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MATHEMATICS M3

Mechanics 3

A.M. WEDNESDAY, 25 January 2006

($1\frac{1}{2}$ hours)

LEGACY SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

INFORMATION FOR CANDIDATES

Graphical calculators may be used for this paper.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves in a straight line with Simple Harmonic Motion, the centre of oscillations being O. When the particle is at a distance 4 m from O, its speed is $4{\cdot}5\ \text{ms}^{-1}$ and the magnitude of its acceleration is $9\ \text{ms}^{-2}$.

(a) Find the time taken to make 5 complete oscillations. [5]

(b) Determine the maximum speed of the particle. [5]

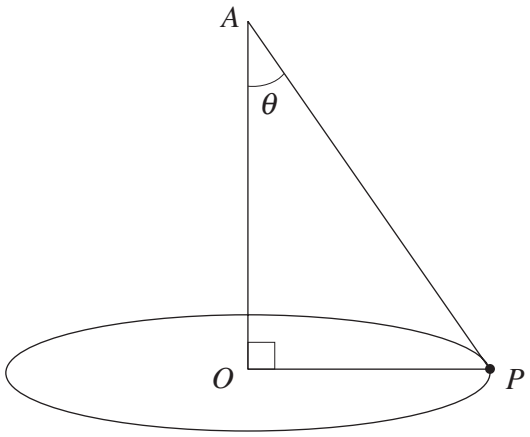
(c) Two points A and B are on different sides of O in the path of the particle. The distances of A and B from O are 2·5 m and 4 m respectively. Calculate the shortest time for the particle to travel from A to B. [4]

2. A particle moves along the x-axis so that at time t s, its displacement x m from the origin satisfies the differential equation.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} - 21x = 42t + 55.$$

Given that when $t = 0$, the particle is at rest at the origin, find its displacement at time t s. [11]

3. One end of a light inextensible string of length l m is attached to a fixed point A. The other end of the string is attached to a particle P of mass 2·5 kg. The particle P moves in a horizontal circle with constant angular speed 2 radians per second about the point O, where O is vertically below A as shown in the diagram. The string makes an angle θ with the downward vertical.



Given that the tension in the string is 28·3 N,

(a) find, correct to the nearest degree, the value of θ, [4]

(b) find the value of l. [6]

4. The foot of a uniform ladder of length 4 m and mass 18 kg rests on rough ground and the top of the ladder rests against a smooth vertical wall. The ladder is inclined at an angle θ to the vertical, where $\tan\theta = \frac{8}{9}$.

(a) Draw a diagram showing the forces acting on the ladder. [1]

(b) Find the magnitude of the frictional force exerted by the ground on the ladder. [6]

(c) Given that the ladder is on the point of slipping, find the value of the coefficient of friction between the ground and the ladder. [3]

5. A body moves in a straight line so that its velocity v ms^{-1} at time t s is given by

$$v = \frac{12}{2 + 5x},$$

where x m is the displacement of the body from a fixed point O on the line at time t s.

Given that $x = 0$ when $t = 1$,

(a) calculate the value of t when $x = 4$, [5]

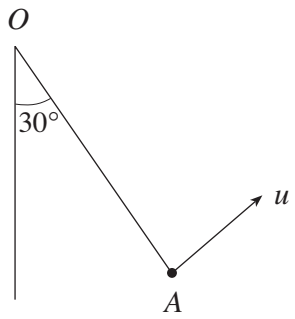
(b) find the expression for the acceleration of the body in terms of x. [2]

6. A stone of mass m kg is projected vertically upwards from a point O with speed 42 ms^{-1} . When the height of the stone above O is x m, the speed of the stone is v ms^{-1} and the resistance to motion of the stone is $0{\cdot}04\,mv^2$ N.

(a) Show that $25v\frac{\mathrm{d}v}{\mathrm{d}x} = -(245 + v^2)$. [3]

(b) Calculate the greatest height of the stone above O. [7]

7. *(a)* One end of a light rigid rod, of length a, is freely pivoted at a fixed point O. A particle, of mass m, is attached to the other end of the rod. Initially, the particle is held at rest at a point A such that OA is inclined at an angle of 30° to the downward vertical through O. The particle is projected from A with speed u in the direction perpendicular to OA in the vertical plane containing OA as shown in the diagram, so that it starts describing a vertical circle with centre O.



(i) When the rod is inclined at an angle θ to the downward vertical the speed of the particle is v and the tension in the rod is T.
Find, in terms of u, a, θ and m, expressions for v^2 and T.

(ii) Given that the particle describes complete circles, show that $u > \sqrt{ag(2+\sqrt{3})}$. [10]

(b) If the rod in part *(a)* is replaced by a light inextensible string and $u^2 = 4ag$, determine whether or not the particle describes complete circles. [3]