WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

CYD-BWYLLGOR ADDYSG CYMRU
Tystysgrif Addysg Gyffredinol
Uwch Gyfrannol/Uwch

493/01

MATHEMATICS P3

Pure Mathematics

P.M. MONDAY, 23 January 2006

($1\frac{1}{2}$ hours)

LEGACY SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

Graphical calculators may be used for this paper.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that $f(x) = \dfrac{7x+4}{(x-2)(x+1)^2}$,

(a) express $f(x)$ in partial fractions, [4]

(b) find $f'(1)$. [3]

2. *(a)* Find the values of θ between 0° and 360° satisfying the equation

$$2\cot^2\theta - \operatorname{cosec}\theta - 4 = 0.$$

[6]

(b) Express $24\sin\theta + 7\cos\theta$ in the form $R\sin(\theta + \alpha)$, where the values of R and α are to be determined.
Hence find all values of θ between 0° and 360° satisfying the equation

$$24\sin\theta + 7\cos\theta = 10.$$ [6]

3. *(a)* Differentiate $\tan^{-1}(2x)$ with respect to x. [2]

(b) Find the equation of the tangent to the curve $x^4 + x^2y + y^3 = 21$ at the point (2, 1). [5]

4. Expand $\dfrac{3}{1+2x} + \dfrac{1}{(1+x)^2}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which your expansion is valid. [6]

5. *(a)* Find $\displaystyle\int x\,e^{-x}\,dx$. [3]

(b) Use the substitution $u = 2 + \ln x$ to evaluate

$$\int_1^e \frac{1}{x(2+\ln x)^3}\,dx.$$

[5]

6. The region R is bounded by the curve $y = \cos x$, the x-axis and the lines $x = 0$, $x = \dfrac{\pi}{4}$. Find the volume generated when R is rotated through four right-angles about the x-axis. [6]

7. A curve has parametric equations $x = 2\cos t$, $y = 3\sin t$.
Show that the normal to the curve at the point P, whose parameter is p, has equation

$$(2\sin p)x - (3\cos p)y + 5\sin p\cos p = 0.$$

The normal at P meets the x-axis at A and the y-axis at B. Write down the values of OA and OB, where O is the origin, when $p = \frac{\pi}{3}$. [8]

8. Given that $y = \frac{\pi}{2}$ when $x = \frac{\pi}{2}$, solve the differential equation

$$\frac{dy}{dx} = \frac{x + 2 + \cos x}{\sin y},$$

expressing your answer in the form $f(x) = g(y)$. [6]

9. *(a)* The vector equation of the line L_1 is

$$\mathbf{r} = (4 + 2\lambda)\,\mathbf{i} + (1 + \lambda)\,\mathbf{j} + (2 + 3\lambda)\,\mathbf{k},$$

where λ is a parameter.

A second line L_2 has vector equation

$$\mathbf{r} = (\mu + 3)\,\mathbf{i} + (2\mu + 5)\,\mathbf{j} + (8 + 4\mu)\,\mathbf{k},$$

where μ is a parameter.

Given that the lines intersect, find the position vector of their point of intersection. [5]

(b) The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$, respectively. Find $\cos A\hat{O}B$, where O is the origin. [5]

10. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assuming that $\sqrt{5}$ is rational, let $\sqrt{5} = \frac{a}{b}$ where a and b are integers that have no common factor.

Then $5b^2 = a^2 \Rightarrow 5$ is a factor of a^2. [5]