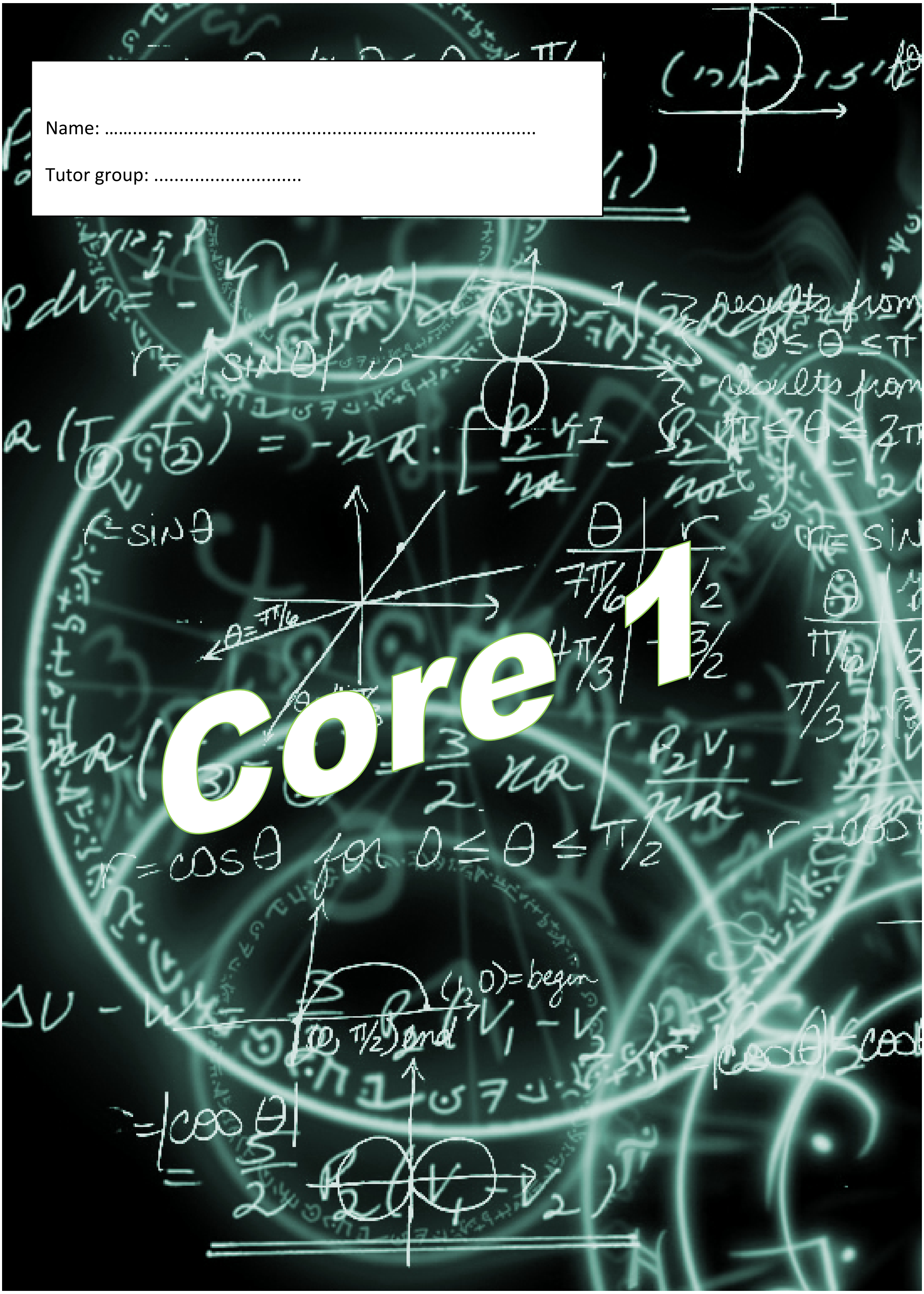
AQA - Core 1 - Revision booklet



Name: ………………………………………………………………………………

Tutor group: ………………………..

Key dates

Core 1 exam: **13th May 2013 am**

Term dates:

Term 1: Monday 3 September 2012 - Wednesday 24 October 2012 (38 teaching days)	Term 4: Monday 18 February 2013 - Friday 22 March 2013 (25 teaching days)
Term 2: Monday 5 November 2012 - Friday 21 December 2012 (35 teaching days)	Term 5: Monday 8 April 2013 - Friday 24 May 2013 (34 teaching days)
Term 3: Monday 7 January 2013 - Friday 8 February 2013 (25 teaching days)	Term 6: Monday 3 June 2013 - Wednesday 24 July 2013 (38 teaching days)

Scheme of Assessment *Mathematics*
Advanced Subsidiary (AS)
Advanced Level (AS + A2)

The Scheme of Assessment has a modular structure. The A Level award comprises four compulsory Core units, one optional Applied unit from the AS scheme of assessment, and one optional Applied unit either from the AS scheme of assessment or from the A2 scheme of assessment.

For the written papers, each candidate will require a copy of the AQA Booklet of formulae and statistical tables issued for this specification.

All the units count for $33^{1}/_{3}$% of the total AS marks
$16^{2}/_{3}$% of the total A level marks

Written Paper
1hour 30 minutes
75 marks

Grading System

The AS qualifications will be graded on a five-point scale: A, B, C, D and E. The full A level qualifications will be graded on a six-point scale: A*, A, B, C, D and E.

To be awarded an A* in Further Mathematics, candidates will need to achieve grade A on the full A level qualification and 90% of the maximum uniform mark on the aggregate of the best three of the A2 units which contributed towards Further Mathematics. For all qualifications, candidates who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

CORE 1 subject content

Algebra

Coordinates geometry

Differentiation

Integration

Core 1 specifications

Candidates will be required to demonstrate:

a) construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;

b) correct understanding and use of mathematical language and grammar in respect of terms such as "equals", "identically equals", "therefore", "because", "implies", "is implied by", "necessary", "sufficient" and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow and \Leftrightarrow$.

Candidates are **not** allowed to use a calculator in the assessment unit for this module.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates **should learn** the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Quadratic equations	$ax^2+bx+c=0 \text{ has roots } \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$
Circles	A circle, centre (a,b) and radius r, has equation $(x-a)^2+(y-b)^2=r^2$
Differentiation	$if\ \ y=ax^n\ \ then\ \dfrac{dy}{dx}=anx^{n-1}$ $if\ y=f(x)+g(x)\ then\ \dfrac{dy}{dx}=f\,'(x)+g\,'(x)$
Integration	$if\ \dfrac{dy}{dx}=ax^n\quad then\ \ y=\dfrac{a}{n+1}x^{n+1}+c$ $if\ \dfrac{dy}{dx}=f\,'(x)=g\,'(x)\quad then\quad y=f(x)+g(x)+c$
Area	$Area\ under\ a\ curve=\int_a^b y\,dx \qquad (y\ge 0)$

Algebra

Use and manipulation of surds.	To include simplification and rationalisation of the denominator of a fraction. $e.g. \sqrt{12}+2\sqrt{27}=8\sqrt{3}$; $\frac{1}{1-\sqrt{2}}=\sqrt{2}+1$; $\frac{2\sqrt{3}+\sqrt{2}}{3\sqrt{2}+\sqrt{3}}=\frac{\sqrt{6}}{3}$
Quadratic functions and their graphs.	To include reference to the vertex and line of symmetry of the graph.
The discriminant of a quadratic function.	To include the conditions for equal roots, for distinct real roots and for no real roots
Factorisation of quadratic polynomials.	E.g. factorisation of $2x^2+x-6$
Completing the square.	$e.g.\ x^2+6x-1=(x+3)^2-10$; $2x^2-6x+2=2(x-1.5)^2-2.5$
Solution of quadratic equations.	Use of any factorisation, $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ *or* completing the square will be accepted.
Simultaneous equations, e.g. one linear and one quadratic, analytical solution by substitution.	
Solution of linear and quadratic inequalities.	$e.g.\ 2x^2+x\geq 6$
Algebraic manipulation of polynomials, including expanding brackets and collecting like terms.	
Simple algebraic division.	Applied to a quadratic or a cubic polynomial divided by a linear term of the form $(x + a)$or $(x - a)$ where a is a small whole number. Any method will be accepted, e.g. by inspection, by equating coefficients or by formal division e.g. $\frac{x^3-x^2-5x+2}{x+2}$.
Use of the Remainder Theorem.	Knowledge that when a quadratic or cubic polynomial f (x) is divided by (x – a) the remainder is f (a) and, that when f (a) = 0 , then (x – a) is a factor and vice versa.
Use of the Factor Theorem.	Greatest level of difficulty as indicated by $x^3 - 5x^2 + 7x - 3$, i.e. a cubic always with a factor $(x + a)$or $(x - a)$ where a is a small whole number but including the cases of three distinct linear factors, repeated linear factors or a quadratic factor which cannot be factorized in the real numbers.
Graphs of functions; sketching curves defined by simple equations.	Linear, quadratic and cubic functions. The *f(x)* notation may be used but only a very general idea of the concept of a function is required. Domain and range are not included. Graphs of circles are included.
Geometrical interpretation of algebraic solution of equations and use of intersection points of graphs of functions to solve equations.	Interpreting the solutions of equations as the intersection points of graphs and vice versa.
Knowledge of the effect of translations on graphs and their equations.	Applied to quadratic graphs and circles, i.e. $y = (x - a)^2 + b$ as a translation of $y = x^2$ and $(x - a)^2 + (y - b)^2 = r^2$ as a translation of $x^2 + y^2 = r^2$.

Coordinates geometry	
Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ *and* $ax + by + c = 0$	To include problems using gradients, mid-points and the distance between two points. The form y = mx + c is also included.
Conditions for two straight lines to be parallel or perpendicular to each other.	Knowledge that the product of the gradients of two perpendicular lines is -1.
Coordinate geometry of the circle.	Candidates will be expected to complete the square to find the centre and radius of a circle where the equation of the circle is for example given as $x^2 + 4x + y^2 - 6y - 12 = 0$.
The equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$.	The use of the following circle properties is required: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the tangent to a circle is perpendicular to the radius at its point of contact.
The equation of the tangent and normal at a given point to a circle.	Implicit differentiation is **not** required. Candidates will be expected to use the coordinates of the centre and a point on the circle or of other appropriate points to find relevant gradients.
The intersection of a straight line and a curve.	Using algebraic methods. Candidates will be expected to interpret the geometrical implication of equal roots, distinct real roots or no real roots. Applications will be to either circles or graphs of quadratic functions.
Differentiation	
The derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a point; the gradient of the tangent as a limit; interpretation as a rate of change.	The notations f '(x) or $\frac{dy}{dx}$ will be used. A general appreciation only of the derivative when interpreting it is required. Differentiation from first principles will **not** be tested.
Differentiation of polynomials.	
Applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions	Questions will not be set requiring the determination of or knowledge of points of inflection. Questions may be set in the form of a practical problem where a function of a single variable has to be optimised.
Second order derivatives.	Application to determining maxima and minima.
Integration	
Indefinite integration as the reverse of differentiation Integration of polynomials.	
Evaluation of definite integrals. Interpretation of the definite integral as the area under a curve.	Integration to determine the area of a region between a curve and the x-axis. To include regions wholly below the x-axis, i.e. knowledge that the integral will give a negative value. Questions involving regions partially above and below the x-axis will not be set. Questions may involve finding the area of a region bounded by a straight line and a curve, or by two curves.

The formulae booklet

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n[2a + (n-1)d]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Summations

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Trigonometry – the Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Binomial Series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$a^x = e^{x \ln a}$$

Complex numbers

$$\{r(\cos\theta + i\sin\theta)\}^n = r^n(\cos n\theta + i\sin n\theta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k i}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1} \frac{x^r}{r} + \ldots \qquad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{for all } x$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \qquad (x \geqslant 1)$$

$$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$

$$\tanh^{-1} x = \tfrac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Asymptotes	none	none	$\frac{x}{a} = \pm \frac{y}{b}$	$x = 0, y = 0$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\tan kx$	$k\sec^2 kx$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\,\mathrm{g}(x)-\mathrm{f}(x)\,\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Integration

(+ constant; $a > 0$ where relevant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert=\ln\lvert\tan(\frac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert=\ln\lvert\tan(\frac{1}{2}x+\frac{1}{4}\pi)\rvert$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$

$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\{x+\sqrt{x^2-a^2}\}$	$(x > a)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\{x+\sqrt{x^2+a^2}\}$	
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Area of a sector

$$A = \frac{1}{2}\int r^2\,\mathrm{d}\theta \quad \text{(polar coordinates)}$$

Arc length

$$s = \int\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \quad \text{(cartesian coordinates)}$$

$$s = \int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \quad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi\int y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \quad \text{(cartesian coordinates)}$$

$$S_x = 2\pi\int y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \quad \text{(parametric form)}$$

Numerical integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h=\frac{b-a}{n}$

The mid-ordinate rule: $\int_a^b y\,\mathrm{d}x \approx h(y_{\frac{1}{2}}+y_{\frac{3}{2}}+\ldots+y_{n-\frac{3}{2}}+y_{n-\frac{1}{2}})$, where $h=\frac{b-a}{n}$

Simpson's rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{3}h\{(y_0+y_n)+4(y_1+y_3+\ldots+y_{n-1})+2(y_2+y_4+\ldots+y_{n-2})\}$

where $h=\frac{b-a}{n}$ and n is even

Content

Algebra

Indices and surds

KEY POINTS

Definition: $for\ n$ integer and $a \neq 0,\ a^n = a \times a \times a ... \times a \quad (n\ factors)$

$$a^1 = a \quad and \quad a^0 = 1$$

(a is called the base)

Multiplication : $a^n \times a^p = a^{n+p}$

Division : $a^n \div a^p = \frac{a^n}{a^p} = a^{n-p}$

Negative index : $a^{-n} = \frac{1}{a^n} \quad and \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Power of a power : $\left(a^n\right)^p = a^{n \times p}$

Fractional index : $a^{\frac{1}{n}} = \sqrt[n]{a} \qquad and \qquad a^{\frac{n}{p}} = \sqrt[p]{a^n} = \left(\sqrt[p]{a}\right)^n$

Different bases : $a^n \times b^n = (ab)^n \quad and \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

KEY POINTS

Definition : $for\ a \geq 0,\ \sqrt{a}$ is the positive number so that $\left(\sqrt{a}\right)^2 = a$

Multiplication : $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

in particular $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = \left(\sqrt{a}\right)^2 = a \quad (for\ a \geq 0)$

Division : $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \qquad (for\ a \geq 0, b > 0)$

Difference of squares: $(x+y)(x-y) = x^2 - y^2$

$$hence\ \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$$

$\sqrt{a} \pm \sqrt{b}$ is called the conjugate (expression) to $\sqrt{a} \mp \sqrt{b}$

Rationalise the denominator:

- $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$
- $\frac{1}{\sqrt{a} \pm \sqrt{b}} = \frac{1}{\sqrt{a} \pm \sqrt{b}} \times \frac{\sqrt{a} \mp \sqrt{b}}{\sqrt{a} \mp \sqrt{b}} = \frac{\sqrt{a} \mp \sqrt{b}}{a - b}$

(Multiple numerator and denominator by the conjugate expression)

Surds - exercises

Question 1:

Simplify the following as far as possible

$a)\sqrt{28} \quad b)\sqrt{63} \quad c)\sqrt{32} \quad d)\ \sqrt{150}$

Question 2:

Expand and simplify

$a)\sqrt{2}\left(\sqrt{8}-2\sqrt{3}\right) \quad b)\sqrt{3}\left(\sqrt{6}-\sqrt{27}\right) \quad c)\left(\sqrt{5}-1\right)^2 \quad d)\left(\sqrt{3}+\sqrt{6}\right)^2 \quad e)\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)$

Question 3:

Rationalise the denominator and simplify

$$a)\frac{10}{\sqrt{5}} \quad b)\frac{2}{\sqrt{3}+1} \quad c)\frac{1}{\sqrt{5}-2} \quad d)\frac{\sqrt{2}}{\sqrt{8}-\sqrt{6}} \quad e)\frac{39}{4-\sqrt{3}} \quad f)\frac{1-\sqrt{2}}{2\sqrt{2}-3}$$

$$g)\frac{\sqrt{6}\times\sqrt{12}}{\sqrt{2}+\sqrt{8}} \quad h)\frac{3-2\sqrt{2}}{(1+\sqrt{2})^2} \quad i)\frac{\sqrt{300}-\sqrt{75}}{\sqrt{12}+\sqrt{3}}$$

Question 4: Exam June 2006

$a)$ Express $\left(4\sqrt{5}-1\right)\left(\sqrt{5}+3\right)$ in the form $p+q\sqrt{5}$, where p *and* q are integers. $(3\,marks)$

b) Show that $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$ is an integer and find its value. $(3\,marks)$

Question 5: Exam Jan 2006

$a)$ *Simplify* $\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)$. $(2\,marks)$

$b)$ *Express* $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer $(2\,marks)$

Question 6: Exam June 2007

$a)$ Express $\dfrac{\sqrt{63}}{3}+\dfrac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. $(3\,marks)$

$b)$ Express $\dfrac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7}+q$, where p *and* q are integers. $(4\,marks)$

Question 7: Exam Jan 2007

$a)$ Express $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5}+q$, where p *and* q are integers. $(4\,marks)$

$b)\,i)$ Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. $(1\,mark)$

$ii)$ Solve the equation $x\sqrt{20}=7\sqrt{5}-\sqrt{45}$

giving your answer in its simplest form. $(3\,marks)$

Surds - exercises' answers

Question 1:

a) $\sqrt{28}=\sqrt{4\times7}=\sqrt{4}\times\sqrt{7}=2\sqrt{7}$

b) $\sqrt{63}=\sqrt{9\times7}=\sqrt{9}\times\sqrt{7}=3\sqrt{7}$

c) $\sqrt{32}=\sqrt{16\times2}=4\sqrt{2}$

d) $\sqrt{150}=\sqrt{25\times6}=5\sqrt{6}$

Question 2:

a) $\sqrt{2}\left(\sqrt{8}-2\sqrt{3}\right)=\sqrt{16}-2\sqrt{6}=4-2\sqrt{6}$

b) $\sqrt{3}\left(\sqrt{6}-\sqrt{27}\right)=\sqrt{18}-\sqrt{81}=3\sqrt{2}-9$

c) $\left(\sqrt{5}-1\right)^2=\left(\sqrt{5}-1\right)\left(\sqrt{5}-1\right)=5-\sqrt{5}-\sqrt{5}+1=6-2\sqrt{5}$

d) $\left(\sqrt{3}+\sqrt{6}\right)^2=\left(\sqrt{3}+\sqrt{6}\right)\left(\sqrt{3}+\sqrt{6}\right)=3+\sqrt{18}+\sqrt{18}+6=9+2\sqrt{18}$

e) $\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)=5-2\sqrt{5}+2\sqrt{5}-4=1$

Question 3:

a) $\dfrac{10}{\sqrt{5}}=\dfrac{10}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{10\sqrt{5}}{5}=2\sqrt{5}$

b) $\dfrac{2}{\sqrt{3}+1}=\dfrac{2}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}=\dfrac{2(\sqrt{3}-1)}{3-1}=\sqrt{3}-1$

c) $\dfrac{1}{\sqrt{5}-2}=\dfrac{1}{\sqrt{5}-2}\times\dfrac{\sqrt{5}+2}{\sqrt{5}+2}=\dfrac{\sqrt{5}+2}{5-2}=\sqrt{5}+2$

d) $\dfrac{\sqrt{2}}{\sqrt{8}-\sqrt{6}}=\dfrac{\sqrt{2}}{\sqrt{8}-\sqrt{6}}\times\dfrac{\sqrt{8}+\sqrt{6}}{\sqrt{8}-\sqrt{6}}=\dfrac{\sqrt{16}-\sqrt{12}}{8-6}=\dfrac{4-2\sqrt{3}}{2}=2-\sqrt{3}$

e) $\dfrac{39}{4-\sqrt{3}}=\dfrac{39}{4-\sqrt{3}}\times\dfrac{4+\sqrt{3}}{4+\sqrt{3}}=\dfrac{156+39\sqrt{3}}{16-3}=12+3\sqrt{3}$

f) $\dfrac{1-\sqrt{2}}{2\sqrt{2}-3}=\dfrac{1-\sqrt{2}}{2\sqrt{2}-3}\times\dfrac{2\sqrt{2}+3}{2\sqrt{2}+3}=\dfrac{2\sqrt{2}+3-4-3\sqrt{2}}{8-9}=\sqrt{2}+1$

g) $\dfrac{\sqrt{6}\times\sqrt{12}}{\sqrt{2}+\sqrt{8}}=\dfrac{6\sqrt{2}}{\sqrt{2}+2\sqrt{2}}=\dfrac{6\sqrt{2}}{3\sqrt{2}}=2$

h) $\dfrac{3-2\sqrt{2}}{(1+\sqrt{2})^2}=\dfrac{3-2\sqrt{2}}{3+2\sqrt{2}}=\dfrac{(3-2\sqrt{2})^2}{(3+2\sqrt{2})(3-2\sqrt{2})}=\dfrac{9+8-12\sqrt{2}}{9-8}=17-12\sqrt{2}$

i) $\dfrac{\sqrt{300}-\sqrt{75}}{\sqrt{12}+\sqrt{3}}=\dfrac{10\sqrt{3}-5\sqrt{3}}{2\sqrt{3}+\sqrt{3}}=\dfrac{5\sqrt{3}}{3\sqrt{3}}=\dfrac{5}{3}$

Question 4: June 2006

a)
$$\begin{aligned}(4\sqrt{5}-1)(\sqrt{5}+3)&=4\times5+12\sqrt{5}-\sqrt{5}-3\\&=20-3+12\sqrt{5}-\sqrt{5}\\&=17+11\sqrt{5}\end{aligned}$$

b)
$$\begin{aligned}\frac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}&=\frac{\sqrt{25\times3}-\sqrt{9\times3}}{\sqrt{3}}\\&=\frac{5\sqrt{3}-3\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{\sqrt{3}}=2\end{aligned}$$

Question 5: Jan 2006

a) $\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)=5-2\sqrt{5}+2\sqrt{5}-4=1$

b) $\sqrt{8}+\sqrt{18}=\sqrt{4\times2}+\sqrt{9\times2}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$

Question 6: June 2007

a)
$$\begin{aligned}\frac{\sqrt{63}}{3}+\frac{14}{\sqrt{7}}&=\frac{\sqrt{9\times7}}{3}+\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{3\sqrt{7}}{3}+\frac{14\sqrt{7}}{7}\\&=\sqrt{7}+2\sqrt{7}=3\sqrt{7}\end{aligned}$$

b)
$$\begin{aligned}\frac{\sqrt{7}+1}{\sqrt{7}-2}&=\frac{\sqrt{7}+1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{7+2\sqrt{7}+\sqrt{7}+2}{7-4}\\&=\frac{9+3\sqrt{7}}{3}\end{aligned}$$

Question 7: Jan 2007

a) $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}=\dfrac{\sqrt{5}+3}{\sqrt{5}-2}\times\dfrac{\sqrt{5}+2}{\sqrt{5}+2}=\dfrac{5+2\sqrt{5}+3\sqrt{5}+6}{5-4}=11+5\sqrt{5}$

b) i) $\sqrt{45}=\sqrt{9\times5}=3\sqrt{5}$

ii)
$$\begin{aligned}x\sqrt{20}&=7\sqrt{5}-\sqrt{45}\\x\times2\sqrt{5}&=7\sqrt{5}-3\sqrt{5}\\x&=\frac{4\sqrt{5}}{2\sqrt{5}}=\frac{4}{2}=2\end{aligned}$$

Quadratic functions

VOCABULARY

- A quadratic function can be written $f(x) = ax^2 + bx + c$, where $a, b,$ *and* c *are* 3 *real numbers*.
- The graph of a quadratic function is called a PARABOLA

  if $a > 0$, the parabola is $\cup$-shaped

  if $a < 0$, the parabola is $\cap$-shaped
- The maximum or minimum point of the parabola is called the VERTEX.
- The parabola is symmetrical around the 'vertical' line going through the vertex.

COMPLETED SQUARE FORM

- $ax^2 + bx + c$ can be re-arrange into $a(x+p)^2 + q$.

  This is the completed square form
- The vertex of the parabola is $V(-p, q)$.
- The axis of symmetry of the parabola has equation $x = -p$.
- Transformation: $y = x^2$ *is mapped onto* $y = (x+p)^2 + q$ *by a* translation with vector $\begin{bmatrix}-p\\q\end{bmatrix}$

QUADRATIC EQUATIONS

A quadratic equation can be written $ax^2 + bx + c = 0$

- The discriminant is the value of the expression $b^2 - 4ac$.

if $b^2 - 4ac < 0$, there is no solution.

if $b^2 - 4ac = 0$, there is a repeated/double root.

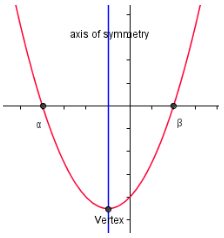
if $b^2 - 4ac > 0$, there are two solutions/roots: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- The roots are the values of x, where the parabola crosses the $x-axis$.

