WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

CYD-BWYLLGOR ADDYSG CYMRU
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502/01

MATHEMATICS S2

STATISTICS 2

P.M. WEDNESDAY, 18 January 2006

($1\frac{1}{2}$ hours)

LEGACY SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

Graphical calculators may be used for this paper.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The heights of men in a certain population can be assumed to be normally distributed with mean 1·8 m and standard deviation 0·1 m. Find the probability that the height of a randomly chosen man in this population lies between 1·6 m and 1·9 m. [6]

2. The distance that Bill can throw a cricket ball is normally distributed with standard deviation 3 metres. Bill claims that the mean distance thrown is 50 metres but his team mates think that it is less than this. To test his claim, Bill throws a cricket ball ten times and the distances travelled (in metres) were as follows.

46·2, 48·9, 49·1, 44·4, 51·3, 48·1, 50·5, 47·4, 53·4, 50·1

(a) State appropriate null and alternative hypotheses. [1]

(b) Test Bill's claim using a significance level of 10%. [7]

3. The continuous random variable X is uniformly distributed on the interval [5, 10].

(a) (i) Sketch the probability density function of X.

(ii) Calculate the interquartile range of X. [5]

(b) (i) Find the standard deviation of X, giving your answer correct to three decimal places.

(ii) Hence find the probability that X lies within one standard deviation of the mean. [4]

4. For a certain breed of chickens, the weights, X kg, of the male birds are normally distributed with mean 2·5 kg and standard deviation 0·2 kg. The weights, Y kg, of the female birds are normally distributed with mean 1·5 kg and standard deviation 0·15 kg.

(a) (i) Determine the distribution of $X - 2Y$.

(ii) Hence find the probability that the weight of a randomly chosen male bird is more than double the weight of a randomly chosen female bird. [7]

(b) Find the probability that the combined weight of a random sample of three female birds is less than the combined weight of a random sample of two male birds. [6]

5. The number of accidents occurring per week along a certain stretch of road can be modelled by a Poisson distribution with mean 0·8. In an attempt to reduce the mean, road calming measures are introduced.

(a) The local council decides to use the total number of accidents occurring in the 15 weeks following the introduction of these measures to assess whether or not they have been successful in reducing the mean.

(i) State appropriate null and alternative hypotheses.

(ii) Given that there was a total of 8 accidents in these first 15 weeks, calculate and interpret the p-value of this result. [5]

(b) In the longer term, there were 70 accidents in the 100 weeks following the introduction of the calming measures. Calculate, approximately, the p-value of this result and interpret your value in the context of the problem. [7]

6. The random variable X has probability density function

$$f(x) = k(1 - x), \qquad \text{for } 0 \leqslant x \leqslant 1,$$
$$f(x) = 0, \qquad \text{otherwise.}$$

(a) Show that $k = 2$. [2]

(b) Find $E(X)$. [3]

(c) (i) Obtain an expression for $F(x)$, valid for $0 \leqslant x \leqslant 1$, where F denotes the cumulative distribution function of X.

(ii) Find $P(0{\cdot}3 \leqslant X \leqslant 0{\cdot}6)$.

(iii) Find the median value of X. [9]

7. A manufacturer of cereals states that 25% of the packets of a certain cereal contain a gift voucher.

(a) A Consumer Council, wishing to investigate this statement, buys 50 of these packets and counts the number X containing a gift voucher. They assume that a proportion p of the packets contain gift vouchers and they set up the hypotheses

$$H_0: p = 0{\cdot}25 \text{ versus } H_1: p \neq 0{\cdot}25.$$

They define the critical region as $X \geqslant 18$ or $X \leqslant 7$.

(i) Calculate the significance level of this procedure.

(ii) Calculate the probability of drawing the correct conclusion if the value of p is actually 0·2. [6]

(b) The Council now decides to buy 500 of these packets. They find that 102 of them contain gift vouchers. Use a normal approximation to find the p-value of this result with respect to the hypotheses defined in *(a)*. [7]