Matrix transformations – exam questions

Question 1: June 2006 – Q2

A transformation is represented by the matrix $\mathbf{A} = \begin{bmatrix} 0.28 & -0.96 & 0 \\ 0.96 & 0.28 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Evaluate det **A**. *(1 mark)*

(b) State the invariant line of the transformation. *(1 mark)*

(c) Give a full geometrical description of this transformation. *(3 marks)*

Question 2: Jan 2006 – Q1

Describe the geometrical transformation defined by the matrix

$$\begin{bmatrix} 0.6 & 0 & 0.8 \\ 0 & 1 & 0 \\ -0.8 & 0 & 0.6 \end{bmatrix}$$ *(3 marks)*

Question 3: Jun 2007 – Q6

The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & t \end{bmatrix}$$

(a) Find, in terms of t, the matrices:

(i) **AB**; *(3 marks)*

(ii) **BA**. *(2 marks)*

(b) Explain why **AB** is singular for all values of t. *(1 mark)*

(c) In the case when $t = -2$, show that the transformation with matrix **BA** is the combination of an enlargement, E, and a second transformation, F. Find the scale factor of E and give a full geometrical description of F. *(6 marks)*

Question 4: Jan 2007 – Q4

The matrices $\mathbf{M}_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\mathbf{M}_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ represent the transformations A and B respectively.

(a) Give a full geometrical description of each of A and B. *(5 marks)*

(b) Transformation C is obtained by carrying out A followed by B.

(i) Find $\mathbf{M}_C$, the matrix of C. *(2 marks)*

(ii) Hence give a full geometrical description of the single transformation C. *(2 marks)*

Question 5: Jan 2008 – Q1

Give a full geometrical description of the transformation represented by each of the following matrices:

(a) $\begin{bmatrix} 0.8 & 0 & -0.6 \\ 0 & 1 & 0 \\ 0.6 & 0 & 0.8 \end{bmatrix}$; *(3 marks)*

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. *(2 marks)*

Question 6: Jun 2009 – Q2

(a) Write down the 3×3 matrices which represent the transformations A and B, where:

(i) A is a reflection in the plane $y = x$; *(2 marks)*

(ii) B is a rotation about the z-axis through the angle θ, where $\theta = \frac{\pi}{2}$. *(1 mark)*

(b) (i) Find the matrix **R** which represents the composite transformation

'A followed by B' *(3 marks)*

(ii) Describe the single transformation represented by **R**. *(2 marks)*

Question 7: Jan 2010 – Q1

The 2×2 matrix **M** represents the plane transformation T. Write down the value of $\det \mathbf{M}$ in each of the following cases:

(a) T is a rotation;

(b) T is a reflection;

(c) T is a shear;

(d) T is an enlargement with scale factor 3. *(4 marks)*

Question 8: Jun 2010 – Q8

The matrix $\begin{bmatrix} 12 & 16 \\ -9 & 36 \end{bmatrix}$ represents the transformation which is the composition, in either order, of the two plane transformations

E: an enlargement, centre O and scale factor k $(k > 0)$

and

S: a shear parallel to the line l which passes through O

Show that $k = 24$ and find a cartesian equation for l. *(7 marks)*

Question 9: Jun 2011 – Q2

The plane transformation T is the composition of a reflection in the line $y = x \tan \alpha$ followed by an anticlockwise rotation about O through an angle β.

Determine the matrix which represents T, and hence describe T as a single transformation. *(6 marks)*

Question 10: Jan 2012 – Q2

Describe the single transformation represented by each of the matrices:

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; *(2 marks)*

(b) $\begin{bmatrix} 0.6 & 0 & -0.8 \\ 0 & 1 & 0 \\ 0.8 & 0 & 0.6 \end{bmatrix}$. *(3 marks)*

Matrix transformations – exam questions MS

Question 1: Jun 2006 – Q2

(a)	$\det \mathbf{A} = 1$	B1	1
(b)	The z-axis $(\text{i.e } x = y = 0)$	B1	1
(c)	Rotation	M1	
	about the z-axis	A1√	
	through $\cos^{-1} 0.28$	A1	3
	Total		**5**