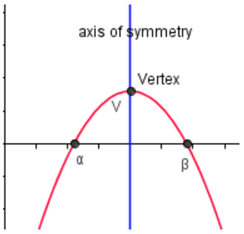
$b^2 - 4ac > 0$

$f(x) = ax^2 + bx + c$

$f(x) = a(x-\alpha)(x-\beta)$



a > 0



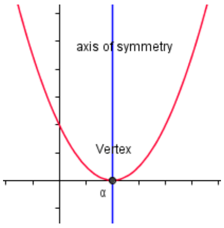
a < 0

$b^2 - 4ac = 0$

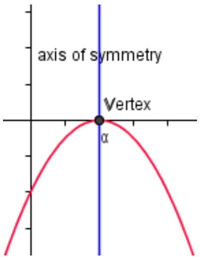
$f(x) = ax^2 + bx + c$

$f(x) = a(x-\alpha)^2$

The parabola is TANGENT to the x-*axis*



a > 0



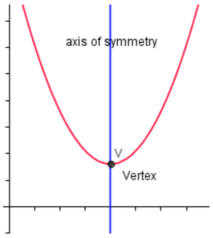
a < 0

$b^2 - 4ac < 0$

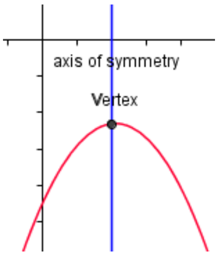
$f(x) = ax^2 + bx + c$

$f(x)$ *can't be factorised*

The parabola does not cross the x-*axis*.



a > 0



a < 0

Quadratic functions - exercises

Question 1:

Find the coordinates of the points where the curve with equation $y = x^2 - 6x + 5$ intersects the coordinate axes.

Question 2:

The curve with equation $y = x^2 - 2x - 8$ cuts the x-axis at the point A and B.

Calculate the distance AB.

Question 3:

$a)$ Find the coordinates of the points where the parabola with equation $y = 4x^2 - 8x$ crosses the x - *axis*.

$b) i)$ Write $4x^2 - 8x$ in the form $a(x+p)^2 + q$ *where* a, p *and* q are integers

$ii)$ *Hence*, work out the coordinates of the vertex.

$iii)$ and give the equation of the axis of symmetry of this parabola.

Question 4:

$a)$ Write $x^2 - 4x + 6$ in the form $(x-p)^2 + q$ where p and q are integers.

$b)$ Hence find

$i)$ the vertex of the parabola $y = x^2 - 4x + 6$,

$ii)$ the equation of the axis of symmetry of this parabola.

Question 5:

The equation $kx^2 + 8x + (k-6) = 0$ has equal roots.

Work out the possible values of k.

Question 6:

Calculate the possible values of k, if $(k+1)x^2 + kx + k + 1 = 0$ has equal roots.

Question 7:

$a)$ Factorise $7x^2 - 12x - 64$

$b)$ The equation $kx^2 - (k+8)x + 2k + 1 = 0$ has a repeated root.

Find the possible values of k.

Question 8:

$a)$ Write $x^2 - 6x + 8$ in the form $(x-p)^2 + q$.

$b)$ Describe the geometric transformation that maps the graph $y = x^2$ onto the graph of $y = x^2 - 4x + 8$.

Question 9:

Find the equation of the graph of $y = x^2$ after it has been translated by the given vectors. Give your answer in the form $y = x^2 + bx + c$.

$a)\begin{bmatrix}1\\0\end{bmatrix}$ $b)\begin{bmatrix}0\\3\end{bmatrix}$ $c)\begin{bmatrix}2\\5\end{bmatrix}$ $d)\begin{bmatrix}-3\\-6\end{bmatrix}$

Quadratic functions – exercises' answers

Question 1:

Intersection with the y-axis.

Substitute x by 0: $\quad y=0^2-6\times0+5=5 \qquad (0,5)$

Intersection with the x-axis.

Solve the equation $y=0$: $\quad x^2-6x+5=0$

$(x-5)(x-1)=0 \qquad (5,0)\ (1,0)$

Question 2:

Solve the equation $y=x^2-2x-8=0$

$(x-4)(x+2)=0 \qquad x=4 \ \ or \ \ x=-2$

$A(4,0)$ *and* $B(-2,0)$ $\quad$ The distance AB=6.

Question 3:

$a)\,4x^2-8x=0$

$4x(x-2)=0 \qquad x=0 \ \ and \ \ x=2$

The parabola crosses the x-axis at (0,0) and (2,0).

$b)\,i)\,4x^2-8x=4(x^2-2x)=4\left((x-1)^2-1\right)=4(x-1)^2-4$

$ii)$ the vertex is $V(1,-4)$ *and* the axis of symmetry of the parabola is $x=1$.

Question 4:

$a)\,x^2-4x+6=\left(x-2\right)^2-4+6=(x-2)^2+2$

$b)i)$ The vertex V(2,2)

$ii)$ The axis of symmetry has equation $x=2$.

Question 5:

$kx^2+8x+(k-6)=0$ *has equal roots*

means the discriminant = 0

$8^2-4\times k\times(k-6)=0 \qquad 64-4k^2+24k=0 \qquad (\div-4)$

$k^2-6k-16=0 \qquad (k-8)(k+2)=0$

$k=8 \ \ or \ \ k=-2$

Question 6:

$(k+1)x^2+kx+(k+1)=0$ *has equal roots*

means the discriminant = 0

$k^2-4\times(k+1)\times(k+1)=0 \qquad k^2-4k^2-8k-4=0$

$3k^2+8k+4=0 \qquad (3k+2)(k+2)=0$

$k=-\frac{2}{3} \ \ or \ \ k=-2$

Question 7:

$a)\,7x^2-12x-64=\left(7x+16\right)\left(x-4\right)$

$b)\,kx^2-(k+8)x+2k+1=0$ has repeated root so

the discriminant = 0

$(-(k+8))^2-4\times k\times(2k+1)=0$

$k^2+16k+64-8k^2-4k=0 \qquad -7k^2+12k+64=0$

$7k^2-12k-64=0 \qquad (7k+16)(k-4)=0$

$k=-\frac{16}{7} \ \ or \ \ k=4$

Question 8:

$a)\,x^2-6x+8=(x-3)^2-9+8=(x-3)^2-1$

$b)$ The transformation is a translation of vector $\begin{pmatrix}3\\-1\end{pmatrix}$

Question 9:

$a)\,y-0=(x-1)^2 \qquad y=x^2-2x+1$

$b)\,y-3=(x-0)^2 \qquad y=x^2+3$

$c)\,y-5=(x-2)^2 \qquad y=x^2-4x+9$

$d)\,y+6=(x+3)^2 \qquad y=x^2+6x+3$

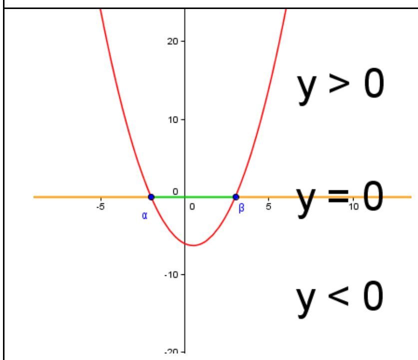
Quadratic inequalities

The purpose of this part is to solve inequalities of the form

$$ax^2+bx+c>0\ (\geq 0)\quad or \quad ax^2+bx+c<0\ (\leq 0)$$

$b^2-4ac>0$	$b^2-4ac=0$	$b^2-4ac<0$
$f(x)=ax^2+bx+c$ $f(x)=a(x-\alpha)(x-\beta)$	$f(x)=ax^2+bx+c$ $f(x)=a(x-\alpha)^2$ The parabola is TANGENT to the x - *axis*	$f(x)=ax^2+bx+c$ $f(x)$ *can't be factorised* The parabola does not cross the x - *axis*.

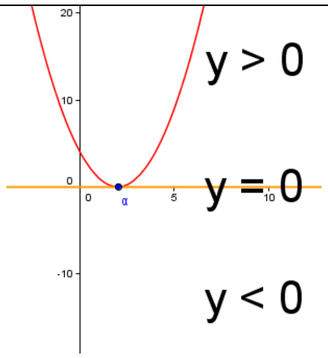
$a>0$



$ax^2+bx+c>0$ *for* $x<\alpha$ *or* $x>\beta$

$ax^2+bx+c<0$ *for* $\alpha<x<\beta$

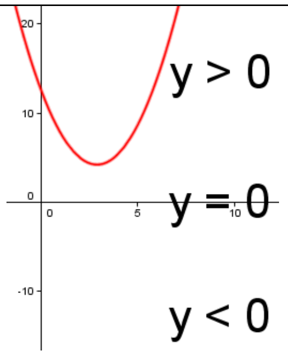
$ax^2+bx+c=0$ *for* $x=\alpha$ *or* $x=\beta$



$ax^2+bx+c>0$ *for* $x<\alpha$ *or* $x>\alpha$

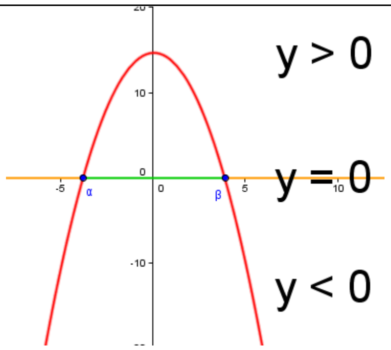
$ax^2+bx+c<0$ *has no solution*

$ax^2+bx+c=0$ *for* $x=\alpha$



$ax^2+bx+c>0$ *for all* x

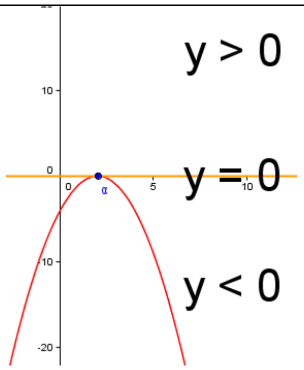
$a<0$



$ax^2+bx+c>0$ *for* $\alpha<x<\beta$

$ax^2+bx+c<0$ *for* $x<\alpha$ *or* $x>\beta$

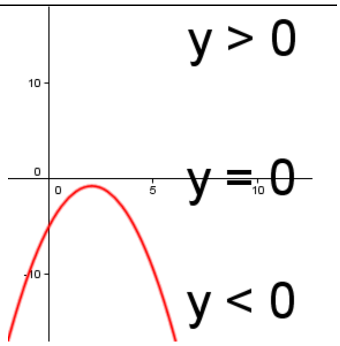
$ax^2+bx+c=0$ *for* $x=\alpha$ *or* $x=\beta$



$ax^2+bx+c>0$ *has no solution*

$ax^2+bx+c<0$ *for* $x<\alpha$ *or* $x>\alpha$

$ax^2+bx+c=0$ *for* $x=\alpha$



$ax^2+bx+c<0$ *for all* x

Quadratic inequalities - exercises

Question 1:

Solve the following inequalities for x.

(In each case, support your answer with a sketch).

a) $\left(x-3\right)\left(x+4\right)<0$ b) $(x-1)(x+3)\geq 0$

c) $(3-x)(x+1)\leq 0$ d) $x^2+3x+2>0$

e) $2x^2-3x-2<0$ f) $6-4x-2x^2<0$

Question 2:

Find the range of values for k that give each of these equations no real roots

a) x^2-2x+k b) $3x^2+kx+7$ c) $kx^2=2x-k$

Question 3:

Calculate the possible values of k, if $(k+1)x^2+kx+k+1=0$ has two distinct roots.

Question 4:

Calculate the possible values of k, if $(k+1)x^2+4kx+9=0$ has no real roots.

Question 5:

The equation $x^2-3kx+7-k$ has two distinct roots.

Find the possible values of k.

Question 6:

Find the condition of k for the equation $(x+1)(x^2+kx+4)=0$ to only have one real root.

Question 7:

a) Simplify $(k+5)^2-12k(k+2)$

b) The quadratic equation $3(k+2)x^2+(k+5)x+k=0$ has real roots.

i) Show that $(k-1)(11k+25)\leq 0$

ii) Hence find the possible values of k.

Question 8:

The quadratic equation $\left(2k-3\right)x^2+2x+(k-1)=0$

where k is a constant, has real roots.

a) Show that $2k^2-5k+2\leq 0$. (3 *marks*)

b) i) Factorise $2k^2-5k+2$. (1 *mark*)

ii) Hence or ortherwise, solve the quadratic inequality $2k^2-5k+2\leq 0$ (3 *marks*)

Quadratic inequalities – exercises' answers

Question 1:

$a)\,(x-3)(x+4)<0 \quad for \ -4<x<3$

$b)\,(x-1)(x+3)\ge 0 \quad for \ x\le -3 \ or \ x\ge 1$

$c)\,(3-x)(x+1)\le 0 \quad for \ x\le -1 \ or \ x\ge 3$

$d)\,x^2+3x+2=(x+2)(x+1)>0 \quad for \ x<-2 \ or \ x>-1$

$e)\,2x^2-3x-2=(2x+1)(x-2)<0 \quad for \ -\frac{1}{2}<x<2$

$f)\,6-4x-2x^2<0$

$2x^2+4x-6>0 \quad (2x-2)(x+3)>0 \quad for \ x<-3 \ or \ x>1$

Question 2:

$a)\,x^2-2x+k \ has \ no \ real \ roots:$

the discriminant < 0

$(-2)^2-4\times1\times k<0$

$4-4k<0 \quad k>1$

$b)\,3x^2+kx+7 \ has \ no \ real \ roots:$

the discriminant < 0

$k^2-4\times3\times7<0$

$k^2-84<0 \quad (k+\sqrt{84})(k-\sqrt{84})<0$

$-\sqrt{84}<k<\sqrt{84}$

$-2\sqrt{21}<k<2\sqrt{21}$

$c)\,kx^2-2x+k=0$ has no real roots:

the discriminant < 0

$(-2)^2-4\times k\times k<0$

$4-4k^2<0$

$k^2-1>0 \quad (k+1)(k-1)>0$

$k<-1 \ or \ k>1$

Question 3:

$(k+1)x^2+kx+k+1=0 \ has \ two \ distinct \ roots:$

the discriminant > 0

$(k)^2-4\times(k+1)^2>0$

$-3k^2-8k-4>0$

$3k^2+8k+4<0$

The roots of this quadratic expression are

$k=\frac{-8+\sqrt{28}}{6}=\frac{-4+\sqrt{7}}{3} \ or \ k=\frac{-8-\sqrt{28}}{6}=\frac{-4-\sqrt{7}}{3}$

$so \ 3k^2+8k+4<0 \quad for \ \frac{-4-\sqrt{7}}{3}<k<\frac{-4+\sqrt{7}}{3}$

Question 4:

$(k+1)x^2+4kx+9=0 \ has \ no \ real \ roots:$

the discriminant = 0

$(4k)^2-4\times(k+1)\times9=0$

$16k^2-36k-36=0$

$4k^2-9k-9=0$

$(4k+3)(k-3)=0$

$k=-\frac{3}{4} \ or \ k=3$

Question 5:

$x^2-3kx+7-k$ has two distinct roots:

Discriminant >0

$(-3k)^2-4\times1\times(7-k)>0$

$9k^2+4k-28>0$

$(9k-14)(k+2)>0$

$k<-2 \ or \ k>\frac{14}{9}$

Question 6:

$(x+1)(x^2+kx+4)=0 \ has \ only \ one \ real \ root.$

This root is -1.

This means that x^2+kx+4 has no real roots:

discriminant < 0

$k^2-4\times1\times4<0$

$k^2-16<0$

$(k+4)(k-4)<0$

$-4<k<4$

Question 7:

$a)\,(k+5)^2-12k(k+2)=$

$k^2+10k+25-12k^2-24k=$

$-11k^2-14k+25$

$b)$ the discriminant ≥ 0

$(k+5)^2-4\times3(k+2)\times k\ge 0$

$(k+5)^2-12k(k+2)\ge 0$

$-11k^2-14k+25\ge 0$

$11k^2+14k-25\le 0$

$(11k+25)(k-1)\le 0$

$so \ -\frac{25}{11}\le k\le 1$

Simultaneous equations

SET OF LINEAR EQUATIONS

Consider the line $\mathrm{L}_1 : ax + by = c$

and the line $\mathrm{L}_2 : dx + ey = f$

To work out the coordinates of the point of INTERSECTION, solve the equations SIMULTANEOUSLY.

Solving by *combination* / elimination:

$$\begin{cases} ax + by = c & (\times d) \\ dx + ey = f & (\times -a) \end{cases} \qquad \begin{cases} adx + bdy = cd \\ -adx - aey = -af \end{cases}$$

Then add the equations to find the value of y. Use any other equation to find the value of x.

Solving by *identification*:

Make y the subject in both equations and identify the values of y:

$L_1 : y = m_1 x + c_1$

$L_2 : y = m_2 x + c_2$ *this gives* $(y =)\, m_1 x + c_1 = m_2 x + c_2$ *and solve.*

Solving by *substitution*:

Make y the subject in one of the equation then substitute y by this expression in the second equation:

$\mathrm{L}_1 : y = mx + c$

$L_2 : dx + ey = f$ *this gives* $dx + e(mx + c) = f$ *then solve.*

SET OF QUADRATIC AND LINEAR EQUATIONS

A parabola C has equation $y = ax^2 + bx + c$,

a line L has equation $y = dx + e$ (*make* y *the subject if it is an implict equation*)

To work out the coordinates of the points of intersection of the parabola and the line, solve these equations simultaneoulsy

Solving by identification:

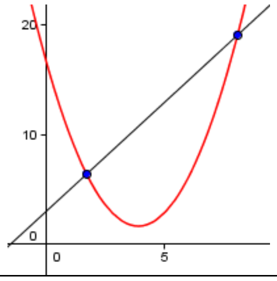
$$(y =)\ ax^2 + bx + c = dx + e \quad \text{then re-arrange into}$$

$$ax^2 + (b - d)x + c - e = 0 \ \ \textit{and solve.}$$

Let's re-write as $Ax^2 + Bx + C = 0$

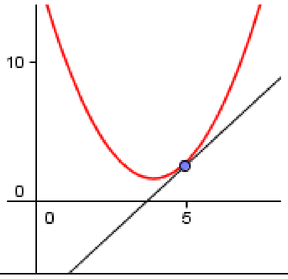
if $B^2 - 4AC > 0$

Two points of intersection



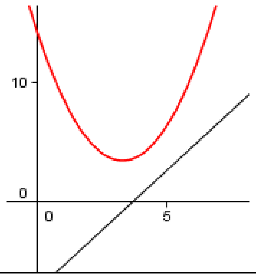
if $B^2 - 4AC = 0$

The line is tangent to the parabola



if $B^2 - 4AC < 0$

There is no intersection



Simultaneous equations - exercises

Question 1:

Solve these simultaneous equations

a) $\begin{cases} 3x+2y=7 \\ 5x+y=7 \end{cases}$ $b)\begin{cases} 4x+3y=5 \\ 3x+2y=4 \end{cases}$ $c)\begin{cases} 4x-5y=1 \\ 2x-3y=1 \end{cases}$

Question 2:

Solve these simultaneous equations

$a)\begin{cases} y=3x-4 \\ y=x^2-4x+6 \end{cases}$ $b)\begin{cases} y=4x+1 \\ y=2x^2-3x+4 \end{cases}$ $c)\begin{cases} y=x^2-6x+5 \\ y=x-1 \end{cases}$

Question 3:

$a)$ Solve simultaneously

$$x+y+3=0 \text{ and } y=2x^2+3x-1$$

$b)$ Interpret your solution graphically

Question 4:

Find the points of intersection of the curve $y=7-x^2$ *and* the line $2x+y=4$.

Question 5:

The line L has equation $y+2x=12$ and the curve C has equation $y=x^2-4x+9$.

$i)$ Show that the x-coordinate of the points of intersection of L and C satisfy the equation:

$$x^2-2x-3=0$$

$ii)$ Hence find the coordinates of the points of intersection of L and C.

Question 6:

Find the value of k such that the line $y=2x+k$ is a tangent to the curve $y=x^2+1$

Question 7:

A line has equation $y=mx+1$, where m is a constant.

A curve has equation $y=x^2-3x+10$.

$a)$ Show that the x-coordinate of any point of intersection of the line and the curve satisfies the equation

$$x^2-(m+3)x+9=0$$

$b)$ Find the values of m for which the equation

$$x^2-(m+3)x+9=0 \quad \text{has equal roots.}$$

$c)$ Describe geometrically the case when m takes either of the values found in part b).

$d)$ Find the set of values of m such that the line $y=mx+1$ intersects the curve $y=x^2-3x+10$ in two distinct points.

Simultaneous equations – exercises'answers

Question 1:

$$a)\begin{cases}3x+2y=7\\5x+y=7\end{cases}\Leftrightarrow\begin{cases}3x+2y=7\\-10x-2y=-14\end{cases}$$ By adding, we have

$-7x=-7,\ x=1\ \ and\ \ y=7-5x=7-5\times1=2$

The solution is $(1,2)$

$$b)\begin{cases}4x+3y=5\\3x+2y=4\end{cases}\Leftrightarrow\begin{cases}8x+6y=10\\-9x-6y=-12\end{cases}$$

$-x=-2\ ,\ x=2\ \ and\ \ 3\times2+2y=4\ \ gives\ \ y=-1$

The solution is $(2,-1)$

$$c)\begin{cases}4x-5y=1\\2x-3y=1\end{cases}\Leftrightarrow\begin{cases}4x-5y=1\\-4x+6y=-2\end{cases}$$

$y=-1\ \ and\ \ 2x-3\times-1=1\ \ gives\ \ x=-2$

The solution is $(-2,-1)$

Question 2:

$$a)\begin{cases}y=3x-4\\y=x^2-4x+6\end{cases}$$ *by subtraction,*

we obtain $x^2-7x+10=0$

$(x-2)(x-5)=0$

$x=2\ \ and\ \ y=3\times2-4=2\ \ or\ \ x=5\ and\ \ y=3\times5-4=11$

The solutions are $(2,2)$ *and* $(5,11)$

$$b)\begin{cases}y=4x+1\\y=2x^2-3x+4\end{cases}$$ *by subtraction,*

we obtain $2x^2-7x+3=0$

$(2x-1)(x-3)=0$

$x=\frac{1}{2}\ \ and\ \ y=4\times\frac{1}{2}+1=3\ \ or\ \ x=3\ and\ \ y=4\times3+1=13$

The solutions are $(\frac{1}{2},3)$ *and* $(3,13)$

$$c)\begin{cases}y=x^2-6x+5\\y=x-1\end{cases}$$ *by subtration ,*

we obtain $x^2-7x+6=0$

$(x-1)(x-6)=0$

$x=1\ \ and\ \ y=1-1=0\ \ or\ x=6\ \ and\ \ y=6-1=5$

The solutions are $(1,0)$ *and* $(6,5)$

Question 3:

$x+y+3=0\ \ and\ \ y=2x^2+3x-1$

By substitution, we obtain

$x+2x^2+3x-1+3=0$

$2x^2+4x+2=0$

$x^2+2x+1=0\qquad (x+1)^2=0$

$x=-1\ (repeated\ root)\ \ and\ \ y=-2$

The solution is $(-1,-2)$

$b)$ The line $x+y+3=0$ is TANGENT

to the parabola $y=2x^2+3x-1$ at the point $(-1,-2)$.

Question 4:

$y=7-x^2\ \ and\ \ 2x+y=4$

By substitution, we have

$2x+7-x^2=4$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3\ \ and\ \ y=7-9=-2$

or $\quad x=-1\ \ and\ \ y=7-1=6$

The solutions are $(3,-2)$ *and* $(-1,6)$

Question 5:

$y+2x=12\ \ and\ \ y=x^2-4x+9$

by substitution, we have

$x^2-4x+9+2x=12$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3\ \ and\ \ y=12-2x=6$

or $\quad x=-1\ and\ \ y=12+2=14$

The solutions are $(3,6)$ *and* $(-1,14)$

Question 6:

$y=2x+k$ is tangent to $y=x^2+1$

by subtraction, we obtain, $x^2-2x+1-k=0$

Because the line is tangent to the parabola,

the discriminant of this equation must be 0:

$(-2)^2-4\times1\times(1-k)=0$

$4-4+4k=0\qquad k=0$

Question 7:

$a)\ y=mx+1\ \ and\ \ y=x^2-3x+10$

By subtraction, we obtain $x^2-(3+m)x+9=0$

$b)$ The equation has equal root when the discriminant=0

$(-(3+m))^2-4\times1\times9=0$

$9+6m+m^2-36=0$

$m^2+6m-27=0$

$(m+9)(m-3)=0$

$m=-9\ \ or\ \ m=3$

$c)$ Two distinct points of intersection when the discriminant > 0

$(3+m)^2-36>0$

$(3+m)^2>36$

$3+m<-6\ \ or\ \ 3+m>6$

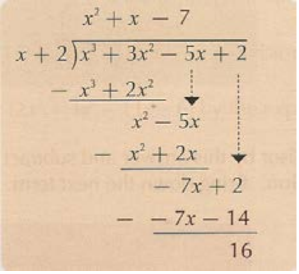
$m<-9\ \ or\ \ m>3$

Polynomials

Algebraic division

We use algebraic division to divide algebraic expression:

$(x^3+3x^2-5x+2)\div(x+2)$



$$\begin{array}{r} x^2+x-7 \\ x+2\overline{)x^3+3x^2-5x+2} \\ -\underline{x^3+2x^2} \\ x^2-5x \\ -\underline{x^2+2x} \\ -7x+2 \\ -\underline{-7x-14} \\ 16 \end{array}$$

Conclusion: $x^3+3x^2-5x+2=(x+2)(x^2+x-7)+16$

The remainder and factor theorem.

a) The remainder theorem.

