

976/01

**MATHEMATICS C4**

**Pure Mathematics**

P.M. MONDAY, 12 June 2006

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$f(x) = \frac{2x^2 + 4}{(x-2)^2(x+4)},$$

(a) express  $f(x)$  in partial fractions, [4]

(b) hence find the value of  $f'(0)$ . [3]

2. Find the equation of the normal to the curve

$$2x^3 + 6xy^2 - y^4 = 27$$

at the point  $(2, 1)$ . [5]

3. Find all values of  $\theta$  in the range  $0^\circ \leq \theta \leq 360^\circ$  satisfying

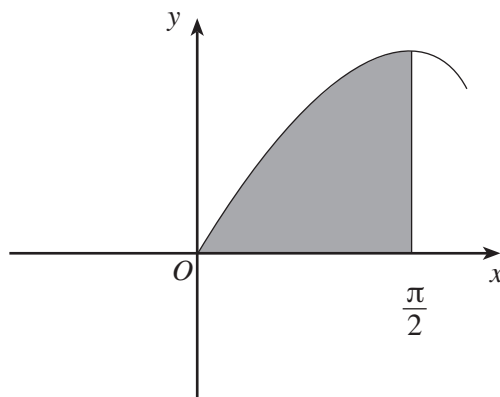
$$2 + 3\cos 2\theta = \cos \theta. [6]$$

4. (a) Express  $4\sin x + 3\cos x$  in the form  $R\sin(x + \alpha)$ , where  $R$  and  $\alpha$  are constants with  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]

(b) Hence find the greatest value of

$$\frac{1}{4\sin x + 3\cos x + 7}. [2]$$

5.



The diagram shows the shaded region bounded by the curve  $y = \sin x$ , the  $x$ -axis and the line  $x = \frac{\pi}{2}$ . The shaded region is rotated through four right-angles about the  $x$ -axis. Find the volume of the solid formed. [5]

6. The parametric equations of the curve  $C$  are

$$x = \frac{1}{t}, \quad y = t^2 .$$

- (a) Show that the tangent to  $C$  at the point  $P$  with parameter  $p$  has equation

$$y + 2p^3x - 3p^2 = 0. \quad [4]$$

- (b) The tangent to  $C$  at the point  $P$  intersects the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ . Show that  $PB = 2PA$ . [5]

7. (a) Find  $\int x \ln x \, dx$  . [5]

- (b) Use the substitution  $u = 2\sin x + 3$  to evaluate

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{(2\sin x + 3)^2} \, dx . \quad [4]$$

8. Water leaks from a hole at the bottom of a large water tank. The depth of the water at time  $t$  minutes is  $x$  metres. The rate of decrease of  $x$  is directly proportional to  $\sqrt{x}$  .

- (a) Write down a differential equation that is satisfied by  $x$ . [1]

- (b) Given that the depth of water in the tank when  $t = 0$  is 9 metres, show that

$$kt = 6 - 2\sqrt{x} ,$$

where  $k$  is a positive constant. [4]

- (c) Given that the depth of water in the tank is 4 metres when  $t = 20$ , find the time taken for the tank to empty. [3]

9. The position vectors of the points  $A$  and  $B$  are given by

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}.$$

- (a) Find the vector equation of the line  $AB$ . [3]

- (b) The vector equation of the line  $L$  is

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + p\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

where  $p$  is a constant. Given that  $AB$  and  $L$  intersect, find the value of  $p$ . [6]

- (c) Given  $\mathbf{c} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$ , find  $\mathbf{b} \cdot \mathbf{c}$ . What does your answer tell you about the vectors  $\mathbf{b}$  and  $\mathbf{c}$ ? [3]

**TURN OVER**

10. Expand  $\left(1 + \frac{x}{8}\right)^{\frac{1}{2}}$  in ascending powers of  $x$  up to and including the term in  $x^2$ . State the range of  $x$  for which the expansion is valid. Hence by writing  $x = 1$  in your expansion, show that  $\sqrt{2} \approx \frac{256}{181}$ . [5]

11. Complete the following proof by contradiction to show that  $\sqrt{2}$  is irrational.

*Assume that  $\sqrt{2}$  is rational. Then  $\sqrt{2}$  may be written in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are positive integers having no common factor.*

$$\therefore a^2 = 2b^2.$$

$$\therefore a^2 \text{ has a factor } 2.$$

$$\therefore a \text{ has a factor } 2 \text{ so that } a = 2k,$$

*where  $k$  is an integer.*

[4]