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MATHEMATICS FP2

Further Pure Mathematics

A.M. FRIDAY, 22 June 2007

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the substitution $x = y^2$ to evaluate the integral

$$\int_1^4 \frac{dx}{\sqrt{x(9-x)}},$$

giving your answer correct to two significant figures. [6]

2. Find the two square roots of the complex number $1 + \sqrt{3}i$. Give your answers in the form $x + iy$. [6]

3. Let

$$f(x) = \frac{(x+1)(x+2)}{(x-1)(x^2+1)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Find $\int f(x)dx$. [4]

4. Find the general solution of the equation

$$\sin 2\theta + \sin 4\theta = \cos \theta. \quad [9]$$

5. The ellipse E has equation

$$16x^2 + 25y^2 = 400.$$

(a) Find the coordinates of the foci of E . [4]

(b) Show that the point P with coordinates $(5\cos\theta, 4\sin\theta)$ lies on E . [1]

(c) (i) Show that the equation of the normal to E at P is

$$4y\cos\theta - 5x\sin\theta + 9\sin\theta\cos\theta = 0.$$

(ii) This normal intersects the x -axis at Q and the y -axis at R . Show that the locus of M , the mid-point of QR , is an ellipse. [10]

6. The function f is defined by

$$f(x) = \frac{x^2 + 4}{x}.$$

- (a) Find the coordinates of the stationary points on the graph of f . [4]
 (b) Find the equation of each of the two asymptotes. [2]
 (c) Sketch the graph of f . [2]
 (d) Find $f(A)$ where A is the interval $[1, 5]$. [4]

7. (a) Given that

$$z = \cos\theta + i\sin\theta,$$

use de Moivre's Theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

for all positive integers n . [3]

(b) Hence by expanding $\left(z + \frac{1}{z}\right)^5$, show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos \theta$$

where a , b and c are constants to be determined. [5]

8. The function f is defined on the domain $(0, 2)$ by

$$f(x) = 4x^2 \quad \text{for } 0 < x < 1,$$

$$f(x) = (x+1)^2 \quad \text{for } 1 \leq x < 2.$$

- (a) Determine whether or not f is continuous when $x = 1$. [2]
 (b) Show that f is a strictly increasing function. [2]
 (c) Obtain an expression for $f^{-1}(x)$ on each part of its domain. [6]