$\boxed{\text{The remainder of the division of } f(x) \text{ by } (x-a) \text{ is } f(a).}$

Example: $f(x)=x^3+2x-5x+3$

- If we divide $f(x)$ by $(x-2)$, the remainder is $f(2)=(2)^3-2\times2-5\times2+3=-3$
- If we divide $f(x)$ by $(2x-3)$, the remainder is $f(\frac{3}{2})=(\frac{3}{2})^3-2\times\frac{3}{2}-5\times\frac{3}{2}+3=-\frac{33}{8}$

$\boxed{\text{The remainder of the division of } f(x) \text{ by } (ax-b) \text{ is } f\left(\frac{b}{a}\right).}$

b) The factor theorem.

The following statements are equivalent:

- a is a root of f
- $f(a)=0$ (The remainder of the division by $(x-a)$ is 0)
- $(x-a)$ is a factor of $f(x)$.

Example: $f(x)=x^3+2x-6x+3$

Show that $(x-1)$ is a factor of f.

$f(1)=1^3+2\times1-6\times1+3=1+2-6+3=0$

1 is a root of f , $(x-1)$ is a factor of f.

Factorising cubic expressions

To factorise a cubic expression, $f(x)$,

1) you need to find or be given a factor or a root of f, *for example* "a".

2) Use the algebraic division to factorise f by $(x-a)$

$f(x)=(x-a)(bx^2+cx+d)$

3) Factorise the quadratic expression bx^2+cx+d.

Example: a) *Show that 2 is a root of* $f(x)=2x^3+3x^2-11x-6$

b) *Factorise fully* $f(x)$.

a) $f(2)=2\times2^3+3\times2^2-11\times2-6=16+12-22-6=0$ $(x-2)$ *is a factor*

b) $f(x)=(x-2)(2x^2+7x+3)=(x-2)(2x+1)(x+3)$.

Polynomials - exercises

Question 1:

Use the long division to divide the cubic below.

In each case state the quotient and the remainder.

$a)\, x^3 - x^2 - 3x + 3 \; by \; (x+3)$

$b)\, x^3 - 3x^2 - 5x + 6 \; by \; (x-2)$

$c)\, x^3 + 2x^2 + 3x + 2 \; by \; (x+2)$

Question 2:

Find the remainder when the following are divided by

$i)\; (x+1) \qquad ii)\,(x-1) \qquad iii)\,(x-2)$

$a)\; f(x) = 6x^3 - x^2 - 3x - 12$

$b)\; f(x) = x^4 + 2x^3 - x^2 + 3x + 4$

$c)\; f(x) = x^5 + 2x^2 - 3$

Question 3:

The remainder when $x^3 + cx^2 + 17x - 10$ is divided by $(x+3)$ is 16.

Use the remainder theorem to the value of c.

Question 4:

$f(x) = (x+5)(x-2)(x-1) + k.$

If $(x+2)$ is a factor of $f(x)$, find the value of k.

Question 5:

Factorise fully $x^3 - 3x^2 + 3x - 1$, given that $(x-1)$ is a factor.

Question 6:

$f(x) = x^3 - 2x^2 - 4x + 8$

$a)$ *Factorise* $f(x)$

$b)$ *Find the solutions of* $f(x) = 0$

Question 7:

Find the roots of the cubic $f(x) = x^3 - x^2 - 3x + 3$

Question 8:

$f(x) = x^3 - px^2 + 17x - 10$ *and* $(x-5)$ *is a factor of* $f(x)$.

$a)$ *Find the value of* p.

$b)$ *Factorise* $f(x)$.

$c)$ *Find all solutions of* $f(x)$.

Question 9: exam question – Jan 2006

The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Hence express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

Polynomials – exercises'answers

Question 1:

$a)\ x^3-x^2-3x+3=(x+3)(x^2-4x+9)-24$

$b)\ x^3-3x^2-5x+6=(x-2)(x^2-x-7)-8$

$c)\ x^3+2x^2+3x+2=(x+2)(x^2+3)-4$

Question 2:

$a)\ i)\ f(-1)=-6-1+3-12=-16$

$ii)\ f(1)=6-1-3-12=-10$

$iii)\ f(2)=48-4-12-12=20$

$b)\ i)\ f(-1)=1-2-1-3+4=-1$

$ii)\ f(1)=1+2-1+3+4=7$

$iii)\ f(2)=16+16-4+6+4=38$

$c)\ i)\ f(-1)=-1+2-3=-2$

$ii)\ f(1)=1+2-3=0$

$iii)\ f(2)=32+8-3=37$

Question 3:

$f(x)=x^3+cx^2+17x-10$

the remiander of the division by $(x+3)$ *is* $f(-3)$

$f(-3)=-27+9c-51-10=16$

$9c=104$

$c=\dfrac{104}{9}$

Question 4:

$f(x)=(x+5)(x-2)(x-1)+k$

$(x+2)$ is a factor of f, means $f(-2)=0$

$f(-2)=3\times-4\times-3+k=0$

$k=-36$

Question 5:

$x^3-3x^2+3x-1=(x-1)(x^2-2x+1)$

(using the algebraic division)

$=(x-1)(x-1)^2=(x-1)^3$

Question 6:

$f(x)=x^3-2x^2-4x+8$

$a)\ f(2)=8-8-8+8=0\ \ so$

$(x-2)\ is\ a\ \ factor\ \ of\ \ f$

$x^3-2x^2-4x+8=(x-2)(x^2-4)$

$f(x)=(x-2)(x+2)(x-2)=(x-2)^2(x+2)$

$b)\ f(x)=0\ \ \ for\ \ x=2\ \ or\ \ x=-2$

Question 7:

$f(x)=x^3-x^2-3x+3$

$f(1)=1-1-3+3=0$

$so\ (x-1)\ is\ a\ \ factor\ of\ \ f$

$f(x)=(x-1)(x^2-3)$

$f(x)=(x-1)(x+\sqrt{3})(x-\sqrt{3})$

Question 8:

$f(x)=x^3-px^2+17x-10$

$(x-5)$ is a factor of f so $f(5)=0$

$f(5)=5^3-25p+85-10=0$

$p=8$

$b)\ f(x)=(x-5)(x^2-3x+2)$

$=(x-5)(x-2)(x-1)$

$c0\ f(x)=0\ \ for\ \ x=5\ or\ \ x=2\ or\ \ x=1$

Question 9:

$a)\ i)\ p(x)=x^3+x^2-10x+8$

$(x-2)$ is a factor of $p\ if\ \ p(2)=0$

$p(2)=8+4-20+8=12-20+8=0$

$(x-2)\ is\ a\ \ factor\ of\ \ p$

$ii)\ p(x)=(x-2)(x^2+3x-4)$

$=(x-2)(x+4)(x-1)$

$b)$ The graph crosses the x-axis at (2,0), (-4,0) and (1,0)

The graph crosses the y-axis at (0,8)

Coordinates geometry

Straight lines

Gradient of a line

Two points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ *form a line AB*

The gradient of the line AB is $\boxed{m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}}$

Equation of a line

An equation of the line going through $A(x_1, y_1)$ with gradeint m is

$$\boxed{y - y_1 = m\left(x - x_1\right)}$$

After re-arranging this equation, you could have the forms:

$y = mx + c$ explicit eqaution

or $ax + by = c$ implicit equation

Parallel and perpendicular lines

Consider two lines $\text{L}_1 : y = m_1 x + c_1$ *and* $L_2 : y = m_2 x + c_2$

- L_1 *and* L_2 *are parallel when* $m_1 = m_2$

- L_1 *and* L_2 *are perpendicular when* $m_1 \times m_2 = -1$

$$m_1 = -\frac{1}{m_2}$$

Mid-point of a line segment

The mid-point of $A(x_1, y_1)$ *and* $B(x_2, y_2)$ *is* $\boxed{I\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)}$

Distance between two points

The distance between $A(x_1, y_1)$ *and* $B(x_2, y_2)$ *is*

$$\boxed{AB = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Straight lines - exercises

Question 1:

Give the equation for the following straight lines AB in the form $y = mx + c$

$a)\ A(4,1)\ \ B(0,-3)$ $b)\ A(12,-3)\ B(14,1)$ $c)\ A(5,7)\ \ B(-2,5)$

Question 2:

Write the following equations in the form $ax + by + c = 0$ *where* a, b *and* c *are* integers.

$a)\ y = 5x + 2$ $b)\ 3y = \frac{1}{2}x + 3$ $c)\ 4y - 1 = 2(x-1)$ $d)\ y - 3 = \frac{1}{3}(x+2)$

Question 3:

Give the equation for the following straight lines AB in the form $ax + by + c = 0$

$a)\ A(5,2)\ \ B(3,4)$ $b)\ A(9,-1)\ B(7,2)$ $c)\ A(-6,1)\ \ B(4,0)$

Question 4:

For each of the following,

$i)$ *find* the distance AB

$ii)$ *find the* midpoint I of AB.

a) $A(3,4)\ B(-2,6)$ $b)\ A(6,2)\ B(-3,-2)$ $c)\ A(2,-4)\ B(-6,-3)$

Question 5:

A line L_1 has equation $y = 2x - 3$. Consider two points A(3,2) and $B(-1,4)$.

a) Work out the equation of the line parallel to L_1 going through A.

Give your answers in the form $y = mx + c$.

b) Work out the equation of the line perpendicular to L_1 going through B.

Give your answers in the form $ax + by + c = 0$.

Question 6: exam question

The triangle ABC has vertices $A(1, 3)$, $B(3, 7)$ and $C(-1, 9)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence show that angle ABC is a right angle. *(2 marks)*

(b) (i) Find the coordinates of M, the mid-point of AC. *(2 marks)*

(ii) Show that the lengths of AB and BC are equal. *(3 marks)*

(iii) Hence find an equation of the line of symmetry of the triangle ABC. *(3 marks)*

Question 7: exam question

The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

Straight lines – exercises’answers

Question 1:

$a)\ m_{AB}=\frac{-3-1}{0-4}=1 \qquad y-y_A=m(x-x_A)$

$y-1=x-4$

$y=x-3$

$b)\ m_{AB}=\frac{1+3}{14-12}=2 \qquad y-y_A=m(x-x_A)$

$y+3=2(x-12)$

$y=2x-27$

$c)\ m_{AB}=\frac{5-7}{-2-5}=\frac{2}{7} \qquad y-y_A=m(x-x_A)$

$y-7=\frac{2}{7}(x-5)$

$y=\frac{2}{7}x+\frac{39}{7}$

Question 2:

$a)\ 5x-y+2=0 \qquad b)\ x-6y+6=0$

$c)\ 2x-4y-1=0 \qquad d)\ x-3y+11=0$

Question 3:

$a)\ m_{AB}=\frac{4-2}{3-5}=-1 \qquad y-y_A=m(x-x_A)$

$y-2=-1(x-5)$

$x+y=7$

$b)\ m_{AB}=\frac{2+1}{7-9}=-\frac{3}{2} \qquad y-y_A=m(x-x_A)$

$y+1=-\frac{3}{2}(x-9)$

$3x+2y-25=0$

$c)\ m_{AB}=\frac{0-1}{4+6}=-\frac{1}{10} \qquad y-y_A=m(x-x_A)$

$y-1=-\frac{1}{10}(x+6)$

$x+10y-4=0$

Question 4:

$a)\ AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}=\sqrt{5^2+2^2}=\sqrt{29}$

$Mid-\text{point}\ I\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right)=\left(\frac{1}{2},5\right)$

$b)\ AB=\sqrt{9^2+4^2}=\sqrt{97} \qquad I\left(\frac{3}{2},0\right)$

$c)\ AB=\sqrt{8^2+1^2}=\sqrt{65} \qquad I\left(-2,\frac{-7}{2}\right)$

Question 5:

$a)$ The gradient of L_1 *is* 2

the line parallel to L_1 has gradient 2

Its equation is $y-y_A=2(x-x_A)$

$y-2=2(x-3)$

$y=2x-4$

$b)$ The line perpendicular to L_1 *has gradient* $-\frac{1}{2}$

Its equation is $y-y_B=-\frac{1}{2}(x-x_B)$

$y-4=-\frac{1}{2}(x+1)$

$x+2y-7=0$

Question 6:

$a)i)\ m_{AB}=\frac{7-3}{3-1}=2$

$ii)$ the gradient of BC is $m_{BC}=\frac{9-7}{-1-3}=-\frac{1}{2}$

$m_{AB}\times m_{BC}=2\times-\frac{1}{2}=-1$

the line AB and BC are perpendicular,

the triangle ABC is a right-angled triangle.

$b)i)\ M(0,6)$

$ii)\ AB=\sqrt{2^2+4^2}=\sqrt{20}=2\sqrt{5}$

$BC=\sqrt{4^2+2^2}=\sqrt{20}=2\sqrt{5}$

$AB=BC$

$iii)$ The triangle ABC is a right-angled isosceles triangle,

Its line of symmetry is the line AM

The gradient of AM $=m_{AM}=\frac{6-3}{0-1}=-3$

The eqaution of AM is $y-y_M=-3(x-x_M)$

$y-6=-3x \qquad y=-3x+6$

Question 7:

$a)\ 3x+4y=7$ *and* B belongs to the line so

$3\times5+4\times k=7 \qquad 15+4k=7$

$k=-2$

$ii)\ A(1,1)$ *and* $B(5,-2) \qquad M(3,-\frac{1}{2})$

$b)\ m_{AB}=\frac{-2-1}{5-1}=\frac{-3}{4}$

$c)i)\ m_{AC}=-\frac{1}{m_{AB}}=\frac{4}{3}$

$ii)$ The equation of AC is : $y-1=\frac{4}{3}(x-1)$

$3y-3=4x-4$ *gives* $4x-3y-1=0$

$iii)\ C(x,0)$ *belongs to the line AC so* $4x-0-1=0$

$x=\frac{1}{4}$

Circles

Equation of a circle.

Consider a circle C with centre $\Omega(a,b)$ and radius r.

Any point $M(x,y)$ on this circle satisfies $\Omega M = r$.

This is equivalent to $\Omega M^2 = r^2$

$$\boxed{(x-a)^2+(y-b)^2=r^2}$$

In particular if the centre is (0,0), the equation becomes $x^2+y^2=r^2$.

Re-arranging circle equations

In its factorised form, it is easy to read centre and radius from the equation of a circle:

$(x-3)^2+(y+1)^2=9$ is the eqation of the circle centre (3,-1) radius 3.

But an equation can be given in its expanded form: $x^2+y^2+2ax+2by+c=0$.

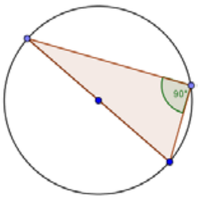
To re-arrange this equation, use the complete square form with x^2+2ax *and* y^2+2bx.

$$\begin{aligned} \textit{example}: \quad & x^2+y^2+6x+4y-3=0 \\ & x^2+6x \quad + \quad y^2+4y \quad -3=0 \\ & (x+3)^2-9 \ + \ (y+2)^2-4 \ -3=0 \\ & (x+3)^2+(y+2)^2=16 \end{aligned}$$

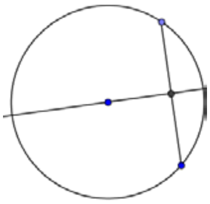
This is the circle centre $(-3,-2)$ radius $r=4$

Circle properties

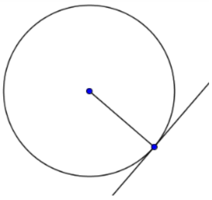
a) Any point joined to the extremities of a diameter form a right-angled triangle.



b) The perpendicular bisector of a chord goes through the centre of the circle.



c) A tangent to the circle is perpendicular to the radius at its point of contact.



Work out exercise: Tangent to a circle

The circle C has centre O(2,1) and radius 25.

The point A(6,4) belongs to the circle.

Work out the equation of the tangent to the circle at A.

_ _

•The tangent at A is PERPENDICULAR to the radius OA.

The gradient of OA is $m_1=\dfrac{4-1}{2-6}=\dfrac{3}{-4}$ The gradient of the tangent is therefore $-\dfrac{1}{m_1}=\dfrac{4}{3}$

The equation of the tangent is : $y-4=\dfrac{4}{3}(x-6)$

$$3y-12=4x-24$$

$$4x-3y-12=0$$

Circles - exercises

Question 1:

For each of the following circles find the radius and the coordinates of the centre.

$a)\, x^2 + y^2 + 2x - 6y - 6 = 0$

$b)\, x^2 + y^2 - 2y - 4 = 0$

$c)\, x^2 + y^2 - 6x - 4y = 12$

$d)\, x^2 + y^2 - 10x + 6y + 13 = 0$

Question 2:

A circle has the equation $(x+1)^2 + (y-2)^2 = 13$.

The circle passess through $A(-3,-1)$.

Find the equation of the tangent at A in the form $ax + by + c = 0$.

Question 3:

A circle has the equation $(x-3)^2 + (y-4)^2 = 25$.

The circle passess through $A(7,1)$.

Find the equation of the tangent at A in the form $ax + by = c$.

Question 4:

A circle has the equation $x^2 + y^2 + 2x - 7 = 0$.

Find the equation of the tangent to the circle at $(-3,2)$.

Question 5:

A circle has the equation $x^2 + y^2 + 2x + 4y = 5$.

Find the equation of the normal to the circle at $(0,-5)$.

Give your answer in the form $ax + by = c$.

Question 6: exam question

A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y-b)^2 = k$$

(2 marks)

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

Question 7: exam question

A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$

(3 marks)

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

Circles – exercises ‘answers

Question 1:

$a)\, x^2+y^2+2x-6y-6=0$

$(x+1)^2-1+(y-3)^2-9-6=0$

$(x+1)^2+(y-3)^2=16=4^2$

The circle has centre $(-1,3)$ and radius 4

$b)\, x^2+y^2-2y-4=0$

$(x)^2+(y-1)^2-1-4=0$

$(x)^2+(y-1)^2=\sqrt{5}^2$

The circle has centre $(0,1)$ and radius $\sqrt{5}$

$c)\, x^2+y^2-6x-4y=12$

$(x-3)^2-9+(y-2)^2-4=12$

$(x-3)^2+(y-2)^2=5^2$

The circle has centre $(3,2)$ and radius 5

$d)\, x^2+y^2-10x+6y+13=0$

$(x-5)^2-25+(y+3)^2-9+13=0$

$(x-5)^2+(y+3)^2=\sqrt{21}^2$

The circle has centre $(5,-3)$ and radius $\sqrt{21}$

Question 2:

The centre of the circle is $I(-1,2)$.

The tangent to the circle at $A(-3,-1)$

is perpendicular to the radius IA

Gradient of $IA\;=\;m_{IA}=\dfrac{-1-2}{-3+1}=\dfrac{3}{2}$

The gradient of the tangent is $m=-\dfrac{1}{m_{IA}}=-\dfrac{2}{3}$

The equation of the tangent is $y-y_A=m(x-x_A)$

$$y+1=-\frac{2}{3}(x+3)$$

$$3y+3=-2x-6$$

$$2x+3y=-9$$

Question 3:

The centre of the circle is $\Omega(3,4)$

The radius $A\Omega$ has gradient $m_{A\Omega}=\dfrac{4-7}{3-1}=-\dfrac{3}{2}$

The tangent to the circle at A is

perpendicular to the radius: its gradient is $\dfrac{2}{3}$

The equation of the tangent is $y-1=\dfrac{2}{3}(x-7)$

$$3y-3=2x-14$$

$$2x-3y=11$$

Question 4:

$x^2+y^2+2x-7=0$

$(x+1)^2+y^2=8$

The centre of the circle is $\Omega(-1,0)$

The gradient of the radius $A\Omega$ is $m_{A\Omega}=\dfrac{0-2}{-1+3}=-1$

The gradient of the tangent is 1

The equation of the tangent is $y-2=1(x+3)$

$$y=x+5$$

Question 5:

$x^2+y^2+2x+4y=5 \quad and \quad A(0,-5)$

$(x+1)^2+(y+2)^2=10$

The centre of the circle is $\Omega(-1,-2)$

The normal is the radius $A\Omega$ and

its the gradient is $m_{A\Omega}=\dfrac{-2+5}{-1-0}=-3$

The equation of the normal is

$y+5=-3(x-0)$

$3x+y=-5$

Question 6:

$x^2+y^2-10y+20=0$

$a)\, x^2+(y-5)^2=5$

$b)$ The centre has coordinates $(0,5)$

The radius is $r=\sqrt{5}$

Question 7:

$x^2+y^2-6x+10y+9=0$

$a)\, (x-3)^2-9+(y+5)^2-25+9=0$

$(x-3)^2+(y+5)^2=25$

$b)\, i)$ The centre is $(3,-5)$

$ii)$ The radius is $r=5$

Transformations of graphs

Translations of graphs

A curve C_f has equation $y = f(x)$.

"a" *is a positive number*.

- The curve with equation $y = f(x) - b$ is the translation of C_f *by vector* $\begin{pmatrix} 0 \\ -b \end{pmatrix}$
- The curve with equation $y = f(x+a)$ is the translation of C_f *by vector* $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

Combined translations

- The curve with equation $y + b = f(x+a)$ is the translation of C_f *by vector* $\begin{pmatrix} -a \\ -b \end{pmatrix}$

Examples: The curve with equation $y = (x-3)^2 + 2$ is the translation of

the curve $y = x^2$ *by vector* $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

The circle $(x-3)^2 + (y+1)^2 = 9$ is the translation of the circle $x^2 + y^2 = 9$

by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Parabolas

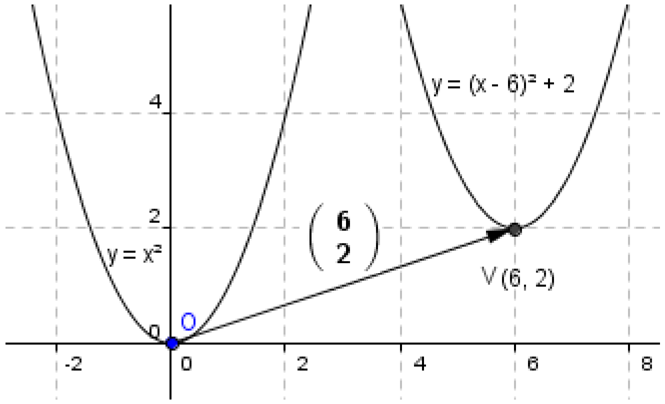
$$\boxed{\text{All parabolas of the form } y = x^2 + bx + c \text{ are the image of the parabola } y = x^2}$$

To work out the vector of this translation, use the completed square form:

$$y = x^2 + bx + c = (x+p)^2 + q$$

The vector of the translation is $\begin{pmatrix} -p \\ q \end{pmatrix}$.

Note: This vector is the vector $\overrightarrow{\text{OV}}$, where $V(-p, q)$ is the vertex of the parabola.



Transforming graphs - exercises

Question 1:

(a) Express $x^2 + 8x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

Question 2:

(a) Express $x^2 - 3x + 4$ in the form $(x - p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)*

Question 3:

(a) Express $(x - 5)(x - 3) + 2$ in the form $(x - p)^2 + q$, where p and q are integers. *(3 marks)*

(b) (i) Sketch the graph of $y = (x - 5)(x - 3) + 2$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(ii) Write down an equation of the tangent to the graph of $y = (x - 5)(x - 3) + 2$ at its vertex. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = (x - 5)(x - 3) + 2$. *(3 marks)*

Question 4:

(a) (i) Express $x^2 + 2x + 5$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence show that $x^2 + 2x + 5$ is always positive. *(1 mark)*

(b) A curve has equation $y = x^2 + 2x + 5$.

(i) Write down the coordinates of the minimum point of the curve. *(2 marks)*

(ii) Sketch the curve, showing the value of the intercept on the y-axis. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 2x + 5$. *(3 marks)*

Transforming graphs – exercises 'answers

Question 1:

$a)\, x^2+8x+19=(x+4)^2+3$

$b)\, x^2+8x+19=0$

$(x+4)^2+3=0$

$(x+4)^2=-3$

No solution

(a squared number is always positive)

$c)$ The minimum point is $(-4,3)$

The graph crosses the y-axis at (0,19)

$d)$ Translation vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Question 2:

$a)\, x^2-3x+4=(x-\frac{3}{2})^2-\frac{9}{4}+4$

$\left(x-\frac{3}{2}\right)^2+\frac{7}{4}$

$b)$ The minimum point is $\left(\frac{3}{2},\frac{7}{4}\right)$

$c)$ Translation vector $\begin{pmatrix} \frac{3}{2} \\ \frac{7}{4} \end{pmatrix}$

Question 3:

$a)\, (x-5)(x-3)+2=x^2-8x+17=(x-4)^2+1$

$b)\, i)$ Minimum point $(4,1)$

The graph crosses the y-axis at (0,17)

$ii)\, y=1$

$c)$ Translation vector $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Question 4:

$a)\, i)\, x^2+2x+5=(x+1)^2+4$

$ii)$ For all x, $(x+1)^2 \geq 0$

$(x+1)^2+4 \geq 4$

$y \geq 4 \geq 0$

$b)\, i)$ Minimum point $(-1,4)$

$ii)$ The graph crosses the y-axis at (0,5)

c) Translation vector $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

Calculus

Differentiation

Notation

- The function you get from differentiating y with respect to x is called the DERIVATIVE of y and it's written $\frac{dy}{dx}$.
- $\frac{dy}{dx}$ is the rate of change of y with respect to x.

  It is the gradient of the curve/the tangent to the curve.
- The notation $f\,'(x)\,(f \text{ prime of } x)$ is sometimes used instead of $\frac{dy}{dx}$.