Question 2: Jan 2006 – Q1

Rotation	B1	
about the y-axis	B1	
through an angle of 53.13°	B1	3
Total		**3**

Question 3: Jun 2007 – Q6

(a)(i)	**AB** = a 3×3 matrix	M1	
	$= \begin{pmatrix} 3 & 2 & t+1 \\ 1 & 2 & t-1 \\ 3 & 2 & t+1 \end{pmatrix}$	A1 A1	3
(ii)	**BA** = a 2×2 matrix	M1	
	$= \begin{pmatrix} 2 & 2 \\ t & t+4 \end{pmatrix}$	A1	2
(b)	$R_1 = R_3 (\Rightarrow \det \mathbf{AB} = 0)$	B1	1
(c)	$\mathbf{BA} = \begin{pmatrix} 2 & 2 \\ -2 & 2 \end{pmatrix} = 2\sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1	
	E: enlargement s.f. $2\sqrt{2}$	B1	
	F: Rotation	M1	
	clockwise (about O) thro' 45°	A1 A1	6
			12

Question 4: Jan 2007 – Q4

4(a)	A is a Rotation thro' 90°	M1 A1	
	about Ox	A1	
	B is a Reflection in $y = 0$ (i.e. x–z plane)	M1 A1	5
(b)(i)	$\mathbf{M}_C = \mathbf{M}_B \mathbf{M}_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	M1 A1	2
(ii)	C is a Reflection in $y = z$	M1	2
		A1	
	N.B. In (i): $\mathbf{M}_A \mathbf{M}_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ scores M0		
	but ft "Reflection in $y = -z$" in (ii)		
			9

Question 5: Jan 2008 – Q1

(a)	Rotation	M1	
	about the y-axis	A1	
	through $\cos^{-1} 0.8$	A1	3
(b)	Reflection in $y = x$	M1A1	2
	Total		**5**

Question 6: Jun 2009 – Q2

(a)(i)	$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B2	2
(ii)	$\mathbf{B} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	1
(b)(i)	$\mathbf{R} = \mathbf{BA} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1 A1 A1	3
(ii)	Reflection in $x = 0$ (or y–z plane)	M1 A1	2
	Note 1: For $\mathbf{R} = \mathbf{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	(B1)	
	Reflection in $y = 0$ (or x–z plane)	(M1) (A1)	
	Note 2: 90° rotation in –ve sense gives $\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	(B1)	
	$\mathbf{R} = \mathbf{BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	(M1) (A1) (A1)	
	Reflection in $y = 0$ (or x–z plane)	(M1) (A1)	
	Total		**8**

Question 7: Jan 2010 – Q1

(a)	1	B1	
(b)	– 1	B1	
(c)	1	B1	
(d)	9	B1	4
	Total		**4**

Question 8: Jun 2010 – Q8

$\det \mathbf{W} = 12.36 + 9.16 = 576 = k^2$	M1	
$\Rightarrow k = 24$	A1	2
$\frac{1}{24}\mathbf{W} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}$	B1	
$\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x + \frac{2}{3}y \\ \frac{3}{2}y - \frac{3}{8}x \end{bmatrix}$	M1 A1	
Equating this to $\begin{bmatrix} x \\ y \end{bmatrix} =$	M1	
$y = \frac{3}{4}x$	A1	
ALT. 1		
$\frac{1}{24}\mathbf{W} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}$	(B1)	
$\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} x \\ mx \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2} + \frac{2}{3}m\right)x \\ \left(\frac{3}{2}m - \frac{3}{8}\right)x \end{bmatrix}$	(M1) (A1)	
Setting $y' = mx'$ and solving for m	(M1)	
$y = \frac{3}{4}x$	(A1)	
ALT. 2		
$\lambda^2 - 2\lambda + 1 = 0$	(M1)	
$\Rightarrow \lambda = 1$ (twice)	(A1)	
$\lambda = 1 \Rightarrow \begin{matrix} -\frac{1}{2}x + \frac{2}{3}y = 0 \\ -\frac{3}{8}x + \frac{1}{2}y = 0 \end{matrix}$	(M1) (A1)	
$y = \frac{3}{4}x$	(A1)	5
Total		**7**

Question 9: Jun 2011 – Q2

$\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$ & $\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$	B1	
Mult'n of these in the correct order	B1	
Use of addition formulae	M1	
$\begin{bmatrix} \cos(2\alpha+\beta) & \sin(2\alpha+\beta) \\ \sin(2\alpha+\beta) & -\cos(2\alpha+\beta) \end{bmatrix}$	A1F	
Reflection ...	A1F	
... in $y = x\tan\left(\alpha + \frac{1}{2}\beta\right)$	A1F	**6**
Total		**6**

Question 10: Jan 2012 – Q2

(a)	Reflection in $x = z$	M1 A1	2
(b)	Rotation about the y-axis	M1 A1	
	Through $\cos^{-1} 0.6$ ($\approx 53.13°$)	A1	3
	Total		**5**