

973/01

**MATHEMATICS C1**

**Pure Mathematics**

P.M. WEDNESDAY, 10 January 2007

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Calculators are **not** allowed for this paper.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points  $A, B, C, D$  have coordinates  $(-5, 0), (0, 5), (3, 4), (4, -3)$ , respectively.
- (a) Show that  $AC$  is perpendicular to  $BD$ . [4]
- (b) Show that  $AD$  is parallel to  $BC$ . [3]
- (c) Show that the equation of  $AC$  is  

$$x - 2y + 5 = 0$$
and find the equation of  $BD$ . [3]
- (d) The lines  $AC$  and  $BD$  intersect at  $E$ .
- (i) Show that the coordinates of  $E$  are  $(1, 3)$ . [2]
- (ii) Find the length of  $AE$ . [2]
2. Simplify **each** of the following expressions, expressing your answers in surd form.
- (a)  $2\sqrt{32} + 3\sqrt{8} - \sqrt{18}$  [3]
- (b)  $\frac{6 + \sqrt{30}}{6 - \sqrt{30}}$  [4]
3. When  $9x^3 + 6x^2 - 5x + p$  is divided by  $x - 1$ , the remainder is 8.
- (a) Show that  $p = -2$ . [2]
- (b) Factorise  $9x^3 + 6x^2 - 5x - 2$ . [5]
4. (a) Expand  $(a + b)^4$ , simplifying your coefficients as much as possible. [2]
- (b) Solve  $(2 + x)^4 = 14 + 33x + 25x^2 + 8x^3 + x^4$ . [4]
5. (a) Given that  $y = 2x^2 - 5x + 3$ , find  $\frac{dy}{dx}$  from first principles. [5]
- (b) Find the equation of the normal to the curve  $y = 2x^2 - 5x + 3$  at the point  $(2, 1)$ . [3]

6. Differentiate **each** of the following with respect to  $x$ .

(a)  $2x^5 + \frac{24}{x^2} - 3\sqrt{x}$  [3]

(b)  $x^2(3x + 1)$  [2]

7. Given that the equation

$$kx^2 - 4x + (k - 3) = 0$$

has real roots, show that

$$k^2 - 3k - 4 \leq 0.$$

Find the range of values of  $k$  satisfying this inequality. [7]

8. (a) Express  $x^2 + 4x + 9$  in the form  $(x + a)^2 + b$ , where the values of  $a$  and  $b$  are to be determined.  
Deduce the maximum value of

$$\frac{1}{x^2 + 4x + 9}.$$
 [4]

(b) Show that the line  $y = x + 2$  touches the curve  $y = x^2 - 5x + 11$ , and find the coordinates of the point of contact. [4]

9. The curve  $C$  has equation

$$y = 4x^3 - 12x + 3.$$

(a) Find the coordinates of the stationary points of  $C$  and determine the nature of each of these points. [7]

(b) Sketch  $C$ , indicating the coordinates of the stationary points. [3]

(c) Given that  $f(x) = 4x^3 - 12x + 3$ , sketch the curve  $y = f(x - 1)$ , indicating the coordinates of **each** of the stationary points. [3]