WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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MATHEMATICS FP1

Further Pure Mathematics

P.M. MONDAY, 12 June 2006

(1 $\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find an expression for the sum of the series

$$1.2.6 + 2.3.7 + ... + n(n + 1)(n + 5).$$

Give your answer as a product of linear factors in n. [6]

2. Differentiate $\dfrac{1}{2x-3}$ from first principles. [6]

3. Solve the following equation for the complex number z.

$$\frac{z}{z+1} = 2 + 3\mathrm{i}$$

Give your answer in its simplest form. [6]

4. Consider the cubic equation

$$x^3 + px^2 + 56x + q = 0.$$

Given that the three roots are all positive and are the first three terms of a geometric series with common ratio 2,

(a) find the three roots of the equation, [4]

(b) find the values of p and q. [2]

5. The matrices **A** and **I** are given by

$$\mathbf{A} = \begin{bmatrix} -4 & -4 & 4 \\ -1 & 0 & 1 \\ -7 & -6 & 7 \end{bmatrix}; \quad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Write down the matrix $\mathbf{A} + \lambda\mathbf{I}$. [2]

(b) Find the values of λ for which the matrix $\mathbf{A} + \lambda\mathbf{I}$ is singular. [7]

6. The transformations T_1 and T_2 in the plane are defined as follows.

T_1 : A translation in which the point (x, y) is transformed to the point $(x - 1, y + 1)$.

T_2 : A reflection in the line $y = x$.

The single transformation T is equivalent to T_1 followed by T_2.

(a) Find the 3×3 matrix representing T. [4]

(b) Find the equation of the locus of the fixed points of T. [4]

7. Use mathematical induction to show that $9^n - 5^n$ is divisible by 4 for all positive integers n. [7]

8. Use reduction to echelon form to solve the equations

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \\ 4 \end{bmatrix}.$$ [7]

9. The curve C has equation

$$y = x^{-x} \text{ for } x > 0.$$

(a) Find the coordinates of the stationary point on C. [6]

(b) (i) Show that

$$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - \frac{y^2}{x}\ .$$

(ii) **Hence** determine the nature of this stationary point. [6]

10. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2.$$

The point P moves along the line $y = x - 1$. Find the Cartesian equation of the locus of Q. [8]