

# CYD-BWYLLGOR ADDYSG CYMRU Tystysgrif Addysg Gyffredinol Uwch Gyfrannol/Uwch

978/01

#### **MATHEMATICS FP2**

## **Further Pure Mathematics**

A.M. MONDAY, 19 June 2006  $(1\frac{1}{2} \text{ hours})$ 

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### INSTRUCTIONS TO CANDIDATES

Answer all questions.

### INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined as follows.

$$f(x) = x for x < 0,$$

$$f(x) = \sin x$$
 for  $x \ge 0$ .

Determine whether or not

- (i) the function f,
- (ii) its derivative f'

is continuous when x = 0. [5]

- 2. Find the three cube roots of the complex number i. Give your answers in the form x + iy. [9]
- **3.** The function f is defined on the domain  $(-\infty, 0) \cup (0, \infty)$  by

$$f(x) = \frac{1}{x(x^2+1)}.$$

- (a) Show that f is strictly decreasing over the interval  $(0, \infty)$ . [3]
- (b) State, giving a reason, whether f is even or odd or neither even nor odd. [2]
- (c) State the equation of each of the asymptotes on the graph of f. [2]
- (d) Sketch the graph of f. [2]
- **4.** A hyperbola has equation

$$2x^2 - 4x - y^2 - 4y = 4.$$

- (a) Find the coordinates of the centre of the hyperbola. [4]
- (b) Find the coordinates of the foci and the equations of the directrices. [5]
- 5. By putting  $t = \tan\left(\frac{\theta}{2}\right)$ , find the general solution of the equation

$$3\cos\theta + 4\sin\theta = 3 - \tan\left(\frac{\theta}{2}\right).$$
 [9]

**6.** (a) Use mathematical induction to prove that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

where n is a positive integer.

[6]

(b) Use the result in (a) with n = 5 to show that

$$\sin 5\theta = a\sin^5\theta - b\sin^3\theta + c\sin\theta$$

where a, b, c are positive integers to be found.

[7]

7. (a) Express

$$\frac{x}{(x+2)(x^2+4)}$$

in partial fractions.

[4]

(b) Hence evaluate the integral

$$\int_{2}^{3} \frac{x}{(x+2)(x^2+4)} \, \mathrm{d}x,$$

giving your answer correct to three decimal places.

[6]

- **8.** The line y = m(x 2) intersects the circle  $x^2 + y^2 = 1$  at the points A and B.
  - (a) Show that the coordinates of M, the mid-point of AB, are

$$\left(\frac{2m^2}{1+m^2}, -\frac{2m}{1+m^2}\right). [5]$$

(b) Find the Cartesian equation of the locus of M as m varies.

[6]