

494/01

MATHEMATICS P4

Pure Mathematics

P.M. MONDAY, 23 January 2006

(1½ hours)

LEGACY SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

Graphical calculators may be used for this paper.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. For each of the following functions state, with a reason, whether it is even, odd or neither even nor odd.

(a) $x^2 + 1$, [2]

(b) $1 - \tan x$, [2]

(c) $x^3 + \frac{1}{x}$. [2]

2. Use mathematical induction to prove that $7^n - 1$ is divisible by 6 for all positive integer values of n . [5]

3. Sum the series

$$3 \times 2 + 6 \times 3 + 9 \times 4 + \dots + 3n(n + 1)$$

giving your answer as a product of linear factors in n . [6]

4. Find the general solution of the equation

$$6 \cos^2 \theta + 5 \sin \theta = 7. \quad [8]$$

5. The function f is defined on the domain $(1, \infty)$ by

$$f(x) = \frac{x^2 + 1}{x^2 - 1}.$$

(a) Show that f is monotonic. [4]

(b) State the range of f . [2]

(c) Evaluate $f^{-1}(A)$ where A is the interval $[2, 3]$. [6]

6. An ellipse has equation

$$4x^2 + 9y^2 = 36.$$

(a) Find

(i) the coordinates of the foci,

(ii) the equations of the directrices. [6]

(b) The line $y = 2x + c$ meets the ellipse at the points R and S .

(i) Find, in terms of c , the coordinates of the mid-point, M , of the chord RS .

(ii) Find the equation of the locus of M as c varies. [7]

7. The curve C has equation

$$y = \frac{x^2 - 2x - 4}{x^2 - 5x + 6}.$$

- (a) Show y cannot take values between 2 and 10. [5]
- (b) By putting y equal successively to 2 and 10, find the coordinates of the stationary points on C . [4]
- (c) Sketch C . [3]

8. The roots of the cubic equation

$$x^3 + ax^2 + bx + c = 0$$

are in geometric progression. Show that

$$b^3 = a^3c.$$

Show that the equation

$$x^3 + 7x^2 - 21x - 27 = 0$$

satisfies this condition.

Hence solve the equation.

[13]