Differentiation from first principle.

Consider two points on a curve $A(x, f(x))$ *and* a point B close to A

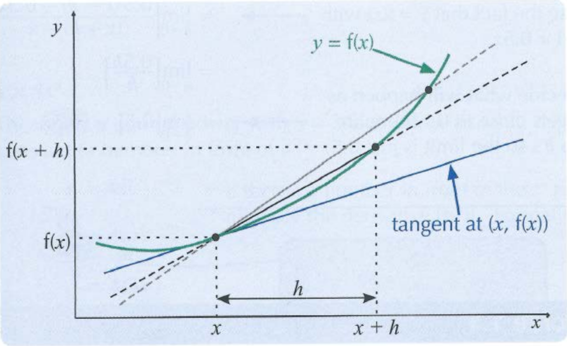
$B(x+h, f(x+h))$ *where H is "small".*

The chord AB has gradient $\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+}{}$

When B get closer and closer to A, h tends to 0.

If $\frac{f(x+h)-f(x)}{h}$ has a value when h tends to 0,

this value is the gradient of the curve at A: $f\,'(x)$.



Example: $f(x) = x^2$

Let's work out the gradient of the curve at $x = 3$.

•$A(3,3^2)$ *and* $B(3+h,(3+h)^2)$

the gradient of AB: $m = \frac{(3+h)^2 - 3^2}{3+h-3} = \frac{9+6h+h^2-9}{h} = 6+h$

When h *tends to* 0, m *tends to* 6:

Conclusion: $\frac{dy}{dx}(x=3) = f\,'(3) = 6$

Differentiating polynomials

- $\boxed{\text{if } y = x^n \text{ then } \frac{dy}{dx} = nx^{n-1}}$
- *if* $y = x^n + x^p$ *then* $\frac{dy}{dx} = nx^{n-1} + px^{p-1}$
- *if* $y = k \times x^n$ *then* $\frac{dy}{dx} = k \times nx^{n-1}$ *where* $k \in \mathbb{R}$.

Example: $y = x^4 \qquad \frac{dy}{dx} = 4x^3$

$y = 5x^6 \qquad \frac{dy}{dx} = 5 \times 6x^5 = 30x^5$

$y = 3x^4 + 5x^3 + x \qquad \frac{dy}{dx} = 12x^3 + 15x^2 + 1$

Differentiation - exercises

Question 1:

Differentiate the following functions

$a)\ f(x) = 4x^3 - x^2$ $\quad b)\ f(x) = x + 1$ $\quad c)\ f(x) = x^4 - x$

$d)\ f(x) = 3x^2 + x - 5$ $\quad e)\ f(x) = -2x^2 + 4x - 6$

Question 2:

Find the gradient of each of the following curves

a) $y = x^4 - x^2 + 2 \quad when \quad x = 3$ $\quad b)\ y = 2x^5 + 4 \quad when \quad x = -2$

$c)\ y = x(x-1)(x-2) \quad at\ (4, 24)$ $\quad d)\ y = 5(x^2 - 1) + x\ at\ (-1, -1)$

Question 3:

For each of the following functions, find the coordinates of the point or points where the gradient is 0

$a)\ y = x^2 - 2x$ $\quad b)\ y = 3x^2 + 4x$ $\quad c)\ y = 5x^2 - 3x$ $\quad d)\ y = 9x - 3x^3$

Differentiation – exercises ‘answers

Question 1:

$a)\frac{df}{dx}=12x^2-2x$ $b)\frac{df}{dx}=1$ $c)\frac{df}{dx}=4x^3-1$

$d)\frac{df}{dx}=6x+1$ $e)\frac{df}{dx}=-4x+4$

Question 2:

$a)\frac{dy}{dx}=4x^3-2x$ $\frac{dy}{dx}(x=3)=4\times3^3-2\times3=102$

$b)\frac{dy}{dx}=10x^4$ $\frac{dy}{dx}(x=-2)=10\times(-2)^4=160$

$c)\,y=x^3-3x^2+2x$ $\frac{dy}{dx}=3x^2-6x+2$ $\frac{dy}{dx}(x=4)=26$

$d)\,y=5x^2+x-5$ $\frac{dy}{dx}=10x+1$ $\frac{dy}{dx}(x=-1)=-9$

Question 3:

$a)\frac{dy}{dx}=2x-2=0 \quad for \quad x=1$

$$y=1^2-2=-1$$

$(1,-1)$

$b)\frac{dy}{dx}=6x+4=0 \quad for \quad x=-\frac{2}{3}$

$$y=3\times\left(-\frac{2}{3}\right)^2+4\times\left(-\frac{2}{3}\right)=-\frac{4}{3}$$

$\left(-\frac{2}{3},-\frac{4}{3}\right)$

$c)\frac{dy}{dx}=10x-3=0 \quad for \quad x=\frac{3}{10}$

$$y=-\frac{9}{20}$$

$\left(\frac{3}{10},-\frac{9}{20}\right)$

$d)\frac{dy}{dx}=9-9x^2=0 \quad for \quad x^2=1$

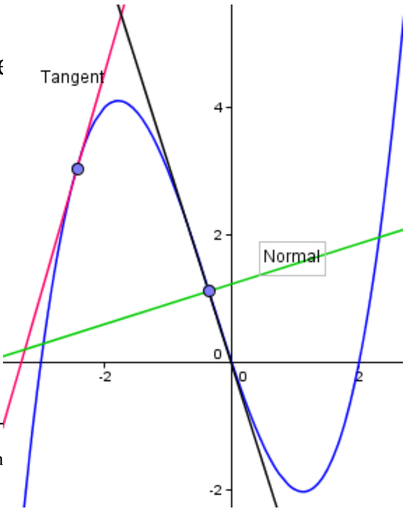
$$x=-1 \;\; or \;\; x=1$$

$$y=-6 \; or \; x=6$$

$(-1,-6) \;\; or \;\; (1,6)$

Using differentiation

Tangents and normals



- A tangent to a curve is a straight line that touches the curve. The gradient of the tangent is the same as the gradient of the curve at the point of contact.

- A normal to a curve at a point A is a straight line which is perpendicular to the tangent to the curve at A.

Consequence: $m_{\text{tangent}} \times m_{normal} = -1$ or $m_{normal} = \dfrac{-1}{m_{\text{tangen}}}$

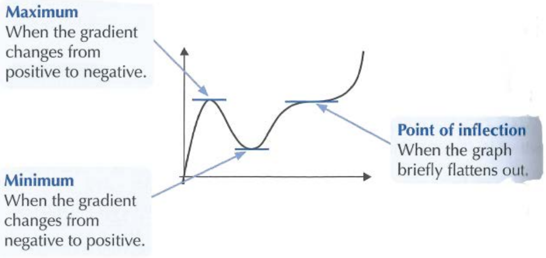
Second order derivatives

- If you differentiate y with respect to x, you get the derivative $\dfrac{dy}{dx}$.
- If you differentiate $\dfrac{dy}{dx}$ with respect to x, you get the second order derivative $\dfrac{d^2y}{dx^2}$.
- The second order derivative gives the rate of change of the gradient of the curve with respect to x.
- If $y = f(x)$, we usethe notation $\dfrac{d^2y}{dx^2} = f\,''(x)$.

Stationary points

Stationary points occur when the gradient of the curve is zero: $\dfrac{dy}{dx} = 0$

There are three kinds of stationary points:



To work out the coordinates of a stationary point:

1) Work out $f\,'(x)$
2) Solve the equation $f\,'(x) = 0$
3) Substitute the x-values found into the original equation to find y-values.

Minimum and maximum points

If the point $A(x_A, f(x_A))$ is a stationary point of the curve $y = f(x)$,
The nature of the point A is determined by the sign of the second order derivative:

If $\dfrac{d^2y}{dx^2}(x = x_A) < 0$, A is a maximum point

If $\dfrac{d^2y}{dx^2}(x = x_A) = 0$, A is point of inflection

If $\dfrac{d^2y}{dx^2}(x = x_A) > 0$, A is a minimum point

Using differentiation - exercises

Question 1:

Find the equation of the tangent to each of these curves at the given point.

Give your answer in the form $y = mx + c$

$a)\ y = 9x - x^2 \quad (1,7)$ $\qquad b)\ y = x^3 - 2x + 3 \quad (2,7)$

$c)\ y = (x+2)(2x-3) \quad (2,4)$

Find the equation of the tangent to each of these curves at the given point.

Give your answer in the form $ax + by + c = 0$ $where\ a, b\ and\ c$ are integers.

$d) y = 3x^2 - 4x + 2 \quad (2,6)$ $\qquad e)\ y = x^2(x+4) - 5x \quad (-1,8)$

Question 2:

The curve with equation $y = x^3 + x^2 + x + 5$ passes through the point A(1,8).

a) Work out the equation of the tangent to the curve at A.

(give your answer in theform $y = mx + c$)

$b)$ Work out the equation of the normal to the curve at A

in the form $ax + by + c = 0$.

Question 3:

Find the stationary points on the graphs of the following functions

and say if they are maximum or minimum turning points.

$a)\ f(x) = 8x^3 + 16x^2 + 8x + 1$

$b)\ f(x) = 2x^4 + x$

$c)\ f(x) = x^3 - 3x^2 + 4$

Question 4:

The curve given by the function $f(x) = x^3 + ax^2 + bx + c$ has a stationary point with coordinates (3,10). If $f''(x) = 0$ at (3,10), find $a, b\ and\ c$.

Question 5:

Given that the curve with equation $y = x^4 + kx^3 + x^2 + 17$ has only one stationary point,

show that $k^2 < \frac{32}{9}$.

Find the coordinates of the stationary point and say if it is a maximum or a minimum point.

Question 6:

A ball is catapulted vertically with an initial speed of 30m/s.

After t seconds the height h of the ball is given by $h = 30t - 7.5t^2$.

Use calculus to find the maximum height the ball reaches.

Using differentiation – exercises ‘answers

Question 1:

$a)\frac{dy}{dx}=9-2x \qquad \frac{dy}{dx}(x=1)=9-2=7$

The equation of the tangent is $y-7=7(x-1)$

$y=7x$

$b)\frac{dy}{dx}=3x^2-2 \qquad \frac{dy}{dx}(x=2)=10$

The equation of the tangent is $y-7=10(x-2)$

$y=10x-13$

$c)\, y=2x^2+x-6 \qquad \frac{dy}{dx}=4x+1$

$\frac{dy}{dx}(x=2)=9$

The equation of the tangent is $y-4=9(x-2)$

$y=9x-14$

$d)\frac{dy}{dx}=6x-4 \qquad \frac{dy}{dx}(x=2)=8$

The equation of the tangent is $y-6=8(x-2)$

$8x-y-10=0$

$e)\ \frac{dy}{dx}=3x^2+8x-5 \qquad \frac{dy}{dx}(x=-1)=-10$

The equation of the tangent is $y-8=-10(x+1)$

$10x+y+2=0$

Question 2:

$a)\frac{dy}{dx}=3x^2+2x+1 \qquad \frac{dy}{dx}(x=1)=6$

The equation of the tangent is $y-8=6(x-1)$

$y=6x+2$

$b)$ The gradient of the normal is $-\frac{1}{6}$

The equation of the normal is $y-8=-\frac{1}{6}(x-1)$

$6y-48=-x+1$

$x+6y-49=0$

Question 3:

$a)\frac{df}{dx}=24x^2+32x+8=0$

$3x^2+4x+1=0$

$(3x+1)(x+1)=0$

$x=-\frac{1}{3} \text{ or } x=-1$

$f(-\frac{1}{3})=-\frac{5}{27} \text{ or } f(-1)=1$

The stationary points are $A(-\frac{1}{3},\frac{5}{27}) \text{ and } B(-1,1).$

$\frac{d^2f}{dx^2}=48x+32$

$\frac{d^2f}{dx^2}(x=-\frac{1}{3})=16>0$ A is a minimum

$\frac{d^2f}{dx^2}(x=-1)=-16<0$ B is a maximum

Question 4:

$f(x)=x^3+ax^2+bx+c$

$\frac{df}{dx}=3x^2+2ax+b$

$\frac{df}{dx}(x=3)=27+6a+b=0$

$6a+b=-27$

$\frac{d^2f}{dx^2}=6x+2a$

$\frac{d^2f}{dx^2}(x=3)=18+2a=0 \qquad a=-9$

$\text{and } b=-27-6a=27$

$f(x)=x^3-9x^2+27x+c$

$f(3)=27-81+81+c=10$

$c=10-27=-17$

$Conclusion: \ f(x)=x^3-9x^2+27x-17$

Question 5:

$\frac{dy}{dx}=4x^3+3kx^2+2x=0$

$x\left(4x^2+3kx+2\right)=0$

$so\ x=0 \ and \ y=17 \ \ give$

THE stationary point

This means that $4x^2+3kx+2$

has no real roots:

The discriminant is <0

$(3k)^2-4\times4\times2<0$

$9k^2-32<0$

$k^2<\frac{32}{9}.$

The stationary point is $(0,17)$

$\frac{d^2y}{dx^2}=12x^2+6kx+2$

$\frac{d^2y}{dx^2}(x=0)=2>0.$

The point $(0,17)$ is a minimum.

Question 6:

$\frac{dh}{dt}=30-15t=0 \quad for \ \ t=2$

$for\ t=2,\ h=60-30=30$

The maximum height reached

is 30m at $t=2$ seconds

Integration

Indefinite integrals

Integration is the "opposite" of differentiation.

If $y = f(x)$ is a given function, to integrate f means finding a function $F(x)$

$$\text{so that } \frac{dF}{dx} = f$$

F is called an INTEGRAL of f and it is noted $\int f(x)dx$

Note: An integral is not unique. If $F(x)$ is an integral, then $F(x) + c$ is also one.

$$\int f(x)dx = F(x) + c \qquad \textit{where c is a constant}$$

Integrating x^n

The formula tells you how to integrate *powers of* x.

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \qquad \textit{for all } n \neq -1$$

Rules of integrations

$f(x)$ *and* $g(x)$ are two functions , a is a constant

$$\int a \times f(x)dx = a \times \int f(x)dx$$

$$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

Examples: $\int x^3 dx = \frac{1}{4}x^4 + c$, $\int (3x^2 - 3x)\,dx = 3 \times \frac{1}{3}x^3 - 3 \times \frac{1}{2}x^2 + c = x^3 - \frac{3}{2}x^2 + c$

Integrating to find the equation of a curve

A curve $y = f(x)$ is going through the point $A(x_A, y_A)$ and $\frac{dy}{dx} = f'(x)$ *is given.*

To find the equation of the curve,

- integrate $f'(x)$: $\int f'(x)dx = F(x) + c$
- find the value of the constant c using the coordinates of A.

Example: The curve $y = f(x)$ goes through A(2,9) and $\frac{dy}{dx} = 3x^2$.

Find the eqaution of the curve.

- $\int 3x^2 dx = x^3 + c$ *so* $y = x^3 + c$
- $A(2,9)$ belongs to the curve so $9 = (2)^3 + c$ $\quad c = 1$
- the eqaution of the curve is $y = x^3 + 1$

Integration - exercises

Question 1:

Find the following

$a)\int x^2dx \qquad b)\int 7x^4dx \qquad c)\int \frac{x}{2}dx \qquad d)\int -\frac{1}{4}xdx$

$e)\int (x^6+x)dx \qquad f)\int (10x^2+4x+1)dx \qquad g)\int x(x+2)dx$

$h)\int \frac{2x^3+4x^2}{x}dx \qquad i)\int \frac{1}{2}x^3+\frac{2x^5}{x^2}dx$

Question 2:

For each of the following, the curve $y=f(x)$ goes through the given point. Find $f(x)$

$a)\ f'(x)=4x^3 \qquad (0,5)$

$b)\ f'(x)=3x^2-4x+3 \qquad (1,-3)$

$c)\ f'(x)=6x(x+2) \qquad (-1,1)$

$d)\ f'(x)=\frac{9x^3+2x^2}{x} \qquad (-1,2)$

Question 3:

Consider $\frac{dy}{dt}=\left(t-3\right)^2$.

Given that $y=9$ *when* $t=4$, find y as a function of t.

Question 4:

The curve $y=f(x)$ has derivative $f'(x)=x^3+\frac{x}{2}+3$ and passes through $(1,-1)$.

Find the equation of the curve.

Integration - exercises

Question 1:

$a)\int x^2dx=\frac{1}{3}x^3+c$

$b)\int 7x^4dx=\frac{7}{5}x^5+c$

$c)\int \frac{x}{2}dx=\frac{1}{4}x^2+c$

$d)\int -\frac{1}{4}xdx=-\frac{1}{8}x^2+c$

$e)\int (x^6+x)dx=\frac{1}{7}x^7+\frac{1}{2}x^2+c$

$f)\int (10x^2+4x+1)dx=\frac{10}{3}x^3+2x^2+x+c$

$g)\int x(x+2)dx=\frac{1}{2}x^2+x^2+c$

$h)\int \frac{2x^3+4x^2}{x}dx=\int (2x^2+4x)dx=\frac{2}{3}x^3+2x^2+c$

$i)\int \frac{1}{2}x^3+\frac{2x^5}{x^2}dx=\int \frac{1}{2}x^3+2x^3dx=\frac{1}{8}x^4+\frac{1}{2}x^4+c$

Question 2:

$a)\ f(x)=x^4+5$

$b)\ f(x)=x^3-2x^2+3x+-5$

$c)\ f(x)=2x^3+6x^2-3$

$d)\ f(x)=3x^3+x^2+4$

Question 3:

$$\frac{dy}{dt}=(t-3)^2=t^2-6t+9$$

$$y=\frac{1}{3}t^3-3t^2+9t+c$$

$for\ \ t=4, y=9\, so$

$$9=\frac{64}{3}-48+36+c$$

$$c=-\frac{1}{3}$$

$$y=\frac{1}{3}t^3-3t^2+9t-\frac{1}{3}$$

Question 4:

$$y=f(x)=\frac{1}{4}x^4+\frac{1}{4}x^2+3x+c$$

$for\ \ x=1,\ y=-1\ so$

$$-1=\frac{1}{4}+\frac{1}{4}+3+c$$

$$c=-\frac{9}{2}$$

$$f(x)=\frac{1}{4}x^4+\frac{1}{4}x^2+3x-\frac{9}{2}$$

Integration and area

Definite integrals

Definite integrals have numbers, a and b, next to the integral sign.

They indicate the range of x-values to integrate the function between.

a is the lower limit, b is the upper limit $\quad a<b$

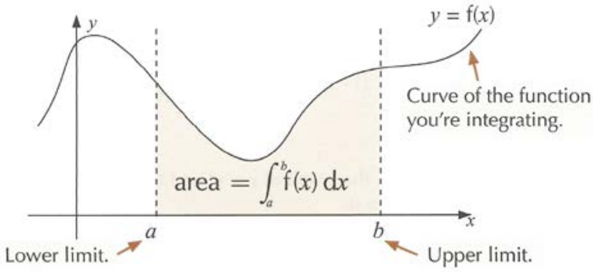
$$\int_a^b f(x)dx=\left[F(x)\right]_a^b=F(b)-F(a)$$ where F is an integral of f.

Example:

$$\int_1^2 x^2dx=\left[\frac{1}{3}x^3\right]_1^2=\left(\frac{1}{3}\times 2^3\right)-\left(\frac{1}{3}\times 1^3\right)=\frac{8}{3}-\frac{1}{3}=\frac{7}{3}$$

Area under a curve

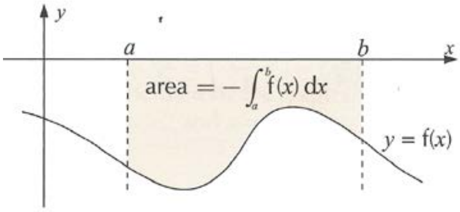
The value of a definite integral represents the area between the curve of the function, the x-axis and the line $x=a$ and $x=b$.



Be careful: if the curve is below the x-axis, i.e if $f(x)<0$,

the integral will give a negative value.

In this case, $Area=-\int_a^b f(x)dx$



Area between two curves

$f(x)$ and $g(x)$ are two functions and a and b are two numbers.

when $a<x<b$, $f(x)>g(x)$.

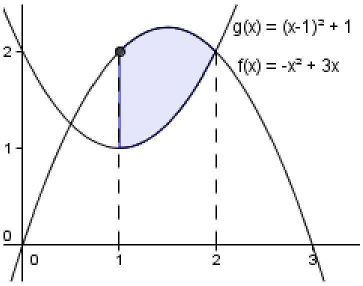
The area between the two curves and the lines $x=a$ and $x=b$ is

$$\int_a^b f(x)dx-\int_a^b g(x)dx \quad or \quad \int_a^b (f(x)-g(x))dx$$

$$Area=\int_1^2 -x^2+3x-\left((x-1)^2+1\right)dx=\int_1^2 -2x^2+5x-2dx$$

$$=\left[-\frac{2}{3}x^3+\frac{5}{2}x^2-2x\right]_1^2$$

$$Area=\left(-\frac{16}{3}+10-4\right)-\left(-\frac{2}{3}+\frac{5}{2}-2\right)=\frac{5}{6}$$



Integration and area - exercises

Question 1:

Work out

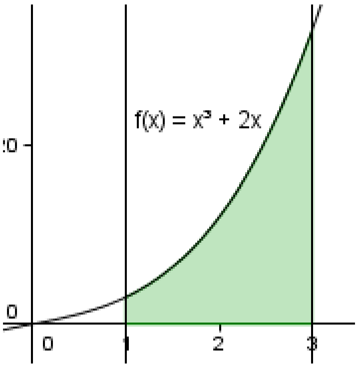
$a)\int_1^3 3x^2 dx \qquad b)\int_{-2}^0 (4x^3+2x)dx \qquad c)\int_0^2 \left(x^3+x\right)dx \qquad d)\int_{-5}^{-2}(x+1)^2 dx$

Question 2:

Given that $\int_0^a x^3 dx = 4,$ work out the value of $a, where\ a > 0.$

Question 3:

Calculate the shaded area in the following diagram:



Question 4:

Find the area between the graph of $y = x^2 + x,$ the x-axis and the lines $x = 1\ and\ \ x = 3$

Question 5:

Find the area between the graph of $y = 5x^3,$ the x-axis and the lines $x = -2\ and\ \ x = -1.$

Question 6:

Find the area enclosed by the curve $y = x^2 + 4$ and the line $y = x + 4.$

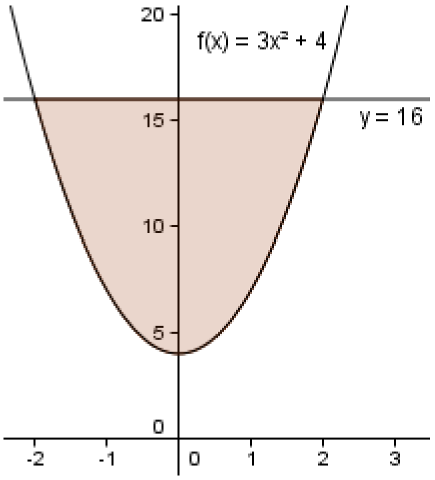
Question 7:

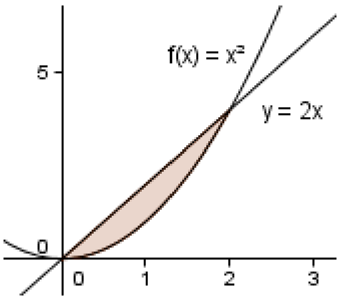
Find the area enclosed by the curve $y = x^2 + x + 1$,

the line $y = 4 - x$ and the $x - axis\ for\ x \geq 0.$

Question 8:

Work out the shaded area in the following diagrams:





Integration and area – exercises ‘answers

Question 1:

$$a)\int_1^3 3x^2dx=\left[x^3\right]_1^3=3^3-1^3=26$$

$$b)\int_{-2}^0 (4x^3+2x)dx=\left[x^4+x^2\right]_{-2}^0=0-\left((-2)^4+(-2)^2\right)=-20$$

$$c)\int_0^2\left(x^3+x\right)dx=\left[\frac{1}{4}x^4+\frac{1}{2}x^2\right]_0^2=(4+2)-(0)=6$$

$$d)\int_{-5}^{-2}(x+1)^2dx=\int_{-5}^{-2}x^2+2x+1dx=\left[\frac{1}{3}x^3+x^2+x\right]_{-5}^{-2}$$

$$=\left(-\frac{8}{3}+4-2\right)-\left(-\frac{125}{3}+25-5\right)=21$$

Question 2:

$$\int_0^a x^3dx=\left[\frac{1}{4}x^4\right]_0^a=\frac{1}{4}a^4=4$$

This gives $a^4=16$ $a=\sqrt[4]{16}=2$

$(because\ a>0)$

Question 3:

$$Area=\int_1^3 x^3+2xdx=\left[\frac{1}{4}x^4+x^2\right]_1^3$$

$$Area=\left(\frac{81}{4}+9\right)-\left(\frac{1}{4}+1\right)=28$$

Question 4:

$$Area=\int_1^3 x^2+xdx=\left[\frac{1}{3}x^3+\frac{1}{2}x^2\right]_1^3$$

$$Area=\left(9+\frac{9}{2}\right)-\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{38}{3}$$

Question 5:

$$I=\int_{-2}^{-1}5x^3dx=\left[\frac{5}{4}x^4\right]_{-2}^{-1}$$

$$I=\left(\frac{5}{4}\right)-(20)=-\frac{75}{4}$$

The area is $\frac{75}{4}=18\frac{3}{4}$

Question 6:

We need to find where the parabola and the line intersect:

$$\begin{cases} y=x^2+4 \\ y=x+4 \end{cases}\quad this\ gives:$$

$$x^2+4=x+4$$

$$x^2-x=0$$

$$x(x-1)=0$$

$$x=0\ or\ x=1$$

$$I=\int_0^1(x+4)-(x^2+4)dx=\int_0^1 x-x^2dx$$

$$I=\left[\frac{1}{2}x-\frac{1}{3}x^3\right]_0^1=\left(\frac{1}{2}-\frac{1}{3}\right)-(0)=\frac{1}{6}$$

The area is $\frac{1}{6}$.

