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MATHEMATICS C3

Pure Mathematics

P.M. THURSDAY, 12 January 2006

(1$\frac{1}{2}$ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_1^2 \sqrt{2 + x^4}\ \mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\tan 2\theta \equiv 2 \tan\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4 \cot^2\theta = 11 - 4 \operatorname{cosec}\theta.$$ [6]

3. *(a)* The curve C is defined by

$$y^4 + x^3y = x^2 + 4x - 3.$$

Find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point (2, 1) [4]

(b) Given that $x = 2t^3$, $y = 3t^4$, find, in terms of t,

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$,

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

[4]

4. *(a)* Show that $2 \tan^{-1} x - 6 \ln (1 + x^2) - 4x^2$ has a stationary value when x satisfies

$$4x^3 + 10x - 1 = 0.$$ [5]

(b) Show that the equation

$$4x^3 + 10x - 1 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \frac{1 - 4x_n^3}{10}$$

with $x_0 = 0{\cdot}1$ may be used to find α. Calculate and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to six decimal places and show that it is the value of α correct to six decimal places. [7]

5. Differentiate each of the following with respect to x, simplifying your answer where possible.

(a) $e^{3x}\cos x$ [3]

(b) $\dfrac{2x^2+1}{3x^2+2}$ [3]

(c) $\tan(5x^2+3)$ [2]

(d) $\ln(2x)$ [2]

(e) $\sin^{-1}(3x)$. [2]

6. *(a)* Solve the inequality $|3x-8| \leqslant 5$. [3]

(b) Given that $f(x) = |x|$, sketch the graph of $y = f(x)$. On the same diagram, sketch the graph of $y = f(x+2)+1$, indicating its position. [4]

7. *(a)* Find

(i) $\displaystyle\int \left(\frac{4}{7x+2} + \frac{5}{(3x+1)^3}\right) dx,$ [4]

(ii) $\displaystyle\int \cos 2x\, dx.$ [2]

(b) Evaluate $\displaystyle\int_0^4 e^{\frac{x}{2}}\, dx.$ [4]

8. The function f is defined for $x \geqslant 0$ by $f(x) = 3x^2 + 4$.

(a) Find an expression for f^{-1}, stating the range and domain of f^{-1}. [6]

(b) Sketch the graphs of f and f^{-1} using the same axes. [3]

9. The function f has domain $(-\infty, \infty)$ and is defined by $f(x) = e^x$.

The function g has domain $(2, \infty)$ and is defined by $g(x) = \ln(x^2 - 4)$.

(a) State the domain of fg. [1]

(b) Solve the equation $fg(x) = 5$. [4]