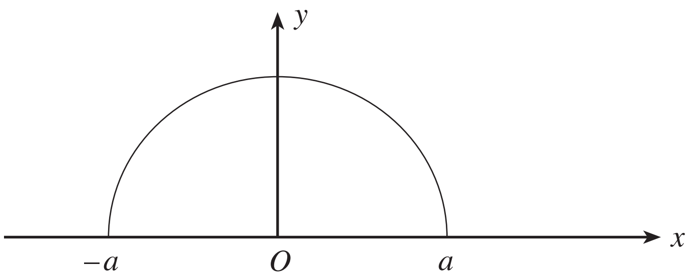
$$f(x) = x^3 + 3x^2 + 6x - 5.$$

(a) Show that f is strictly increasing for all values of x. Deduce the number of real roots of the equation $f(x) = 0$. [4]

(b) (i) Show that the equation $f(x) = 0$ has a root in the interval [0, 1].

(ii) Use the Newton-Raphson method to find the value of this root correct to four decimal places. [7]

3.



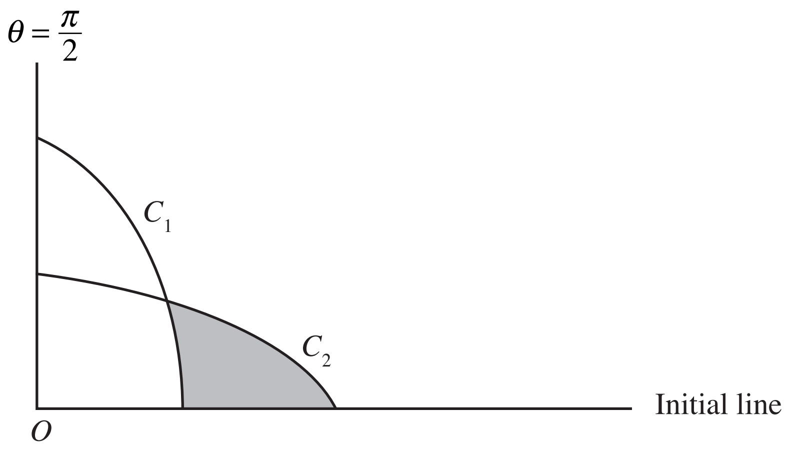
The above diagram shows the upper half of the circle with equation $x^2 + y^2 = a^2$.

(a) Show that, on this curve,

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{a^2}{y^2}.$$ [4]

(b) Hence show that the curved surface area of a sphere with radius a is equal to $4\pi a^2$. [4]

4.



The diagram shows the initial line, the line $\theta = \frac{\pi}{2}$ and the curves C_1, C_2 with equations

$$C_1 : r = e^{\theta} \quad \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right),$$

$$C_2 : r = 2e^{-\theta} \quad \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right).$$

(a) Find the polar coordinates of the point of intersection of C_1 and C_2. [4]

(b) Find the area of the shaded region. [5]

5. *(a)* Given that

$$a\cosh x + b\sinh x \equiv r\cosh(x + \alpha) \text{ where } a > b > 0,\ r > 0,$$

show that

$$\alpha = \frac{1}{2}\ln\left(\frac{a+b}{a-b}\right)$$

and find an expression for r in terms of a and b. [8]

(b) Hence, or otherwise, solve the equation

$$5\cosh x + 3\sinh x = 4,$$

giving your answer as a natural logarithm. [4]

6. The function *f* is defined by

$$f(x) = \ln\tan\left(\frac{\pi}{4} + x\right).$$

(a) Show that

$$f'(x) = 2\sec 2x. \qquad [4]$$

(b) Find the first two non-zero terms in the Maclaurin expansion of *f*. [7]

(c) The equation

$$f(x) = 10x^3$$

has a small positive root. Find its approximate value. [3]

7. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^1 x^n (1-x)^{\frac{3}{2}}\,\mathrm{d}x.$$

(a) Show that, for $n \geqslant 1$,

$$I_n = \left(\frac{2n}{2n+5}\right) I_{n-1}. \qquad [7]$$

(b) Evaluate I_2. [6]