Question 7:

We solve simultaneously $\begin{cases} y=x^2+x+1 \\ y=4-x \end{cases}$

$$x^2+x+1=4-x$$

$$x^2+2x-3=0$$

$$(x+3)(x-1)=0$$

$$x=-3\ or\ x=1$$

But we want $x\geq 0$ so we work out

$$I=\int_0^1(x^2+x+1)-(4-x)dx=\int_0^1 x^2+2x-3dx$$

$$I=\left[\frac{1}{3}x^3+x^2-3x\right]_0^1=\left(\frac{1}{3}+1-3\right)-(0)=-\frac{5}{3}$$

The area comprised between the curve is $\frac{5}{3}$.

Question 8:

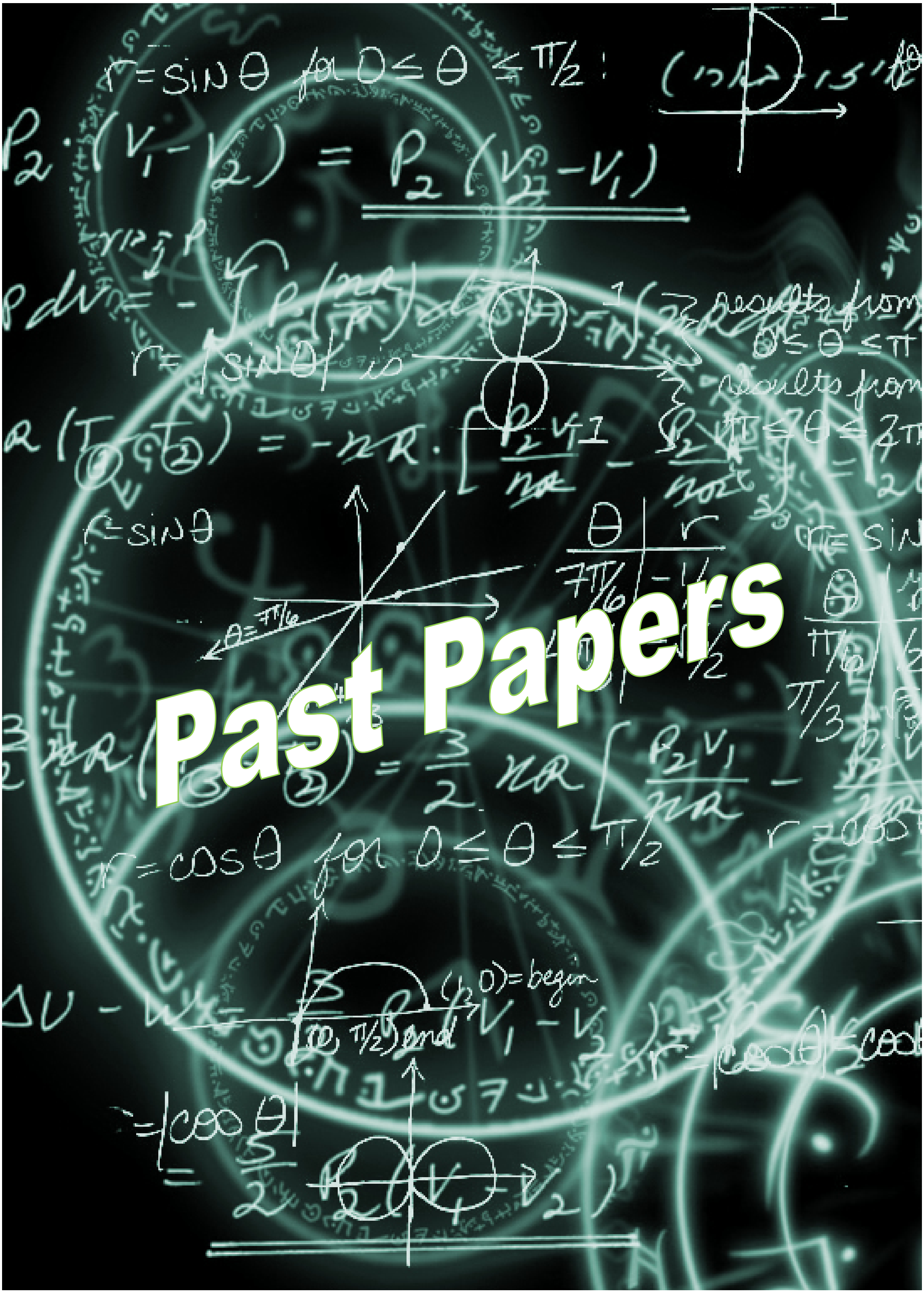
$$a)\ I=\int_{-2}^2 16-\left(3x^2+4\right)dx=\int_{-2}^2 12-3x^2dx$$

$$I=\left[12x-x^3\right]_{-2}^2=(24-8)-(-24+8)=48$$

$$b)\ I=\int_0^2 2x-x^2dx=\left[x^2-\frac{1}{3}x^3\right]_0^2$$

$$I=\left(4-\frac{8}{3}\right)-(0)=\frac{4}{3}$$

The area is $\frac{4}{3}$.



Past Papers

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

ASSESSMENT and QUALIFICATIONS ALLIANCE

Tuesday 10 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

1 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x+4y=7$.

(a) (i) Show that $k=-2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

3 (a) (i) Express x^2-4x+9 in the form $(x-p)^2+q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y=x^2-4x+9$. *(2 marks)*

(b) The line L has equation $y+2x=12$ and the curve C has equation $y=x^2-4x+9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2-2x-3=0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

4 The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO. *(2 marks)*

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

6 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Hence express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

7 The volume, $V\,\mathrm{m}^3$, of water in a tank at time t seconds is given by

$$V = \tfrac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{\mathrm{d}V}{\mathrm{d}t}$; *(3 marks)*

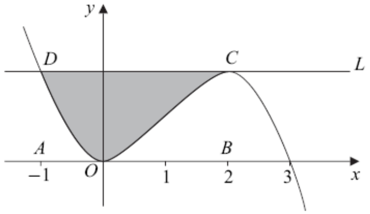
(ii) $\dfrac{\mathrm{d}^2V}{\mathrm{d}t^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in $\mathrm{m}^3\,\mathrm{s}^{-1}$, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,\mathrm{d}x$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

AQA – Core 1 - Jan 2006 – Answers

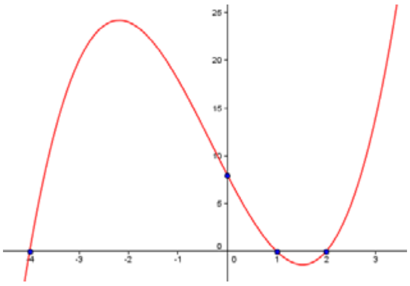
Question 1:	**Exam report**
$a)\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)=5-2\sqrt{5}+2\sqrt{5}-4=1$ $b)\sqrt{8}+\sqrt{18}=\sqrt{4\times 2}+\sqrt{9\times 2}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$	Many candidates earned full marks on this introductory question. (a) Most candidates multiplied out the two brackets to obtain four terms. The most common error occurred in the last term, which was sometimes seen as -2 instead of -4. Very few candidates recognised that it was the difference of two squares. (b) This part was less well done. Some candidates had problems simplifying $\sqrt{8}$ and $\sqrt{18}$ and wrote $2\sqrt{4}$ *and* $2\sqrt{9}$, for example. Some, having correctly converted both surds, added them incorrectly and so $6\sqrt{6}$ was quite common. A few candidates thought $\sqrt{8}+\sqrt{18}$ were equal to $\sqrt{26}$.

Question 2:	**Exam report**
$A(1,1)\quad B(5,k)$ $AB: 3x+4y=7$ $a)\,i)\,B$ belongs to the line so its coordinates satify the equation: $3\times 5+4\times k=7 \qquad 15+4k=7 \qquad k=-2$ $ii)\,I\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right)=I\left(3,-\frac{1}{2}\right)$ $b)\,m_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{-2-1}{5-1}=-\frac{3}{4}$ $c)\,i)\,m_{AC}=-\frac{1}{m_{AB}}=\frac{4}{3}$ $ii)\,AC: y-y_A=m_{AC}\left(x-x_A\right)$ $y-1=\frac{4}{3}(x-1)$ $3y-3=4x-4$ $4x-3y=1$ $iii)\,C$ belongs to the x-axis so $C(x_C,0)$ $4x_C-3\times 0=1 \qquad x_C=\frac{1}{4}$	(a)(i) Candidates used various methods to prove that k=-2. Some used the most direct method of substituting x = 5 into the given line equation and solving for y; some chose to verify that x = 5 and y =-2 satisfied the equation of the straight line. Others took a longer route; they found the gradient using (1,1) and (5,-2) and then found the equation passing through one of the points and proved it to be the given one. (a)(ii) Most candidates knew how to find the midpoint of a line. A few made a simplification error and wrote $\left(3,\frac{1}{2}\right)$ instead of $\left(3,-\frac{1}{2}\right)$.The common error amongst the weaker candidates was to subtract the coordinates instead of adding them. (b) Many candidates gave fully correct answers here. However, some, having obtained $\frac{-2-1}{5-1}$ wrote $\frac{3}{4}$ as a final answer. A few candidates used $\frac{x_1-x_2}{y_1-y_2}$. (c)(i) Most knew the gradient rule for perpendicular lines. However, not all could implement it since it involved the reciprocal of a fraction. (c)(ii) At least half of the candidates found the equation of the line passing through the midpoint of AB instead of through C. (iii) Most realised the need to substitute y = 0 into their AC equation and solve for x, so they at least earned the method mark. Even those with the correct equation did not always earn two marks. Some had difficulty in simplifying $\frac{1}{3}\div\frac{4}{3}$

Question 3:	**Exam report**
$i)\ x^2-4x+9=\left(x-2\right)^2-4+9=(x-2)^2+5$ $ii)$ for all $x,(x-2)^2\ge 0\ \ so\ \ (x-2)^2+5\ge 5$ The minimum y value is 5, obtained for $x=2$ $Min\left(2,5\right)$ $b)$ *Solve the equations* simultaneously: $\begin{cases} y=-2x+12 \\ y=x^2-4x+9 \end{cases}$ *This gives* $(y=)\ x^2-4x+9=-2x+12$ $x^2-2x-3=0$ $ii)\ x^2-2x-3=\left(x-3\right)\left(x+1\right)=0$ $x=3\ or\ x=-1$ *and* $\quad y=12-2x=6\ or\ y=14$ $(3,6)$ *and* $(-1,14)$	(a)(i) Most candidates were familiar with the idea of .completing the square. and answered this part satisfactorily. There were occasional sign errors and +9-4 was not always evaluated correctly. (a)(ii) There were several correct answers although some wrote (5,-2) instead of (5,2). Some did not recognise the link between parts (i) and (ii) and chose to differentiate instead. This was a satisfactory, though more time-consuming, alternative method. Some earned no marks here as they wrote comments such as ".5 is the minimum", with no link to the y-coordinate being 5. (b)(i) This simple proof was usually well done. Occasionally the mark was lost due to the omission of " = 0"... (b)(ii) Many scored full marks here. Most factorised the equation and obtained the correct x-values. Some made no further progress, while a few substituted into the given quadratic equation and obtained y = 0, instead of using the equation of the line or curve to find the values of y. It was encouraging to see many factorising the quadratic correctly. Those who used the quadratic equation formula or completion of the square often made more errors than those who factorised

Question 4:	**Exam report**
$a)\ x^2+(m+4)x+(4m+1)=0\ has\ equal\ roots$ *so the* discriminant =0: $(m+4)^2-4\times 1\times\left(4m+1\right)=0$ $m^2+8m+16-16m-4=0$ $m^2-8m+12=0$ $b)\ m^2-8m+12=0$ $(m-6)(m-2)=0$ $m=6\ \ or\ \ m=2$	(a) There were several completely correct proofs here. Some lost the last mark by concentrating on the discriminant but failing to equate it to zero. There was a little fudging by some; for example, some who wrote -4(4*m*+1) = -16*m*+4 still managed to obtain the correct printed equation. Some of the weaker candidates found b^2-4ac using numerical values from the equation they were supposed to establish. (b) Almost all candidates found both values of *m* successfully. A few spotted just one answer and some factorised correctly and then wrote *m* = -2, *m* =-6, but they were in the minority.

Question 5:	**Exam report**
$a)\ x^2+y^2-8x+6y=11$ $(x-4)^2-16+(y+3)^2-9=11$ $(x-4)^3+(y+3)^2=36$ $b)i)\ Centre\ \ C(4,3)$ $ii)\ Radius\ \ r=\sqrt{36}=6$ $c)\ O(0,0)\quad C(4,3)$ $i)\ Length\ CO=\sqrt{(x_O-x_C)^2+(y_O-y_C)^2}$ $CO=\sqrt{4^2+3^2}=\sqrt{25}=CO=5$ $ii)\ Because\ CO<6, O$ lies INSIDE the circle.	(a) Completion of the squares in the circle equation was carried out well once more. The most common error was a sign slip usually in the second term. Another error lay in combining the constant terms, so answers such as -14 and 11 were seen for r^2. (b) Most earned the mark for the coordinates of the centre of the circle as this was a follow through mark. The mark for the radius was not always earned as some failed to take the square root or had an inappropriate answer such as a negative value for r^2. (c)(i) Most found *CO* to be 5. However, a few neglected to square -3 and 4 before adding and some subtracted 9 from 16. (c)(ii) This part was answered well with most realising the need to explain, using both lengths, why *O* lay inside or outside the circle. Some accompanied their explanations with diagrams, although this was not necessary.

Question 6:	**Exam report**
$a)i)\ p(x)=x^3+x^2-10x+8$ $p(2)=2^3+2^2-10\times2+8=8+4-20+8=0$ $2\ is\ a\ root\ of\ \ p\ so\ (x-2)\ is\ a\ \ factor\ of\ \ p.$ $ii)\ x^3+x^2-10x+8=(x-2)(x^2+3x-4)=(x-2)(x+4)(x-1)$ $\begin{array}{r\|l} & x^2+3x-4 \\ \hline & x^3+x^2-10x+8 \\ & \underline{x^3-2x^2} \\ & \quad 3x^2-10x \\ x-2 & \quad \underline{3x^2-6x} \\ & \qquad -4x+8 \\ & \qquad \underline{-4x+8} \\ & \qquad\qquad 0 \end{array}$ $b)$ The graph cuts the axes at (2,0), (-4,0), (1,0) and (8,0).  	(a)(i) It was good to see that almost all candidates started correctly by evaluating p(2), though a few thought they needed to find p(-2) and others wrongly assumed that long division was the “factor theorem”. It was necessary to write a conclusion or statement after showing that p(2) = 0 , in order to earn the second mark. (a)(ii) The most successful approach here was by using a quadratic factor (ax^2+bx+c), though long division also worked well for many. A surprising number who found the correct quadratic then factorised it wrongly. Those who tried the factor theorem again rarely spotted both factors. A few lost the final mark by failing to write p(x) as a product of factors. (b) Although there were many correct sketches, many lost a mark by failing to mark the point (0,8) on the *y*-axis. Candidates were expected to draw a cubic through their intercepts, to use an approximately linear scale and to continue the graph beyond the intercepts on the *x*-axis. It was common to see the negative values in the factors wrongly taken to be the roots and hence the intercepts on the *x*-axis.

Question 7:	**Exam report**
$a)i)\dfrac{dV}{dt}=\dfrac{1}{3}\times6t^5-2\times4t^3+3\times2t=2t^5-8t^3+6t$ $ii)\dfrac{d^2V}{dt^2}=10t^4-24t^2+6$ $b)$ The rate of change is $\dfrac{dV}{dt}$ $so\ \dfrac{dV}{dt}(t=2)=2\times2^5-8\times2^3+6\times2=64-64+12=12m^3/s$ $c)i)\dfrac{dV}{dt}(t=1)=2\times1^5-8\times1^3+6\times1=2-8+6=0$ V has a stationary point when $t=1$ $ii)\dfrac{d^2V}{dt^2}(t=1)=10\times1^4-24\times1^2+6=10-24+6=-8<0$ The stationary point is a MAXIMUM.	(a)(i) The differentiation was generally well done, though some candidates found the fractional coefficient problematic. Those who wrote $\dfrac{6}{3}t^5$ were not penalised in this part but writing $6.\dfrac{1}{3}$ generally led to errors later in the question. Some tried to avoid the fraction by considering 3V throughout the question, making errors, and suffered a heavy penalty. (a)(ii) Again, most applied the method correctly and here simplification of coefficients was necessary. A few failed to differentiate 6t or omitted it altogether. (b) This part was poorly done with many not recognising that the rate of change was $\dfrac{dV}{dt}$ and substituted t = 2 into the expression for V or $\dfrac{d^2V}{dt^2}$. Quite a lot of candidates made arithmetic errors. A few found two values of the expression and averaged them. (c)(i) Again, many failed to evaluate $\dfrac{dV}{dt}$ in order to verify that a stationary point occurred, but those who did generally obtained a value of zero. It was essential to include a relevant statement to earn both marks. (c)(ii) This required evaluation of the second derivative at t = 1 or an appropriate test. Candidates who tried to test the gradient on either side of 1 almost invariably failed, as the values used were too far away from the stationary point. A surprising number evaluated

	10 -24+6 to be -20 thus losing the accuracy mark. Some appeared to be guessing and drew wrong conclusions about maxima or minima after evaluating the second derivative.

Question 8:	**Exam report**
$y=3x^2-x^3 \qquad A(-1,0)\ B(2,0)$ $a)\ y(2)=3\times2^2-2^3=12-8=4 \qquad C(2,4)\ \ D(-1,4)$ The area of the rectangle is $3\times4=12$ $b)i)\int\left(3x^2-x^3\right)dx=x^3-\frac{1}{4}x^4+c$ $ii)\ Area=12-\int_{-1}^{2}(3x^2-x^3)dx=12-\left[x^3-\frac{1}{4}x^4\right]_{-1}^{2}$ $Area=12-(8-4)+\left(-1-\frac{1}{4}\right)=6\frac{3}{4}$ $c)i)\frac{dy}{dx}=6x-3x^2$ $ii)\frac{dy}{dx}(x=1)=6-3=3$ $y(1)=3-2=2$ Equation of the tangent : $y-2=3(x-1)$ $y=3x-1$ $iii)$ y is decreasing when $\frac{dy}{dx}<0$ $6x-3x^2<0$ $(\div-3) \qquad x^2-2x>0$ $d)\ x^2-2x=x(x-2)>0 \qquad$ for $x<0$ or $x>2$ MY SCORE FOR THIS PAPER IS ..... / 75 MY GRADE :	(a) Not everyone recognised that the height of the rectangle was the value of *y* when *x* = -1 or *x* = 2. Some who did made numerical errors. Even having found the height 4, some obtained the wrong area by taking *AB* as 4 or 2 (sometimes using Pythagoras' .Theorem) or by finding the perimeter instead. (b)(i) The integral was generally correct, though sometimes incorrect simplification occurred subsequently. A few integrated x^3 to $\frac{1}{3}x^4$ and a surprising number misread the integrand as $3x^2-x^2$. There were also candidates who confused integration with differentiation or whose process was a hybrid of the two. (b)(ii) Almost everyone recognised that they should firstly evaluate the integral from -1 to 2 , but most stopped there, instead of going on to subtract the value of the integral from the area of the rectangle. There were a lot of sign errors in the work with some adding instead of subtracting or putting the two parts the wrong way round. A few wrongly substituted in the original function. Those who chose to work with the differences of two integrals seldom completed it correctly. (c)(i) Differentiation was done well on the whole. (c)(ii) Many substituted *x* = 1 into the derivative to find the gradient of the tangent and went no further. Most did not find the *y* coordinate of the point and so made no attempt at the equation of the tangent. A few non-linear equations were seen with a .gradient. of $6x-3x^2$. (c)(iii) Very few candidates completed this part. Many made no attempt, and those who did tended to test a few values of *x* or to find the second derivative, which was of no value. Only the strongest candidates realised that $\frac{dy}{dx}<0$ was the condition for *y* to be decreasing and that, after a couple of lines of algebra, the given inequality could be obtained. (d) Although most made an attempt at the quadratic inequality, few obtained both parts of the solution. It was imperative that candidates wrote *x* > 2, *x* < 0 and not 0 > *x* > 2 as many incorrectly stated. It was disappointing to see how many candidates at this level could not solve the equation $x^2-2x=0$, obtaining values such as -2, √2, 1+√2. Using the formula or completing the square sometimes led to 1±√1, which many candidates failed to simplify.

GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	61	53	45	38	31

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

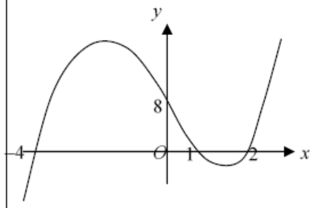
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a)	$\left(\sqrt{5}\right)^2 + 2\sqrt{5} - 2\sqrt{5} - 4 = 1$	M1		Multiplying out or difference of two squares attempted
		A1	2	Full marks for correct answer /no working
(b)	$\sqrt{8} = 2\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$	M1		Either correct
	Answer $= 5\sqrt{2}$	A1	2	Full marks for correct answer /no working
	Total		**4**	
2(a)(i)	$15 + 4k = 7 \quad \Rightarrow 4k = -8 \quad \Rightarrow k = -2$	B1	1	**AG** (condone verification or $y = -2$)
(ii)	$\frac{1}{2}(x_1 + x_2)$ or $\frac{1}{2}(y_1 + y_2)$	M1		
	Midpoint coordinates $\left(3, -\frac{1}{2}\right)$	A1	2	One coordinate correct implies M1
(b)	Attempt at $\Delta y / \Delta x$ or $y = -\frac{3}{4}x + \frac{7}{4}$	M1		(Not *x* over *y*)(may use *M* instead of *A*/*B*)
	Gradient $AB = -\frac{3}{4}$	A1	2	–0.75 etc any correct equivalent
(c)(i)	$m_1 m_2 = -1$ used or stated	1		
	Hence gradient $AC = \frac{4}{3}$	A1√	2	Follow through their gradient of *AB* from part (b)
(ii)	$y - 1 = \frac{4}{3}(x - 1)$ or $3y = 4x - 1$ etc	B1√	1	Follow through their gradient of *AC* from part (c) (i) must be **normal** & (1,1) used
(iii)	$y = 0 \qquad \Rightarrow x - 1 = -\frac{3}{4}$	M1		Putting $y = 0$ in their *AC* equation and attempting to find *x*
	$x = \frac{1}{4}$	A1	2	**CSO**. *C* has coordinates $\left(\frac{1}{4}, 0\right)$
	Total		**10**	
3(a)(i)	$(x - 2)^2$	B1		$p = 2$
	$+ 5$	B1	2	$q = 5$
(ii)	Minimum point (2, 5) or $x = 2$, $y = 5$	B2√	2	B1 for each coordinate correct or ft **Alt method** M1, A1 sketch, differentiation
(b)(i)	$12 - 2x = x^2 - 4x + 9$			Or $x^2 - 4x + 9 + 2x = 12$
	$\Rightarrow x^2 - 2x - 3 = 0$	B1	1	**AG** (be convinced) (must have = 0)
(ii)	$(x - 3)(x + 1) = 0$	M1		Attempt at factors or quadratic formula or one value spotted
	$x = 3, \quad -1$	A1		Both values correct & simplified
	Substitute one value of *x* to find *y*	M1		May substitute into equation for *L* or *C*
	Points are (3, 6) and (–1, 14)	A1	4	*y*-coordinates correct linked to *x* values
	Total		**9**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$(m+4)^2=m^2+8m+16$	B1		Condone $4m+4m$
	$b^2-4ac=(m+4)^2-4(4m+1)=0$	M1		b^2-4ac (attempted and involving m's and no x's) or $b^2-4ac=0$ stated
	$m^2+8m+16-16m-4=0$ $\Rightarrow m^2-8m+12=0$	A1	3	**AG** (be convinced – all working correct- $=0$ appearing more than right at the end)
(b)	$(m-2)(m-6)=0$	M1		Attempt at factors or quadratic formula
	$m=2$, $m=6$	A1	2	**SC B1** for 2 or 6 only without working
	Total		**5**	
5(a)	$(x-4)^2+(y+3)^2$	B2		B1 for one term correct
	$(11+16+9=36)$ RHS $=6^2$	B1	3	Condone 36
(b)(i)	Centre $(4,-3)$	B1√	1	Ft their a and b from part (a)
(ii)	Radius = 6	B1√	1	Ft their r from part (a)
(c)(i)	$CO^2=(-4)^2+3^2$	M1		Accept + or – with numbers but must add
	$CO=5$	A1√	2	Full marks for answer only
(ii)	Considering CO and radius	M1		
	$CO<r \Rightarrow O$ is **inside** the circle	A1√	2	Ft outside circle when 'their CO' $>r$ or on the circle when 'their CO' $=r$ **SC** B1√ *if no explanation given*
	Total		**9**	
6(a)(i)	$p(2)=8+4-20+8$	M1		Finding p(2) M0 *long division*
	$=0$, $\Rightarrow x-2$ is a factor	A1	2	Shown = 0 **AND** conclusion/ statement about $x-2$ being a factor
(ii)	Attempt at quadratic factor	M1		or factor theorem again for 2nd factor
	x^2+3x-4	A1		or $(x+4)$ or $(x-1)$ proved to be a factor
	$p(x)=(x-2)(x+4)(x-1)$	A1	3	
(b)		B1		Graph through (0,8) 8 marked
		B1√		Ft "their factors" 3 roots marked on x-axis
		M1		Cubic curve through their 3 points
		A1	4	Correct including x- intercepts correct Condone max on y-axis etc or slightly wrong concavity at ends of graph
	Total		**9**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{\mathrm{d}V}{\mathrm{d}t}=2t^5-8t^3+6t$	M1 A1 A1	3	One term correct unsimplified Further term correct unsimplified All correct unsimplified (no + c etc)
(ii)	$\frac{\mathrm{d}^2V}{\mathrm{d}t^2}=10t^4-24t^2+6$	M1 A1	2	One term FT correct unsimplified **CSO**. All correct simplified
(b)	Substitute $t=2$ into their $\frac{\mathrm{d}V}{\mathrm{d}t}$	M1		
	(= 64 – 64 + 12) = 12	A1	2	**CSO**. Rate of change of volume is $12\mathrm{m}^3\ \mathrm{s}^{-1}$
(c)(i)	$t=1\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}t}=2-8+6$	M1		Or putting their $\frac{\mathrm{d}V}{\mathrm{d}t}=0$
	$=0\Rightarrow$ Stationary value	A1	2	**CSO**. Shown to = 0 **AND** statement (If solving equation must obtain $t=1$)
(ii)	$t=1\Rightarrow\frac{\mathrm{d}^2V}{\mathrm{d}t^2}=-8$	M1		Sub $t=1$ into their second derivative or equivalent full test.
	Maximum value	A1√	2	Ft if their test implies minimum
	Total		**11**	
8(a)	$y_D=3+1=4$ or $y_C=12-8=4$	M1		Attempt at either y coordinate
	Area $ABCD$ $=3\times4=12$	A1	2	
(b)(i)	$x^3-\frac{x^4}{4}$ $(+C)$	M1 A1 A1	3	Increase one power by 1 One term correct unsimplified All correct unsimplified (condone no +C)
(ii)	Sub limits –1 and 2 into their (b) (i) ans	M1		May use both –1, 0 and 0, 2 instead
	$[8-4]-\left[-1-\frac{1}{4}\right]$ $=5\frac{1}{4}$	A1		
	Shaded area = "their" (rectangle– integral)	M1		Alt method: difference of two integrals
	$=12-5\frac{1}{4}=6\frac{3}{4}$	A1	4	**CSO**. Attempted M2, A2
(c)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x-3x^2$	M1 A1	2	One term correct All correct (no +C etc)
(ii)	When $x=1$, $y=2$ when $x=1$,	B1		May be implied by correct tgt equation
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3$ as 'their' grad of tgt	M1√		Ft their derivative when $x=1$
	Tangent is $y-2=3(x-1)$	A1	3	Any correct form $y=3x-1$ etc
(iii)	Decreasing when $\frac{\mathrm{d}y}{\mathrm{d}x}=6x-3x^2<0$	M1		Watch no fudging here!! May work backwards in proof.
	$3(2x-x^2)<0 \Rightarrow x^2-2x>0$	A1	2	**AG** (be convinced no step incorrect)
(d)	Two critical points 0 and 2	M1		Marked on diagram or in solution
	$x>2$, $x<0$ ONLY	A1	2	or M1 A0 for $0<x<2$ or $0>x>2$ **SC** B1 for $x>2$ (or $x<0$)
	Total		**18**	
	TOTAL		**75**	

General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Monday 22 May 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The point A has coordinates $(1, 7)$ and the point B has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x + 2y = 17$. *(2 marks)*

(b) The line AB intersects the line with equation $x - 4y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

(c) Find an equation of the line through A which is perpendicular to AB. *(3 marks)*

2 (a) Express $x^2 + 8x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

3 A curve has equation $y = 7 - 2x^5$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

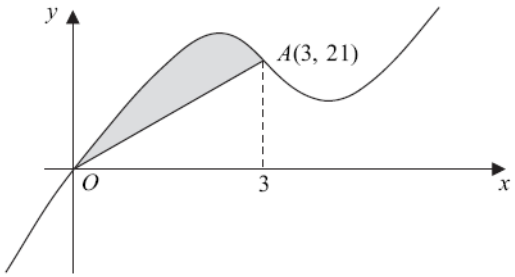
(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

4 (a) Express $\left(4\sqrt{5} - 1\right)\left(\sqrt{5} + 3\right)$ in the form $p + q\sqrt{5}$, where p and q are integers. *(3 marks)*

(b) Show that $\dfrac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*

5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\,dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\,dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when $p(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $p(x)$ in the form

$$(x-2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

7 A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

END OF QUESTIONS

AQA – Core 1 – June 2006 – Answers

Question 1:

$$a)\,i)\,Gradient\ of\ \ AB = m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1-7}{5-1} = \frac{-6}{4} = -\frac{3}{2}$$

$$ii)\,Equation\ of\ \ AB: y - y_A = m_{AB}(x - x_A)$$

$$y - 7 = -\frac{3}{2}(x-1)$$

$$2y - 14 = -3x + 3$$

$$3x + 2y = 17$$

$$b)\,\text{solve simultaneously}\ \begin{cases} 3x + 2y = 17 & (\times 2) \\ x - 4y = 8 \end{cases}$$

$$\begin{cases} 6x + 4y = 34 \\ x - 4y = 8 & (add) \end{cases}$$

$$This\ \ gives\qquad 7x \quad = 42$$

$$x = 6$$

$$AND\qquad x - 4y = 8$$

$$6 - 4y = 8 \qquad y = -\frac{1}{2}$$

$$\text{The lines intersect at } (6, -\frac{1}{2})$$

$$c)\,\text{The gradient of the line perpendicular to AB is } -\frac{1}{m_{AB}} = \frac{2}{3}$$

$$\text{Equation of the line}: y - 7 = \frac{2}{3}(x-1)$$

$$3y - 21 = 2x - 2$$

$$2x - 3y = -19$$

Exam report

Part (a)(i) Although most obtained the correct gradient, some omitted the negative sign (particularly those who relied on a sketch for their evaluation) and some had a fraction with the change in *x* as the numerator which immediately scored no marks. Quite a few found mid-points (possibly since that had appeared on previous examinations) and others added the coordinates instead of finding the differences in their quotient expression for the gradient.

Part (a)(ii) The use of their gradient to obtain the given equation was the most successful method. Those using $y = mx + c$ had a tendency to introduce a new 'c' by doubling both sides but then substituted their value back into the original equation. The most successful candidates used the formula $y - y_1 = m(x - x_1)$. Some re-arranged the given equation to check the gradient then checked one set of co-ordinates; others checked two points and indicated that a straight line has the form $ax + by = c$.

Part (b) Those using substitution often began by using an incorrect rearrangement of one of the equations. If they attempted elimination, sometimes only part of an equation was multiplied by the appropriate constant. Many added the equations instead of subtracting. Of those who wrote $14y = –7$, just as many obtained an incorrect answer of $y = –2$ as the correct answer of $y = -1/2$.

Part (c) The condition for perpendicularity was generally known but some were unable to evaluate –1 divided by – 1.5. A few omitted the – sign while some referred to the equation $3x + 2y = 17$ and gave a gradient of -1/3. Once again, those determined to use $y = mx + c$ often made errors in the constant due to the fractional coefficient of *x*. Quite a few did not use the point *A* as instructed, choosing to use the point *C* instead.

Question 2:

$$a)\,x^2 + 8x + 19 = (x+4)^2 - 16 + 19 = (x+4)^2 + 3$$

$$b)\,x^2 + 8x + 19 = 0\ \ is\ \ equivalent\ \ to\ \ (x+4)^2 + 3 = 0$$

$$(x+4)^2 = -3$$

For all x real, $(x+4)^2 \geq 0$, so this equation has no solution.

$c)$ The minimum point is $(-4, 3)$

The graph crosses the y-axis at (0,19)

$$d)\,x \xrightarrow[\text{units in } x-direction]{translation -4} x + 4 \xrightarrow{f} (x+4)^2 \xrightarrow[\text{units in } y-direction]{tranlation\ 3} (x+4)^2 + 3$$

$$Tranlation\ vector \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

Exam report

Part (a) Many candidates began by finding the correct values of *p* and *q* . A few wrote $(x – 4)^2$ and some added 16 instead of subtracting 16 so $q = 35$ was sometimes seen.

Part (b) Very few chose to consider the expression they had in part (a). Practically all candidates decided to find the discriminant instead but its evaluation was often incorrect. Not everyone quoted the expression for the discriminant, $b^2 – 4ac$, correctly. Some attempted to refer to the fact that the curve was completely above the *x*-axis but did not, in general, complete their argument.

Part (c) The graphs here were disappointing. Although most drew a quadratic shape, there seemed to be little reference to their part (a) and many just tried to plot a few points. Most were able to state the intercept on the *y*-axis. However, sometimes the point (0.19) was shown as the minimum point or a straight line intercept. Several curves were drawn only in the first quadrant, regardless of whether the quoted minimum point was (–4.3) or (4.3).

Part (d) This was not well answered. The term **translation** was required but generally the wrong word was used or it was accompanied by another transformation such as a stretch. The most common incorrect vector stated was $\begin{bmatrix} 8 \\ -19 \end{bmatrix}$.

Question 3:	**Exam report**
$y=7-2x^5$ $a)\,\frac{dy}{dx}=0-2\times5x^4=-10x^4$ $b)$ Gradient of the tangent is $\frac{dy}{dx}(x=1)=-10\times1^4=-10$ $for\;\; x=1,\; y=7-2\times1^5=5$ the equation of the tangent is : $y-5=-10(x-1)$ $10x+y=15$ $c)\,\frac{dy}{dx}(x=-2)=-10\times(-2)^4=-160<0$ y is decreasing	Part (a) Most candidates answered this part correctly. A few included the 7 or thought the derivative of the first term was 7x and the – sign was sometimes lost. Part (b) Many substituted x = 1 correctly, though it was apparent that they did not recognise this value of –10 to be the gradient of the tangent. Many correctly found y as 5 but stopped there. Again, some correct attempts at the tangent equation using y = mx + c foundered and quite a large number attempted to find the equation of the normal. Part (c) Use of the value of $\frac{dy}{dx}$ was the only acceptable method here. Evaluations of y at different points or finding the second derivative were common but earned no marks.

Question 4:	**Exam report**
$a)\,(4\sqrt{5}-1)(\sqrt{5}+3)=4\times5+12\sqrt{5}-\sqrt{5}-3$ $=20-3+12\sqrt{5}-\sqrt{5}$ $=17+11\sqrt{5}$ $b)\,\frac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}=\frac{\sqrt{25\times3}-\sqrt{9\times3}}{\sqrt{3}}$ $=\frac{5\sqrt{3}-3\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{\sqrt{3}}=2$	Part (a) Almost everyone recognised that multiplication of the two brackets was required but there were numerous errors with $7\sqrt{5}$ instead of $12\sqrt{5}$ being common and –2 or –4 instead of –3. Although most dealt with the first term correctly and obtained 20, many added $12\sqrt{5}$ and $-\sqrt{5}$ wrongly to get $-11\sqrt{5}$. Part (b) This part was answered more successfully with $\sqrt{\frac{75}{3}}-\sqrt{\frac{27}{3}}$ being the neatest method. Some failed to complete correctly from $\frac{2\sqrt{3}}{\sqrt{3}}$ to 2 and gave an answer of $\sqrt{3}$. A few went 'all round the houses' but got there eventually. Some tried to cancel out $\sqrt{3}$ but only considered one term in the denominator. Multiplying top and bottom by $\sqrt{3}$ caused some problems. A few attempted to combine the 2 terms in the numerator and wrote $\frac{\sqrt{48}}{\sqrt{3}}$ which of course is also an integer!

Question 5:	**Exam report**
a) i) $\frac{dy}{dx}=3x^2-20x+28$ ii) $\frac{dy}{dx}=0 \qquad 3x^2-20x+28=0$ $(3x-14)(x-2)=0$ $x=\frac{14}{3}$ or $x=2$ There are two stationary points: $x=2$ and $x=\frac{14}{3}$ b) i) $\int(x^3-10x^2+28x)dx=\frac{1}{4}x^4-\frac{10}{3}x^3+14x^2+c$ ii) $\int_0^3(x^3-10x^2+28x)dx=\left[\frac{1}{4}x^4-\frac{10}{3}x^3+14x^2\right]_0^3$ $=\left(\frac{1}{4}\times3^4-\frac{10}{3}\times3^3+14\times3^2\right)-(0-0+0)$ $=\frac{225}{4}=56\frac{1}{4}$ iii) Area shaded $=56\frac{1}{4}-$ Area of the triangle $=56\frac{1}{4}-\frac{1}{2}\times3\times21=\frac{99}{4}=24\frac{3}{4}$	Part (a)(i) Most candidates differentiated correctly. However a few made a slip or misread one of the terms. Part (a)(ii) Although most managed to substitute 2 into their derivative some made numerical errors and some used y or the second derivative. Most who realised that they should equate their derivative to zero (or at least showed their intention though never inserting the = 0) then tried to factorise or use the formula (though it was clear that some did not recognise the quadratic equation as such). It was disappointing that the bracket (3x–14) often produced the solution x = 14 instead of 14/3, and the solution of x = 2 did not always appear. Part (b)(i) The integration was also completed correctly by most candidates, although the 28x was occasionally wrong and some 'hybrid' processes led to terms such as $-\frac{20}{3}$. In part (b)(ii) almost everyone attempted to substitute 3 into their integral but their problems with the ensuing fractions often took pages to resolve, and although most ended 'magically' with the required answer there were many errors en route. A few substituted into the original expression for y instead of the integrated expression. Part (b)(iii) This part was quite well done although again there were errors in evaluating both $\frac{1}{2}\times21\times3$ and $56\frac{1}{4}-31\frac{1}{2}$. Some candidates confused length with area and merely used Pythagoras`s Theorem to find the length of the hypotenuse of the triangle. Those who chose to integrate the equation of the straight line were sometimes successful but many made arithmetic errors.

Question 6:	**Exam report**
$p(x)=x^3-4x^2+3x$ a) $p(3)=3^3-4\times3^2+3\times3=27-36+9=0$ 3 is a root of p so $(x-3)$ is a factor of p b) $x^3-4x^2+3x=x\left(x^2-4x+3\right)=x(x-1)(x-3)$ c) i) $r=p(2)=2^3-4\times2^2+3\times2=8-16+6=-2$ ii) $\begin{array}{r\|l} & x^2-2x-1 \\ \hline x-2 & x^3-4x^2+3x+0 \\ & \underline{x^3-2x^2} \\ & \quad -2x^2+3x \\ & \quad \underline{-2x^2+4x} \\ & \qquad -x+0 \\ & \qquad \underline{-x+2} \\ & \qquad\quad -2 \end{array}$ $x^3-4x^2+3x=(x-2)\left(x^2-2x-1\right)-2$	Part (a) Although many candidates showed that p(3) = 0, many lost a mark for failing to include a statement of the implication. Some candidates appeared ignorant of the Factor Theorem and used long division and therefore earned no marks in this part. Part (b) Only about half of the candidates were able to complete this part, although most made an attempt. The term x^2-x confused some. A few failed to write a product of factors even though this was requested. Part(c)(i) As the question requested the use of the Remainder Theorem, finding p(2) was the only acceptable method here. Many attempted long division and scored no marks. Part (c)(ii) There were many full solutions either by multiplying out and comparing coefficients they are both valid methods or by using long division. The majority of candidates showed poor algebraic skills and were unable to find the correct values of *a* and *b*. No credit was given for stating the value of r obtained in part (i) unless the values of a and b were correct. Full marks were earned by able candidates who simply wrote down the correct values of a, b and r by inspection.

Question 7:	**Exam report**
$x^2+y^2-4x-14=0$ $a)\,i)\,(x-2)^2-4+y^2-14=0$ $(x-2)^2+y^2=18$ The centre C has coordinates (2,0)	Part (a)(i) It was apparent that some candidates had not covered this part of the specification and they made no progress. Most who had done so, earned the marks here. However a few wrote $(x+2)^2$ or even $(x-2)^2$ and then gave the centre as (– 2,0) Some managed to incorporate the 7 with the y term so wrote the coordinates of the centre as (2,–7).
$ii)$ the radius is $r=\sqrt{18}=\sqrt{9\times2}=3\sqrt{2}$	Part (a)(ii) Most candidates were successful in finding the correct radius. However some forfeited one mark by 'meddling' with their equation and putting 18^2 or $\sqrt{18}$ on the right hand side of the equation.
$b)$ Call the chord AB and the mid-point of the chord I then the triangle CIA is a right-angled triangle. The perpendicular distance is the length CI. Using Pythagoras' theorem, $CI^2=CA^2-IA^2$ $=r^2-4^2$ $CI=\sqrt{2}$	Part(b) This part was rarely attempted. Even where a correct diagram was drawn, few recognised that the chord would be bisected. Many assumed that the triangle was right-angled at the centre of the circle. Others drew tangents instead of a chord.
$c)\,i)$ By solving the equations simultaneously: $\begin{cases} y=2k-x \\ x^2+y^2-4x-14=0 \end{cases}$ *we obtain, by substitution* $x^2+(2k-x)^2-4x-14=0$ $x^2+4k^2+x^2-4kx-4x-14=0$ $2x^2-4(k+1)x+4k^2-14=0\quad(\div2)$ $x^2-2(k+1)+2k^2-7=0$	Part (c)(i) Many made little progress here. However, it was good to see more able candidates coping well. A few fell at the final line writing (k–1) instead of (k+1); a few lost a mark by not introducing '= 0' as part of the equation of the circle and simply added '= 0' at the end of several lines of working so as to match the printed answer. Many made a slip in squaring (2k–x) and some made gross errors such as writing this as $4k^2+x^2$ or $4k^2-x^2$. Others 'simplified' the equation to $(x–2) + (2k–x) = \sqrt{18}$.
$ii)$ This equation has equal roots when the discriminant $=0$ $(-2(k+1))^2-4\times1\times(2k^2-7)=0$ $4(k^2+2k+1)-8k^2+28=0$ $-4k^2+8k+32=0$ $k^2-2k-8=0$ $(k-4)(k+2)=0$ $k=4$ *or* $k=-2$	Part (c)(ii) Those candidates who made progress here needed both knowledge and algebraic skills and only a small minority completed this part correctly. However more earned some method marks. Use of the correct condition on the discriminant was required but some just tried to solve the equation using the quadratic formula or used ' > 0' instead of ' = 0'. A few attempts at completing the square were seen but most failed to equate the expression to zero.
$iii)$ The line will be tangent to the circle when $k=4$ *or* $k=-2$	Part (c)(iii) Many candidates who had made no progress in the rest of the question stated that the line would be a tangent to the circle; however several candidates wrote at length about various transformations and completely missed the point.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

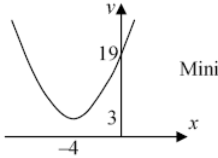
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

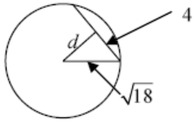
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)(i)	Gradient $AB = \frac{1-7}{5-1}$	M1		Must be y on top and subtr'n of cords
	$= -\frac{6}{4} = -\frac{3}{2} = -1.5$	A1	2	Any correct equivalent
(ii)	$y-7 = m(x-1)$ or $y-1 = m(x-5)$	M1		Verifying 2 points or $y = -\frac{3}{2}x + c$
	leading to $3x+2y = 17$	A1	2	AG (or grad & 1 point verified)
(b)	Attempt to eliminate x or y : $7x = 42$ etc	M1		Solving $x-4y=8$; $3x+2y=17$
	$x = 6$	A1		
	$y = -\frac{1}{2}$	A1	3	C is point $(6, -\frac{1}{2})$
(c)	Grad of perp = – 1 / *their* gradient AB	M1		Or $m_1m_2 = -1$ used or stated
	$= \frac{2}{3}$	A1√		ft their gradient AB
	$y-7 = \frac{2}{3}(x-1)$ or $3y - 2x = 19$	A1	3	CSO Any correct form of equation
	Total		**10**	
2(a)	$(x+4)^2$	B1		$p = 4$
	$+3$	B1	2	$q = 3$
(b)	$(x+4)^2 = -3$ or "their" $(x+p)^2 = -q$	M1		Or discriminant = 64 – 76
	No real square root of –3	A1	2	Disc < 0 so no real roots (all correct figs)
(c)	 Minimum (– 4, 3)	B1√		ft their $-p$ and q (or correct)
	graph	B1		Parabola (vertex roughly as shown)
		B1	3	Crossing at y = 19 marked or (0, 19) stated
(d)	Translation (and no additional transf'n)	E1		Not shift, move, transformation, etc
	through $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$	M1		One component correct eg 3 units up
		A1	3	All correct – if not vector – must say 4 units in negative x- direction, to left etc
	Total		**10**	
3(a)	$\frac{dy}{dx} = -10x^4$	M1		kx^4 condone extra term
		A1	2	Correct derivative unsimplified
(b)	When $x = 1$, gradient = –10	B1√		FT their gradient when $x = 1$
	Tangent is	M1		Attempt at y & tangent (**not** normal)
	$y-5 = -10(x-1)$ *or* $y + 10x = 15$ etc	A1	3	CSO Any correct form
(c)	When $x = -2$ $\frac{dy}{dx} = -160$ (or < 0)	B1√		Value of their $\frac{dy}{dx}$ when $x = -2$
	($\frac{dy}{dx} < 0$ hence) y is **decreasing**	E1√	2	ft Increasing if their $\frac{dy}{dx} > 0$
	Total		**7**	

Q	Solution	Marks	Total	Comments
4(a)				Multiplied out
	$4\left(\sqrt{5}\right)^2+12\sqrt{5}-\sqrt{5}-3$	M1		At least 3 terms with $\sqrt{5}$ term
	$4\left(\sqrt{5}\right)^2=4\times 5 \quad (=20)$	B1		
	Answer $=17+11\sqrt{5}$	A1	3	
(b)	Either $\sqrt{75}=\sqrt{25}\sqrt{3}$ or $\sqrt{27}=\sqrt{9}\sqrt{3}$	M1		Or multiplying top and bottom by $\sqrt{3}$
	Expression $=\dfrac{5\sqrt{3}-3\sqrt{3}}{\sqrt{3}}$	A1		or $\dfrac{\sqrt{225}-\sqrt{81}}{3}$ or $\sqrt{25}-\sqrt{9}$ or 5–3
	$=2$	A1	3	CSO
	Total		**6**	
5(a)(i)	$\dfrac{dy}{dx}=3x^2-20x+28$	M1 A1 A1	3	One term correct Another term correct All correct (no + c etc)
(ii)	Their $\dfrac{dy}{dx}=0$ for stationary point	M1		Or realising condition for stationary pt
	$(x-2)(3x-14)=0$	m1		Attempt to solve using formula/ factorise
	$\Rightarrow x=2$	A1		Award M1, A1 for verification that
	or $x=\dfrac{14}{3}$	A1	4	$x=2\Rightarrow\dfrac{dy}{dx}=0$ then may earn m1 later
(b)(i)	$\dfrac{x^4}{4}-\dfrac{10x^3}{3}+14x^2 \quad (+c)$	M1 A1 A1	3	One term correct unsimplified Another term correct unsimplified All correct unsimplified (condone missing + c)
(ii)	$\left[\dfrac{81}{4}-90+126\right] \quad (-0)$	M1		Attempt to sub limit 3 into their (b)(i)
	$=56\frac{1}{4}$	A1	2	AG Integration, limit sub'n all correct
(iii)	Area of triangle = $31\frac{1}{2}$	B1		Correct unsimplified $\frac{1}{2}\times 21\times 3$
	Shaded Area = $56\frac{1}{4}-\textit{triangle area}$	M1		
	$=24\frac{3}{4}$	A1	3	Or equivalent such as $\dfrac{99}{4}$
	Total		**15**	

Q	Solution	Marks	Total	Comments
6(a)	p(3) = 27 – 36 + 9	M1		Finding p(3) and **not** long division
	p(3) = 0 ⇒ $x-3$ is a factor	A1	2	Shown = 0 **plus a statement**
(b)	$x(x^2-4x+3)$ or $(x-3)(x^2-x)$ attempt	M1		Or p(1) = 0 ⇒ $x-1$ is a factor attempt
	$p(x)=x(x-1)(x-3)$	A1	2	Condone $x+0$ or $x-0$ as factor
(c)(i)	$p(2)=8-16+6$	M1		Must use p(2) and **not** long division
	(Remainder is) -2	A1	2	
(ii)	Attempt to multiply out and compare	M1		Or long division (2 terms of quotient)
	coefficients $a=-2$	A1		$x^2-2x...$
	$b=-1$	A1		-1
	$r=-2$	A1	4	Withhold final A1 for long division unless written as $(x-2)(x^2-2x-1)-2$
	SC B1 for r = –2 if M0 scored			
	Total		**10**	
7(a)(i)	$(x-2)^2$	M1		Attempt to complete square for x
	centre has x-coordinate = 2	A1		M1 implied if value correct or –2
	and y-coordinate = 0	B1	3	Centre (2,0)
(ii)	RHS = 18	B1		Withhold if circle equation RHS incorrect
	Radius = $\sqrt{18}$	M1		Square root of RHS of equation (if > 0)
	Radius = $3\sqrt{2}$	A1	3	
(b)	Perpendicular bisects chord so need to use Length of 4	B1		
	$d^2=(\text{radius})^2-4^2$	M1		
	$d^2=18-16$ so perpendicular distance = $\sqrt{2}$	A1	3	
(c)(i)	$x^2+(2k-x)^2-4x-14=0$	M1		
	$(2k-x)^2=4k^2-4kx+x^2$	B1		
	$\Rightarrow 2x^2+4k^2-4kx-4x-14=0$ $(\Rightarrow x^2+2k^2-2kx-2x-7=0)$ $\Rightarrow x^2-2(k+1)x+2k^2-7=0$	A1	3	AG (be convinced about algebra and = 0)
(ii)	$4(k+1)^2-4(2k^2-7)$	M1		"b^2-4ac" in terms of k (either term correct)
	$4k^2-8k-32=0$ or $k^2-2k-8=0$	A1		$b^2-4ac=0$ correct quadratic equation in k
	$(k-4)(k+2)=0$	m1		Attempt to factorise, solve equation
	$k=-2$, $k=4$	A1	4	SC B1, B1 for –2 , 4 (if M0 scored)
(iii)	Line is a tangent to the circle	E1	1	Line touches circle at one point etc
	Total		**17**	
	TOTAL		**75**	

January 2007

1 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of $\mathrm{p}(x)$, show that $k = 10$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

2 The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B. Find the coordinates of B. *(3 marks)*

(c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k. *(3 marks)*

3 (a) Express $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5} + q$, where p and q are integers. *(4 marks)*

(b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. *(3 marks)*

4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

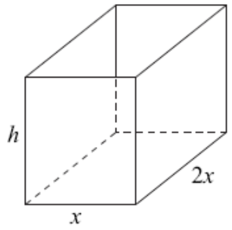
$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*

Turn over for the next question

5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, $V\ \text{m}^3$, that the tank can hold when full is given by

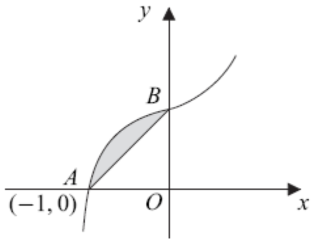
$$V = 18x - \frac{2x^3}{3}$$ *(1 mark)*

(b) (i) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

(ii) Find $\int (3x^5 + 2x + 5)\ \mathrm{d}x$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

7 The quadratic equation $(k+1)x^2 + 12x + (k-4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - – Jan 2007 – Answers

Question 1:	**Exam report**
$p(x)=x^3-4x^2-7x+k$ $a)i)\ (x+2)$ is a factor of $p(x)$ This means that $p(-2)=0$ $p(-2)=(-2)^3-4\times(-2)^2-7\times(-1)+k=$ $=-8-16+14+k=0$ $-10+k=0 \qquad k=10$ $ii)\ x^3-4x^2-7x+10=(x+2)(x^2-6x+5)$ $=(x+2)(x-5)(x-1)$ $b)$ The remainder of the division by $(x-3)$ *is* $p(3)$ $p(3)=3^3-4\times3^2-7\times3+10$ $=27-36-21+10=-20$ $c)$ The graph crosses the x-axis at $(-2,0),(5,0)$ *and* $(1,0)$ crosses the y-axis at $(0,10)$	Part (a)(i) Most candidates found p(-2) but often failed to convince examiners that they had really shown that *k* = 10. Many substituted *k* = 10 from the outset and then drew no conclusion from the fact that p(-2) = 0. Those using long division often made sign errors. Part (a)(ii) Factorisation of a cubic seems well understood and, apart from those who could not factorise $x^2 - 6x + 5$, candidates usually scored full marks. Some still confuse factors and roots. Part (b) Many ignored the request to use the Remainder Theorem and scored no marks for long division. A few who correctly found that p(3) = -20 concluded that the remainder was +20. Part (c) The sketch was generously marked with regard to the position of the stationary points but it was expected that candidates would indicate the values where the curve crossed the coordinate axes and often these values, particularly the 10 on the *y*-axis, were omitted.

Question 2:	**Exam report**
$a)i)\ 3x+5y=8 \qquad A(6,-2)$ Make y the subject: $y=-\frac{3}{5}x+\frac{8}{5}$ The gradient is $m_{AB}=-\frac{3}{5}$ $ii)$ The gradient of the line perpendicular to AB is $-\frac{1}{m_{AB}}=\frac{5}{3}$ The equation of the line is : $y-(-2)=\frac{5}{3}(x-6)$ $3y+6=5x-30$ $5x-3y=36$ $b)$ Solve simultaneously $\begin{cases}3x+5y=8 & (\times2)\\ 2x+3y=3 & (\times-3)\end{cases}$ This gives $\begin{cases}6x+10y=16\\ -6x-9y=-9\end{cases}$ *by adding*, $y=7$ *then* $3x+5y=8 \qquad 3x+35=8 \qquad x=-9$ $B(-9,7)$ $c)\ C(2,k)$ *and* $A(6,-2)$ $AC=\sqrt{(2-6)^2+(k+2)^2}=5$ *so* $AC^2=16+(k+2)^2=25$ $(k+2)^2=9$ $k+2=3 \ \ or \ \ k+2=-3$ $k=1 \ \ or \ \ k=-5$	Part (a)(i) It was disappointing to see many candidates unable to rearrange 3x + 5y = 8 to make y the subject in order to find the gradient. Some were successful in finding a second point on the line such as (1, 1) and then using the coordinates of A to find the gradient of AB. Part (a)(ii) Most candidates knew how to find the gradient of a perpendicular line, but those using y = mx + c made more arithmetic slips than those using the more appropriate form $y - y_1 = m(x - x_1)$. Part (b) Apart from those who used the wrong pair of equations, this part was usually answered correctly. Part (c) Although this part was meant to be challenging, there were many successful attempts, particularly by those who used a sketch and reasoned on a 3, 4, 5 triangle. It had been intended that candidates would have formed an equation such as $16 + (k + 2)^2 = 25$, but more commonly something such as $16 + y^2 = 25$ was seen, resulting in the incorrect values of ±3 .

Question 3:	**Exam report**
$a)\ \frac{\sqrt{5}+3}{\sqrt{5}-2}=\frac{\sqrt{5}+3}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{5+2\sqrt{5}+3\sqrt{5}+6}{5-4}=11+5\sqrt{5}$ $b)\ i)\ \sqrt{45}=\sqrt{9\times5}=3\sqrt{5}$ $ii)\ x\sqrt{20}=7\sqrt{5}-\sqrt{45}$ $x\times2\sqrt{5}=7\sqrt{5}-3\sqrt{5}$ $x=\frac{4\sqrt{5}}{2\sqrt{5}}=\frac{4}{2}=2$	Part (a) It was not uncommon to see the denominator and numerator multiplied by different surds and the usual errors occurred as candidates tried to multiply out brackets. This part of the question did not seem to be answered as well as in previous years. Part (b)(i) was usually correct but very few were successful in solving the equation in part (b)(ii), even though they reached forms of the correct equation such as $2\sqrt{5}x=4\sqrt{5}\ \ or\ \ x=\frac{4\sqrt{5}}{2\sqrt{5}}$.

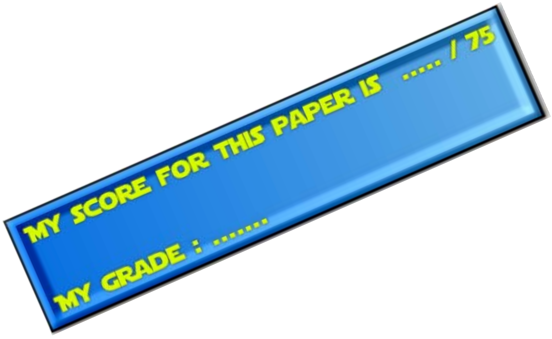
Question 4:	**Exam report**
$x^2+y^2+2x-12y+12=0$ $a)\ (x+1)^2-1+(y-6)^2-36+12=0$ $(x+1)^2+(y-6)^2=25$ $b)\ i)$ The centre $C(-1,6)$ $ii)\ r=\sqrt{25}=5$ $c)$ On the x-axis, $y=0$, The equation becomes $(x+1)^2+36=25$ $(x+1)^2=-11$ No solution as $(x+1)^2>0\ \ for\ \ all\ \ x$. $d)$ Consider simultaneously $\begin{cases}x^2+y^2+2x-12y+12=0\\y=4-x\end{cases}$ by substitution, we have $x^2+(4-x)^2+2x-12(4-x)+12=0$ $x^2+16+x^2-8x+2x-48+12x+12=0$ $2x^2+6x-20=0$ $x^2+3x-10=0$ $ii)\ x^2+3x-10=0$ $(x-2)(x+5)=0 \qquad x=2\ \ or\ \ x=-5$ $and\ \ y=4-x\ : \qquad y=2\ \ or\ \ y=9$ $P(2,2)\ \ and\ \ Q(-5,9)$ $iii)$ The mid-point of PQ is $\left(\frac{x_P+x_Q}{2},\frac{y_P+y_Q}{2}\right)=\left(-\frac{3}{2},\frac{11}{2}\right)$	Part (a) The + 2x term was ignored by many who wrote the left hand side of the circle equation as $(x-1)^2+(y-6)^2$ but most candidates were able to complete the square correctly. The right hand side was often seen as 49 and since this was a perfect square it did not cause candidates to doubt their poor arithmetic. Part (b) Many who had the correct circle equation in part (a) wrote the coordinates of the centre with incorrect signs. Generous follow through marks were awarded in this part provided the right hand side of the equation had a positive value. Part (c) Almost all candidates reasoned correctly by considering the y-coordinate of the centre and the radius of the circle, although a number were successful in showing that the quadratic resulting from substituting y = 0 into the circle equation does not have real roots. Some simply drew a diagram and this alone was not regarded as sufficient to prove that the circle did not intersect the x-axis. Others using an algebraic approach found a quadratic that they said did not factorise and concluded incorrectly that the equation had no real roots. Part (d) The algebra proved too difficult for the weaker candidates and many who had shown good algebraic skills rather casually forgot to include "= 0" on their final line of working. Sadly, many were unable to factorise the quadratic or wrote the coordinates of Q as (-(-5, 2). It was good, however, to see more candidates being able to find the correct mid-point, where in previous years too many had found the difference of the coordinates before dividing by 2.

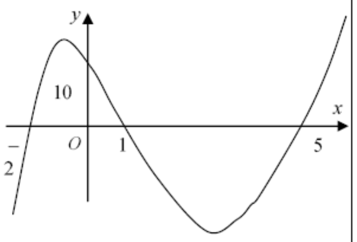
Question 5:	**Exam report**
a) i) Surface area : $x\times 2x+2hx+2h\times 2x=54$ $2x^2+2h(x+2x)=54\quad(\div 2)$ $x^2+3xh=27$ ii) $h=\frac{27-x^2}{3x}=\frac{9}{x}-\frac{x}{3}$ iii) $V=x\times 2x\times h=2x^2\left(\frac{9}{x}-\frac{x}{3}\right)=18x-\frac{2x^3}{3}$ b) i) $\frac{dV}{dx}=18-\frac{2}{3}\times 3x^2=18-2x^2$ ii) $\frac{dV}{dx}=0$ means $18-2x^2=0$ $x^2=9$ $x=3$ or $x=-3$ (x must be positive) c) $\frac{d^2V}{dx^2}=-4x$ and for $x=3$, $\frac{d^2V}{dx^2}=-12<0$ This point is a maximum.	Part (a) Candidates did not seem confident at working on this kind of problem and algebraic weaknesses were evident. Many worked backwards from the result in part (a)(i) and did not always convince the examiner that they were considering the surface area of four faces and the base. The inability of most candidates to rearrange the formula to make h the subject in part (a)(ii) was alarming. Consequently few, without considerable fudging, could establish the printed formula for the volume. Part (b) Basic differentiation is well understood and most candidates found $\frac{dV}{dx}$ correctly. Some tried to substitute x = 3 into the expression for V in order to show there was a stationary point, but usually this part was answered well. Part (c) It was not uncommon to see the second derivative as 4x even though the first derivative was correct. A generous follow through was given here provided candidates could interpret the value of their second derivative.

Question 6:	**Exam report**
a) i) $B(0,y_B)$ belongs to the curve so $y_B=3\times 0^5+2\times 0+5=5$ $B(0,5)$ Area AOB $=\frac{1}{2}\times 1\times 5=\frac{5}{2}$ ii) $\int(3x^5+2x+5)dx=\frac{3}{6}x^6+\frac{2}{2}x^2+5x+c$ $=\frac{1}{2}x^6+x^2+5x+c$ iii) Area of shaded region $=\int_{-1}^{0}(3x^5+2x+5)dx-\frac{5}{2}$ $=\left[\frac{1}{2}x^6+x^2+5x\right]_{-1}^{0}-\frac{5}{2}=(0)-\left(\frac{1}{2}+1-5\right)-\frac{5}{2}$ $=\frac{7}{2}-\frac{5}{2}=1$ b) i) the gradient of the curve at A is $\frac{dy}{dx}(x=-1)$ $\frac{dy}{dx}=15x^4+2$ and for $x=-1$, $m=17$ ii) The equation of the tangent at A is $y-0=17(x+1)$ $y=17x+17$	Part (a)(i) Some candidates ignored the request to state the coordinates of B even though they were using the height of the triangle as 5. The negative x-coordinate of A caused quite a few to feel that the triangle had a negative area. Far too many when finding $\frac{1}{2}\times 1\times 5$ wrote the answer as 3. Part (a)(ii) Practically every candidate found the correct integral although some made errors when cancelling fractions. Part (a)(iii) It was necessary here to have the lower limit as .1 and the upper limit as 0. Many reversed the order and by some trickery arrived at a positive value. This was penalised and so very few, even though many had an answer of 1 for the area, scored full marks for this part of the question. Part (b) Most candidates differentiated correctly but, because of poor understanding of negative signs, many wrong values of -13 were seen for the gradient. There is obviously confusion for many between tangents and normals and several thought the gradient of the tangent was $-\frac{1}{17}$.

Question 7:	Exam report
$(k+1)x^2+12x+(k-4)=0$ *has real roots* *means* the discriminant ≥ 0 a)$12^2-4\times(k+1)\times(k-4)\geq 0$ $144-4k^2+12k+16\geq 0$ $-4k^2+12k+160\geq 0 \quad (\div -4)$ $k^2-3k-40\leq 0$ $b) \quad (k-8)(k+5)\leq 0$ $-5\leq k\leq 8$	Part (a) The condition for real roots was not widely known and the form of the printed answer caused many to write the condition as $b^2 - 4ac \leq 0$. Part (b) It was disappointing to see many unable to factorise the quadratic correctly. Far too many guessed at answers and an approach using a sign diagram or sketch is recommended. Candidates also need to realise that the final form of the answer cannot be written as $"k\geq -5" or "k\leq 8"$

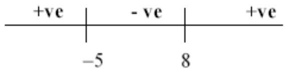
GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	59	51	43	35	28



Q	Solution	Marks	Total	Comments
1(a)(i)	p(–2) = –8 –16 +14 + k	M1		or long division or $(x+2)(x^2-6x+5)$
	p(–2) = 0 $\Rightarrow -10+k=0 \quad \Rightarrow k=10$	A1	2	**AG** likely withhold if p(–2) = 0 not seen
	Must have statement if k=10 substitute			
(ii)	$\text{p}(x)=(x+2)(x^2+\ldots.\quad 5)$	M1		Attempt at quadratic or second linear
	$\text{p}(x)=(x+2)(x^2-6x+5)$	A1		factor $(x-1)$ or $(x-5)$ <u>from factor theorem</u>
	$\Rightarrow \text{p}(x)=(x+2)(x-1)(x-5)$	A1	3	Must be written as product
(b)	$\text{p}(3)=27-36-21+k$	M1		long division scores M0
	(Remainder) = $k-30$ = <u>–20</u>	A1	2	Condone $k-30$
(c)		B1		Curve thro' 10 marked on y-axis
		B1√		**FT** their 3 roots marked on x-axis
		M1		Cubic shape with a max and min
		A1	4	Correct graph (roughly as on left) going beyond –2 and 5 (condone max anywhere between $x=-2$ and 1 and min between 1 and 5)
	Total		**11**	
2(a)(i)	$y=-\frac{3}{5}x+\ldots;$ Gradient $AB=-\frac{3}{5}$	M1		**Attempt** to find $y=$ or $\Delta y/\Delta x$ or $\frac{3}{5}$ or $3x/5$
		A1	2	Gradient correct – condone slip in $y=\ldots$
(ii)	$m_1m_2=-1$	M1		Stated or used correctly
	Gradient of perpendicular = $\frac{5}{3}$	A1√		**ft** gradient of AB
	$\Rightarrow y+2=\frac{5}{3}(x-6)$	A1	3	**CSO** Any correct form eg $y=\frac{5}{3}x-12$, $5x-3y=36$ etc
(b)	Eliminating x or y (unsimplified)	M1		Must use $3x+5y=8;\ 2x+3y=3$
	$x=-9$	A1		
	$y=7$	A1	3	B (–9,7)
(c)	$4^2+(k+2)^2 \quad (=25)$ or $16+d^2=25$	M1		Diagram with 3,4, 5 triangle
	$k=1$	A1		Condone slip in one term (or $k+2=3$)
	or $k=-5$	A1	3	**SC1** with no working for spotting one correct value of k. Full marks if both values spotted with no contradictory work
	Total		**11**	

Q	Solution	Marks	Total	Comments
3(a)	$\frac{\sqrt{5}+3}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$	M1		Multiplying top & bottom by $\pm\left(\sqrt{5}+2\right)$
	Numerator = $5+3\sqrt{5}+2\sqrt{5}+6$	M1		Multiplying out (condone one slip) $\pm\left(\sqrt{5+3}\right)\left(\sqrt{5+2}\right)$
	$= 5\sqrt{5}+11$	A1		
	Final answer = $5\sqrt{5}+11$	A1	4	With clear evidence that denominator =1
(b)(i)	$\sqrt{45}=3\sqrt{5}$	B1	1	
(ii)	$\sqrt{20}=\sqrt{4}\sqrt{5}$ or $4\sqrt{5}=\sqrt{4}\times\sqrt{20}$ or attempt to have equation with $\sqrt{5}$ or $\sqrt{20}$ only	M1		Both sides
	$\left[x\ 2\sqrt{5}=7\sqrt{5}-3\sqrt{5}\right]$ **or** $x\sqrt{20}=2\sqrt{20}$	A1		or $x=\sqrt{4}$
	$x=2$	A1	3	**CSO**
	Total		**8**	
4(a)	$(x+1)^2+(y-6)^2$	B2		B1 for one term correct or missing + sign
	$(1+36\ -12=25)$ RHS $=5^2$	B1	3	Condone 25
(b)(i)	Centre (–1 , 6)	B1√	1	**FT** their a and b from part (a) or correct
(ii)	Radius = 5	B1√	1	**FT** their r from part (a) RHS must be > 0
(c)	Attempt to solve "their" $x^2+2x+12=0$	M1		Or comparing "their" $y_c=6$ and their $r=5$ may use a diagram with values shown
	(all working correct) so no real roots or statement that does not intersect	A1	2	$r<y_c$ so does not intersect condone ± 1 or $\pm\ 6$ in centre for A1
(d)(i)	$(4-x)^2=16-8x+x^2$	B1		Or $(-2-x)^2=4+4x+x^2$
	$x^2+(4-x)^2+2x-12(4-x)+12=0$ or $(x+1)^2+(-2-x)^2=25$	M1		Sub $y=4-x$ in circle eqn (condone slip) or "their" circle equation
	$\Rightarrow 2x^2+6x-20=0 \quad \Rightarrow x^2+3x-10=0$	A1	3	**AG CSO** (must have = 0)
(ii)	$(x+5)(x-2)=0 \quad \Rightarrow x=-5, x=2$	M1		Correct factors or unsimplified solution to quadratic
	Q has coordinates (–5, 9)	A1	2	(give credit if factorised in part (i)) <u>SC2</u> if Q correct. Allow x = –5 $y=9$
(iii)	Mid point of 'their' (–5, 9) and (2,2)	M1		Arithmetic mean of **either** x or y coords
	$\left(-1\frac{1}{2}, 5\frac{1}{2}\right)$	A1	2	Must follow from correct value in (ii)
	Total		**14**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$2x^2+2xh+4xh \quad (=54)$ $\Rightarrow x^2+3xh=27$	M1 A1	2	Attempt at surface area (one slip) **AG CSO**
(ii)	$h=\dfrac{27-x^2}{3x}$ or $h=\dfrac{9}{x}-\dfrac{x}{3}$ etc	B1	1	Any correct form
(iii)	$V=2x^2h \quad = \; 18x-\dfrac{2x^3}{3}$	B1	1	**AG** (watch fudging) condone omission of brackets
(b)(i)	$\dfrac{\mathrm{d}V}{\mathrm{d}x}=18-2x^2$	M1 A1	2	One term correct "their" V All correct unsimplified $18 - 6x^2/3$
(ii)	Sub $x = 3$ into their $\dfrac{\mathrm{d}V}{\mathrm{d}x}$	M1		Or attempt to solve their $\dfrac{\mathrm{d}V}{\mathrm{d}x}=0$
	Shown to equal 0 plus **statement** that this implies a stationary point if verifying	A1	2	**CSO** Condone $x=\pm3$ or $x = 3$ if solving
(c)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}=-4x$ $(=-12)$	B1√		**FT** their $\dfrac{\mathrm{d}V}{\mathrm{d}x}$
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}<0$ at stationary point $\Rightarrow$ maximum	E1√	2	**FT** their second derivative conclusion If "their" $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}>0 \Rightarrow$ minimum etc
	Total		**10**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$B\,(0,5)$ Area $AOB=\frac{1}{2}\times1\times5$ $=2\frac{1}{2}$	B1 M1 A1	3	 Condone slip in number or a minus sign
(ii)	$\dfrac{3x^6}{6}+\dfrac{2x^2}{2}+5x$ or $\dfrac{x^6}{2}+x^2+5x$ (may have $+\,c$ or not)	M1 A1 A1	3	Raise one power by 1 One term correct All correct unsimplified
(iii)	Area under curve = $\displaystyle\int_{-1}^{0}\mathrm{f}(x)\,\mathrm{d}x$	B1		Correctly written or F(0) – F(–1) correct
	$[0]-\left[\dfrac{1}{2}+1-5\right]$ Area under curve = $3\frac{1}{2}$	M1 A1		Attempt to sub limit(s) of –1 (and 0) Must have integrated **CSO** (no fudging)
	Area of shaded region = $3\frac{1}{2}-2\frac{1}{2}=1$	B1√	4	**FT** their integral and triangle (very generous)
(b)(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=15x^4+2$	M1 A1		One term correct All correct (no +c etc)
	when $x=-1$, gradient = 17	A1	3	**cso**
(ii)	$y=$"their gradient"$(x+1)$	B1√	1	Must be finding **tangent** – not normal any form e.g. $y=17x+17$
	Total		**14**	
7(a)	$b^2-4ac \; = \; 144-4(k+1)(k-4)$	M1		Clear attempt at b^2-4ac Condone slip in one term of expression
	Real roots when $b^2-4ac \geqslant 0$ $36-(k^2-3k-4)\geqslant 0$	B1		Not just a statement, must involve k
	$\Rightarrow k^2-3k-40\leqslant 0$	A1	3	**AG** (watch signs carefully)
(b)	$(k-8)(k+5)$ Critical points 8 and –5	M1 A1		Factors attempt or formula
	Sketch or sign diagram **correct,** must have 8 and –5 $-5\leqslant k\leqslant 8$ A0 for $-5<k<8$ or two separate inequalities unless word AND used	M1 A1	4	
	Total		**7**	
	TOTAL		**75**	

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

2 (a) Express $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. *(3 marks)*

(b) Express $\frac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7} + q$, where p and q are integers. *(4 marks)*

3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x+p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

5 A circle with centre C has equation $(x+3)^2 + (y-2)^2 = 25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates (2, 6).

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

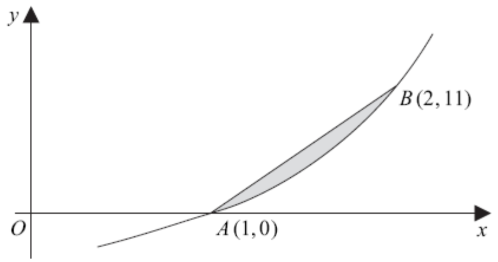
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

7 The quadratic equation

$$(2k-3)x^2 + 2x + (k-1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

END OF QUESTIONS

AQA – Core 1 - – June 2007 – Answers

Question 1:

$A(6,-1)\quad B(2,5)$

$$a)i)\ m_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{5+1}{2-6}=\frac{6}{-4}=-\frac{3}{2}$$

$$ii)\ Equation\ of\ \ AB:\ y+1=-\frac{3}{2}(x-6)$$

$$2y+2=-3x+18$$

$$3x+2y=16$$

$$b)i)\ \text{the line perpendicular to AB has gradient }-\frac{1}{m_{AB}}=\frac{2}{3}$$

$$\text{The equation of this line : }y-5=\frac{2}{3}(x-2)$$

$$3y-15=2x-4$$

$$2x-3y=-11$$

$ii)\ C(k,7)$. the angle ABC is a right angle so the point C belongs to the perpendicular to AB.

By substituting : $2\times k-3\times 7=-11$

$$2k=10 \qquad k=5$$

Exam report

Part (a)(i) Most candidates were able to show that the gradient was $-\frac{3}{2}$. However, examiners had to be vigilant since fractions such as $\frac{6}{4}$ *and* $\frac{-4}{6}$ were sometimes equated to $-\frac{3}{2}$.

Part (a)(ii) Many candidates did not heed the request for integer coefficients and left their answer as $y=-\frac{3}{2}x+8$. Many who attempted to express the equation in the required form were unable to double the 8 and wrote their final equation as$3x + 2y = 8$.

Part (b)(i) Most candidates realised that the product of the gradients should be -1. However, not all were able to calculate the negative reciprocal. Others used the incorrect point and therefore found an equation of the wrong line.

Part (b)(ii) Many candidates made no attempt at this part of the question. The most successful method was to substitute $y = 7$ into the answer to part (b)(i) or to equate the gradient to $\frac{2}{3}$. There were also some good answers using a diagrammatic approach. Those using Pythagoras usually made algebraic errors and so rarely reached a solution.

Question 2:

$$a)\ \frac{\sqrt{63}}{3}+\frac{14}{\sqrt{7}}=\frac{\sqrt{9\times 7}}{3}+\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{3\sqrt{7}}{3}+\frac{14\sqrt{7}}{7}$$

$$=\sqrt{7}+2\sqrt{7}=3\sqrt{7}$$

$$b)\ \frac{\sqrt{7}+1}{\sqrt{7}-2}=\frac{\sqrt{7}+1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{7+2\sqrt{7}+\sqrt{7}+2}{7-4}$$

$$=\frac{9+3\sqrt{7}}{3}$$

Exam report

Part (a) Some candidates found this part more difficult than part (b) and revealed a lack of understanding of surds. Some managed to express the first term as $\sqrt{7}$ but were unable to deal with the second term. Those who attempted to find a common denominator often multiplied the terms in the numerator and/or added those in the denominator. Very few obtained the correct answer of $3\sqrt{7}$.

Part (b) Most candidates recognised the first crucial step of multiplying the numerator and denominator by $\sqrt{7}+2$ and many obtained $\frac{9+3\sqrt{7}}{3}$, but poor cancellation led to a very common incorrect answer of $3\sqrt{7}+3$.

Question 3:	**Exam report**
$a)\,i)\,x^2+10x+19=(x+5)^2-25+19=(x+5)^2-6$ $ii)$ The minimum point has coordinates $(-5,-6)$ $iii)$ The graph is symmetrical around the line $x=-5$ $iv)\ x\xrightarrow[\text{units in } x-\text{direction}]{\text{translation}-5}x+5\xrightarrow{f}(x+5)^2$ $\xrightarrow[\text{units in } y-\text{direction}]{\text{translation}-6}(x+5)^2-6$ *Translation vector* $\begin{bmatrix}-5\\-6\end{bmatrix}$ $b)$ solve simultaneously $\begin{cases}y=x+11\\y=x^2+10x+19\end{cases}$ *by identifying the y's* $(y=)x^2+10x+19=x+11$ $x^2+9x+8=0$ $(x+8)(x+1)=0$ $x=-8\ \ or\ \ x=-1$ *and* $y=x+11$ *gives* $y=3$ *or* $y=10$ The points of intersection have coordinates $(-8,3)$ *and* $(-1,10)$	Part (a)(i) The completion of the square was done successfully by most candidates, although occasionally + 44 was seen instead of .-6 for q. Part (a)(ii) Most candidates were able to write down the correct minimum point, although some wrote (5, -6) as the vertex. A few chose to use differentiation but often made arithmetic slips in finding the coordinates of the stationary point. Part (a)(iii) Although there were correct answers for the equation, the term . line of symmetry. was not well understood by many; typical wrong answers were y = -6, the y-axis and even y = $-x^2$ +10x+19 or other quadratic curves. Part (a)(iv) The more able candidates earned full marks here. The term translation was required but generally the wrong word was used or it was accompanied by another transformation such as a stretch. The most common (but incorrect) vector stated was $\begin{bmatrix}10\\19\end{bmatrix}$. Part (b) There were a number of complete correct solutions here. The errors that did occur usually stemmed from sign slips in rearranging the equations. Some candidates found the x-coordinates and made no attempt at the y-coordinates. A few candidates wrote down the coordinates of at least one point without any working.

Question 4:	**Exam report**
$a)\,i)\,\frac{d}{dt}=\frac{1}{4}\times4t^3-26\times2t+96=t^3-52t+96$ $ii)\,\frac{d^2y}{dt^2}=3t^2-52$ $b)$ Let's verify that for $t=2,\ \frac{dy}{dt}=0$ $\frac{dy}{dt}(t=2)=2^3-52\times2+96=8-104+96=0$ There is a stationary point when $t=2$. Let's work out $\frac{d^2y}{dt^2}(t=2)=3\times2^2-52=12-52=-40<0$ The stationary point is a Maximum. c) The rate of change is $\frac{dy}{dt}(t=1)=1-52+96=45\,cm/s$ $d)$ The sign of $\frac{dy}{dt}$ *at* $t=3$ will indicate if the height is increasing or decreasing. $\frac{dy}{dt}(t=3)=3^3-52\times3+96=27-156+96=-33<0$ The height is decreasing when $t=3$	Part (a) Almost all candidates were able to find the first and second derivatives correctly, although there was an occasional arithmetic slip; some could not cope with the fraction term, others doubled 26 incorrectly. Part (b) Those who substituted t = 2 into $\frac{dy}{dt}$ did not always explain that $\frac{dy}{dt}=0$ is the condition for a stationary point. Many used the second derivative test and concluded that the point was a maximum. Some assumed that a stationary point occurred when t = 2 and went straight to the test for maximum or minimum and only scored half of the marks. A few tested $\frac{dy}{dt}$ on either side of t = 2 correctly, but those who only considered the gradient on one side of the stationary value scored no marks for the test. Part (c) The concept of .rate of change. was not understood by many. Approximately equal numbers of candidates substituted into t = 1 into the expression for y, $\frac{dy}{dt}$ *or* $\frac{d^2y}{dt^2}$ and so only about one third of the candidates were able to score any marks on this part. Those who used $\frac{dy}{dt}$ often made careless arithmetic errors when adding three numbers. Part (d) As in part (c), candidates did not realise which expression to use and perhaps the majority wrongly selected the second derivative. It is a general weakness that candidates do not realise that the sign of the first derivative indicates whether a function is increasing or decreasing at a particular point.

Question 5:	**Exam report**
$(x+3)^2+(y-2)^2=25$ $a)\,i)\,C(-3,2)$ $ii)\ Radius\ \ r=\sqrt{25}=5$ $b)\,i)$ N belongs to the circle if its coodinates satify the equation $(0+3)^2+(-2-2)^2=3^2+4^2=9+16=25$ $N(0,-2)$ belongs to the circle $ii)$ $iii)$ The equation of the normal is the equation of the line CN $\mathrm{m}_{CN}=\frac{-2-2}{0+3}=-\frac{4}{3}$ $Equation:\ \ y+2=-\frac{4}{3}\left(x-0\right)$ $3y+6=-4x$ $4x+3y=-6$ $c)\,P(2,6)\quad C(-3,2)$ $i)\,PC=\sqrt{(-3-2)^2+\left(2-6\right)^2}=\sqrt{25+16}=\sqrt{41}$ $ii)$ If we call T the point of contact of the tangent from P then the triangle PTC is a right-angled triangle. $\mathrm{PT}^2=PC^2-TC^2=\left(\sqrt{41}\right)^2-r^2=41-25=16$ $PT=\sqrt{16}=4$	Part (a) Most candidates found the correct coordinates of the centre, although some wrote these as (3, -2) instead of (-3, 2). Those who multiplied out the brackets were often unsuccessful in writing down the correct radius of the circle. Part (b)(i) Most candidates were able to verify that the point N was on the circle, although some, who had perhaps worked a previous examination question, were keen to show that the distance from C to N was less than the radius and that N lay inside the circle. Part (b)(ii) Most sketches were correct, though some were very untidy with several attempts at the circle so that the diagram resembled a chaotic orbit of a planet. Some candidates omitted the axes and scored no marks. Part (b)(iii) The majority of candidates found the gradient of CN and then assumed they had to find the negative reciprocal of this since the question asked for the normal at N. Reference to their diagram might have avoided this incorrect assumption. Part (c)(i) Most wrote $PC^2=5^2+4^2$, provided they had the correct coordinates of C. However, the length of PC was often calculated incorrectly with answers such as $\sqrt{31}$ and $\sqrt{36}=6$ seen quite often. Part (c)(ii) Although there were many correct solutions seen, Pythagoras' Theorem was often used incorrectly. A large number of candidates wrote the answer as a difference of two lengths such as $\sqrt{41}-5$. Candidates need to realise that obtaining the correct answer from incorrect working is not rewarded; quite a few wrote $\sqrt{41}-\sqrt{25}=\sqrt{16}=4$ and scored no marks. Many who drew a good diagram realised that a tangent from (2,6) touched the circle at (2,2) and so the vertical line segment was of length 4 units.

Question 6:	**Exam report**
a) $f(x)=x^3+4x-5$ i) Let's work out $f(1)$ $f(1)=1^3+4\times1-5=1+4-5=0$ 1 is a root of f so $(x-1)$ is a factor of f. ii) $f(x)=(x-1)(x^2+x+5)$ iii) The discriminant of x^2+x+5 is $1^2-4\times1\times5=-19<0$ x^2+x+5 has no real roots The only real root of f is 1. b) i) $\int(x^3+4x-5)dx=\frac{1}{4}x^4+2x^2-5x+c$ ii) Mark the point C(2,0). Area of the shaded region is Area of ABC $-\int_1^2 f(x)dx$ $A=\frac{1}{2}\times1\times11-\left[\frac{1}{4}x^4+2x^2-5x\right]_1^2=\frac{11}{2}-\left[(4+8-10)-\left(\frac{1}{4}+2-5\right)\right]$ $A=\frac{11}{2}-2-\frac{11}{4}=\frac{22-8-11}{4}=\frac{3}{4}$	Part (a)(i) Most candidates realised the need to find the value of f(x) when x = 1. However, it was also necessary, after showing that f(1) = 0, to write a statement that the zero value implied that x -1 was a factor. Part (a)(ii) Those who used inspection were the most successful here. Methods involving long division or equating coefficients usually contained algebraic errors. Part (a)(iii) This section seemed unclear to some candidates. Many tried to find the discriminant but used the coefficients of the cubic equation. Many who used the quadratic thought that in order to have one real root the discriminant had to be zero, no doubt thinking the question was asking about equal roots. Some correctly stated that 1 was the only real root but many were obviously confused by the terms "factor" and "root" and stated that "x-1 was a root". Part (b)(i) Most candidates were well versed in integration and earned full marks here. Part (b)(ii) The correct limits were usually used, although many sign/arithmetic slips occurred after substitution of the numbers 1 and 2. Very few candidates realised the need to find the area of a triangle as well and so failed to subtract the value of the integral from the area of the triangle in order to find the area of the shaded region.

Question 7:	**Exam report**
$(2k-3)x^2+2x+(k-1)=0$ *has real roots* This means that the discriminat ≥ 0 $2^2-4\times(2k-3)\times(k-1)\geq0$ $4-8k^2+20k-12\geq0$ $-8k^2+20k-8\geq0 \quad (\div-4)$ $2k^2-5k+2\leq0$ b) i) $2k^2-5k+2=(2k-1)(k-2)$ ii) $(2k-1)(k-2)\leq0$ *critical values* : $\frac{1}{2}$ *and* 2 $\frac{1}{2}\leq k\leq2$	Part (a) Only the more able candidates were able to complete this proof correctly. Many began by stating that the discriminant was less than or equal to zero, no doubt being influenced by the printed answer. Part (b)(i) The factorisation was usually correct. Part (b)(ii) Most candidates found the critical values, but many then either stopped or wrote down a solution to the inequality without any working. Many candidates wrongly thought the solution was $k\leq\frac{1}{2}, k\leq2$. Candidates are advised to draw an appropriate sketch or sign diagram so they can deduce the correct interval for the solution.

GRADE BOUNDARIES

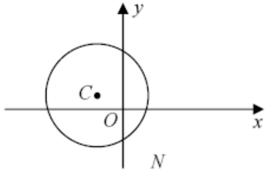
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	60	52	44	37	30

MY SCORE FOR THIS PAPER IS ..... / 75

MY GRADE : ....

Q	Solution	Marks	Total	Comments
1(a)(i)	Gradient $AB = \frac{-1-5}{6-2}$ or $\frac{5--1}{2-6}$	M1		$\pm\frac{6}{4}$ implies M1
	$=\frac{-6}{4}=-\frac{3}{2}$	A1	2	AG
(ii)	$\left.\begin{matrix}y-5\\y+1\end{matrix}\right\}=-\frac{3}{2}\left\{\begin{matrix}(x-2)\\(x-6)\end{matrix}\right.$	M1		or $y=-\frac{3}{2}x+c$ and attempt to find c
	$\Rightarrow 3x+2y=16$	A1	2	OE; must have integer coefficients
(b)(i)	Gradient of perpendicular $=\frac{2}{3}$	M1		or use of $m_1m_2=-1$
	$\Rightarrow y-5=\frac{2}{3}(x-2)$	A1	2	$3y-2x=11$ (no misreads permitted)
(ii)	Substitute $x=k$, $y=7$ into their (b)(i)	M1		or grads $\frac{7-5}{k-2}\times\frac{-3}{2}=-1$
	$\Rightarrow 2=\frac{2}{3}(k-2) \Rightarrow k=5$	A1	2	or Pythagoras $(k-2)^2=(k-6)^2+8$
	Total		**8**	
2(a)	$\frac{\sqrt{63}}{3}=\sqrt{7}$ or $\frac{3\sqrt{7}}{3}$	B1		or $\frac{\left(\sqrt{7}\sqrt{63}+14\times3\right)}{3\sqrt{7}}$
	$\frac{14}{\sqrt{7}}=2\sqrt{7}$ or $\frac{14\sqrt{7}}{7}$	B1		or $\frac{\sqrt{7}}{\sqrt{7}}(\quad)$ M1
	$\Rightarrow$ sum $=3\sqrt{7}$	B1	3	$\Rightarrow$ correct answer with all working correct A2
(b)	Multiply by $\frac{\sqrt{7}+2}{\sqrt{7}+2}$	M1		
	Denominator = 7 – 4 = 3	A1		
	Numerator $=\left(\sqrt{7}\right)^2+\sqrt{7}+2\sqrt{7}+2$	m1		multiplied out (allow one slip) $9+3\sqrt{7}$
	Answer $=\sqrt{7}+3$	A1	4	
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$(x+5)^2$	B1		$p=5$
	-6	B1	2	$q=-6$
(ii)	$x_{\text{vertex}}=-5$ (or their $-p$)	B1√		may differentiate but must have $x=-5$
	$y_{\text{vertex}}=-6$ (or their q)	B1√	2	and $y=-6$. Vertex $(-5,-6)$
(iii)	$x=-5$	B1	1	
(iv)	Translation (not shift, move etc)	E1		and NO other transformation stated
	through $\begin{bmatrix}-5\\-6\end{bmatrix}$ (or 5 left, 6 down)	M1 A1	3	either component correct M1, A1 independent of E mark
(b)	$x+11=x^2+10x+19$			quadratic with all terms on one side of equation
	$\Rightarrow x^2+9x+8=0$ or $y^2-13y+30=0$	M1		
	$(x+8)(x+1)=0$ or $(y-3)(y-10)=0$	m1		attempt at formula (1 slip) or to factorise
	$\left.\begin{matrix}x=-1\\y=10\end{matrix}\right\}$ or $\left.\begin{matrix}x=-8\\y=3\end{matrix}\right\}$	A1 A1	4	both x values correct both y values correct and linked
				SC $(-1,10)$ B2, $(-8,3)$ B2 no working
	Total		**12**	
4(a)(i)	$t^3-52t+96$	M1		one term correct
		A1		another term correct
		A1	3	all correct (no $+c$ etc)
(ii)	$3t^2-52$	M1		ft one term correct
		A1√	2	ft all "correct"
(b)	$\frac{\mathrm{d}y}{\mathrm{d}t}=8-104+96$	M1		substitute $t=2$ into their $\frac{\mathrm{d}y}{\mathrm{d}t}$
	$=0 \Rightarrow$ stationary value	A1		CSO; shown $=0$ + statement
	Substitute $t=2$ into $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ $(=-40)$	M1		any appropriate test, e.g. $y'(1)$ and $y'(3)$
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}<0 \Rightarrow$ max value	A1	4	all values (if stated) must be correct
(c)	Substitute $t=1$ into their $\frac{\mathrm{d}y}{\mathrm{d}t}$	M1		must be their $\frac{\mathrm{d}y}{\mathrm{d}t}$ NOT $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$
	Rate of change $=45$ $\left(\mathrm{cm\,s^{-1}}\right)$	A1√	2	ft their $y'(1)$
(d)	Substitute $t=3$ into their $\frac{\mathrm{d}y}{\mathrm{d}t}$	M1		interpreting their value of $\frac{\mathrm{d}y}{\mathrm{d}t}$
	$(27-156+96=-33<0)$			
	$\Rightarrow$ decreasing when $t=3$	E1√	2	allow increasing if their $\frac{\mathrm{d}y}{\mathrm{d}t}>0$
	Total		**13**	

Q	Solution	Marks	Total	Comments
5(a)(i)	Centre $(-3, 2)$	M1		± 3 or ± 2
		A1	2	correct
(ii)	Radius = 5	B1	1	accept $\sqrt{25}$ but not $\pm\sqrt{25}$
(b)(i)	$3^2+(-4)^2=9+16=25$ $\Rightarrow N$ lies on circle	B1	1	must have $9+16=25$ or a statement
(ii)		M1		must draw axes; ft their centre in correct quadrant
		A1	2	correct (reasonable freehand circle enclosing origin)
(iii)	Attempt at gradient of CN	M1		withhold if subsequently finds tangent
	grad $CN = -\frac{4}{3}$	A1		CSO
	$y=-\frac{4}{3}x-2$ (or equivalent)	A1√	3	ft their grad CN
(c)(i)	$P(2,6)$ Hence $PC^2=5^2+4^2$	M1		"their" PC^2
	$\Rightarrow PC = \sqrt{41}$	A1	2	
(ii)	Use of Pythagoras correctly	M1		
	$PT^2 = PC^2 - r^2 = 41-25$, where T is a point of contact of tangent	A1√		ft their PC^2 and r^2
	$\Rightarrow PT = 4$	A1	3	**Alternative** sketch with vertical tangent M1 showing that tangent touches circle at point (2, 2) A1 hence $PT=4$ A1
	Total		**14**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{f}(1)=1+4-5$	M1		must find $\mathrm{f}(1)$ NOT long division
	$\Rightarrow \mathrm{f}(1)=0 \Rightarrow (x-1)$ is factor	A1	2	shown $=0$ plus a statement
(ii)	Attempt at x^2+x+5	M1		long division leading to $x^2 \pm x + \ldots$ or equating coefficients
	$\mathrm{f}(x)=(x-1)(x^2+x+5)$	A1	2	$p=1,\ q=5$ by inspection scores B1, B1
(iii)	$(x=)\ 1$ is real root	B1		
	Consider b^2-4ac for their x^2+x+5	M1		not the cubic!
	$b^2-4ac=1^2-4\times 5=-19<0$			
	Hence no real roots (or only real root is 1)	A1	3	CSO; all values correct plus a statement
(b)(i)	$\int \ldots \,\mathrm{d}x = \frac{x^4}{4}+2x^2-5x\ (+c)$	M1 A1 A1	 3	one term correct unsimplified second term correct unsimplified all correct unsimplified
(ii)	$[4+8-10]-\left[\frac{1}{4}+2-5\right]$	M1		correct use of limits 1 and 2; $\mathrm{F}(2)-\mathrm{F}(1)$ attempted
	$=4\frac{3}{4}$	A1		
	Area of $\Delta = \frac{1}{2}\times 11 = 5\frac{1}{2}$	B1		correct unsimplified
	$\Rightarrow$ shaded area $=5\frac{1}{2}-4\frac{3}{4}$			combined integral of $7x-6-x^3$ scores M1 for limits correctly used then
	$=\frac{3}{4}$	A1	4	A3 correct answer with all working correct
	Total		**14**	
7(a)	$b^2-4ac=4-4(k-1)(2k-3)$	M1		(or seen in formula) condone one slip
	Real roots when $b^2-4ac \geqslant 0$	E1		must involve $\mathrm{f}(k) \geqslant 0$ (usually M1 must be earned)
	$4-4(2k^2-5k+3) \geqslant 0$			
	$\Rightarrow -2k^2+5k-3+1 \geqslant 0$			at least one step of working justifying $\leqslant 0$
	$\Rightarrow 2k^2-5k+2 \leqslant 0$	A1	3	AG
(b)(i)	$(2k-1)(k-2)$	B1	1	
(ii)	(Critical values) $\frac{1}{2}$ and 2	B1√		ft their factors or correct values seen on diagram, sketch or inequality or stated
		M1		use of sketch / sign diagram
	$\Rightarrow 0.5 \leqslant k \leqslant 2$	A1	3	M1A0 for $0.5<k<2$ or $k \geqslant 0.5,\ k \leqslant 2$
	Total		**7**	
	TOTAL		**75**	

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.

(a) Find the coordinates of the mid-point of BC. *(2 marks)*

(b) (i) Find the gradient of AB, in its simplest form. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $x + qy = r$, where q and r are integers. *(2 marks)*

(iii) Find an equation of the line passing through C which is parallel to AB. *(2 marks)*

(c) Prove that angle ABC is a right angle. *(3 marks)*

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

3 (a) Express $5\sqrt{8} + \frac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. *(3 marks)*

(b) Express $\frac{\sqrt{2} + 2}{3\sqrt{2} - 4}$ in the form $c\sqrt{2} + d$, where c and d are integers. *(4 marks)*

4 A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k$$ *(2 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

(c) A line has equation $y = 2x$.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$. *(2 marks)*

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P. *(3 marks)*

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. *(2 marks)*

5 (a) Factorise $9 - 8x - x^2$. *(2 marks)*

(b) Show that $25 - (x + 4)^2$ can be written as $9 - 8x - x^2$. *(1 mark)*

(c) A curve has equation $y = 9 - 8x - x^2$.

(i) Write down the equation of its line of symmetry. *(1 mark)*

(ii) Find the coordinates of its vertex. *(2 marks)*

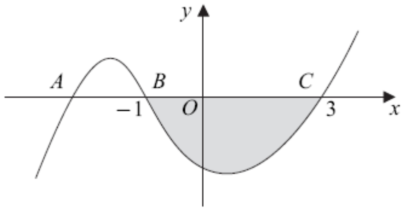
(iii) Sketch the curve, indicating the values of the intercepts on the x-axis and the y-axis. *(3 marks)*

6 (a) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x + 1)$, where k is a constant.

(a) Show that the x-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$ *(3 marks)*

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - – Jan 2008 – Answers

Question 1:	Exam report
$A(-2,3)\ \ B(4,1)\ \ C(2,-5)$ $a)\text{ Mid-point of BC}\left(\frac{4+2}{2},\frac{1-5}{2}\right)=(3,-2)$ $b)\,i)\,m_{AB}=\frac{1-3}{4+2}=-\frac{2}{6}=-\frac{1}{3}$ $ii)\,Eq:\ y-3=-\frac{1}{3}(x+2)$ $3y-9=-x-2$ $x+3y=7$ $iii)$ this line has the same gradient $-\frac{1}{3}$ $Eq:\ y+5=-\frac{1}{3}(x-2)$ $3y+15=-x+2$ $x+3y=-13$ $c)$ Let's work out the gradient of the line BC: $m_{BC}=\frac{-5-1}{2-4}=\frac{-6}{-2}=3$ $m_{AB}\times m_{BC}=-\frac{1}{3}\times 3=-1$ *Conclusion*: the line AB and BC are perpendicular the triangle ABC is a right-angled triangle.	In part (a), apart from a few sign errors, it was pleasing to see that most candidates were able to find the correct mid-point. However, those who insisted on subtracting the coordinates before dividing by 2 would do well to learn the formula in the first bullet point above. Quite a few candidates found the mid-point of *AB* instead of *BC*, and this was generously treated as a misread. In part (b)(i), many ignored the request to simplify the gradient, but most were successful in writing the gradient of *AB* as -1/3. In part (b)(ii), almost all candidates managed to write down a correct equation for the line *AB*, but careless arithmetic prevented many from obtaining the required form of *x* + 3*y* = 7 . Some were content to give a final answer that was not in the required form, thus losing a mark. In part (b)(iii), some candidates immediately used *m*1 ×*m*2 = −1 to find the gradient of the parallel line and scored no marks. Many who used the formula *y* = *mx* + *c* for the equation of the straight line through *C* parallel to *AB* made arithmetic slips and did not obtain a correct final equation. In part (c), the most common approach, and the one expected, was to use gradients in order to prove that angle *ABC* was a right angle. