

981/01

**MATHEMATICS M2**

**Mechanics 2**

P.M. TUESDAY, 6 June 2006

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A car of mass 1200 kg is towing a trailer of mass 800 kg up a slope inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{1}{28}$ . The resistance to motion acting on the car is 150 N and that acting on the trailer is 100 N. The car's engine is working at 45 kW.

(a) Calculate the acceleration of the car and trailer when the speed is  $25 \text{ ms}^{-1}$ . [6]

(b) Determine the tension in the rigid tow-bar connecting the car and the trailer when the speed is  $25 \text{ ms}^{-1}$ . [4]

2. Particle  $A$  is moving with constant velocity  $-2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ , and at time  $t = 0$  s it has position vector  $\mathbf{i} - 10\mathbf{k}$ . Particle  $B$  is moving with constant velocity  $\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}$ , and at time  $t = 0$  s it has position vector  $7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$ .

(a) Write down the position vectors of  $A$  and  $B$  at time  $t$  s. [2]

(b) Find the distance between  $A$  and  $B$  when  $t = 2$  s. [3]

3. A particle  $P$ , of mass 3 kg, moves along the horizontal  $x$ -axis under the action of a resultant force  $F$  N. Its velocity  $v \text{ ms}^{-1}$  at time  $t$  seconds is given by

$$v = 12t - 3t^2.$$

(a) Given that the particle is at the origin  $O$  when  $t = 1$ , find an expression for the displacement of the particle from  $O$  at time  $t$  s. [4]

(b) Find the acceleration of the particle at time  $t$  s. [2]

(c) Find the power of the force  $F$  when  $t = 1.5$ . [3]

4. A light elastic string, of natural length 0.8 m and modulus of elasticity 35.4 N, has one end  $A$  attached to a fixed point and the other end  $B$  attached to a particle  $P$  of mass 3 kg. Initially  $P$  is held at rest at  $A$ . It is then released and allowed to fall. Calculate the speed of  $P$  when the length of the string is 1.2 m. [7]

5. A stone is projected in a direction which makes an angle of  $45^\circ$  above the horizontal. It strikes a small target whose horizontal and vertical distances from the point of projection are 120 m and 41.6 m respectively. The target is above the level of the point of projection.

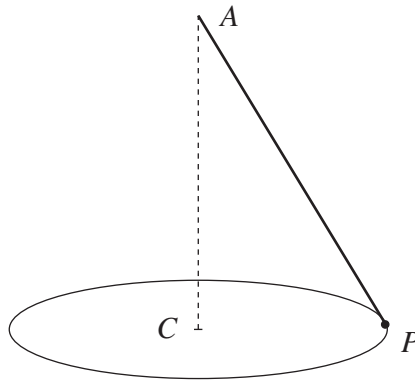
(a) Find the speed of projection and show that the time taken for the stone to reach the target is 4 s. [8]

(b) Determine, correct to two decimal places, the speed and direction of motion of the stone as it hits the target. [7]

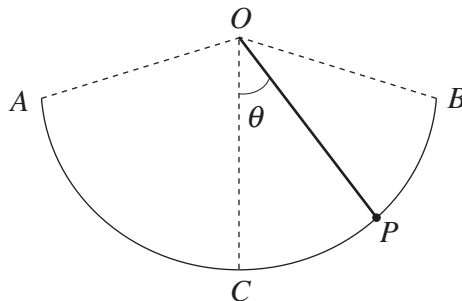
6. A particle  $P$  moves such that its position vector  $\mathbf{r}$  with respect to the origin  $O$  at time  $t$  is given by

$$\mathbf{r} = \cos 3t\mathbf{i} + \sin 3t\mathbf{j}.$$

- (a) Find an expression for  $\mathbf{v}$ , the velocity of  $P$  at time  $t$ . [3]
- (b) Show that the direction of  $\mathbf{v}$  is perpendicular to that of  $\mathbf{r}$  for all values of  $t$ . [3]
- (c) Find the speed of  $P$ . [3]
7. The diagram shows a small body  $P$ , of mass 3 kg, attached by means of a light inextensible string, of length 1.3 m, to a fixed point  $A$ . The point  $C$  is vertically below  $A$ , and  $P$  describes a horizontal circle, with centre  $C$  and radius 0.5 m, with a uniform angular speed of  $\omega$  radians per second about  $C$ .



- (a) Find the tension in the string. [3]
- (b) Calculate, correct to two decimal places, the value of  $\omega$ . [4]
8. One end of a light rod of length  $l$  m is attached to a fixed point  $O$  and the other end is attached to a particle  $P$  of mass  $m$  kg. The particle  $P$  is set in motion so that it moves back and forth along the minor arc  $AB$  of a vertical circle with centre  $O$  and radius  $l$  m, as shown in the diagram.



When  $P$  is at its lowest point  $C$ , its speed is  $u \text{ ms}^{-1}$  and the tension in the rod is  $2mg \text{ N}$ .

- (a) Show that  $u = \sqrt{gl}$ . [4]
- (b) The speed of  $P$  when  $OP$  makes an angle  $\theta$  with the vertical is denoted by  $v \text{ ms}^{-1}$ . Show that  $v^2 = gl(2\cos\theta - 1)$ . [3]
- (c) Find the greatest value of  $\theta$ . [2]
- (d) Find the value of  $\theta$  when the tension in the rod is  $mg \text{ N}$ . [4]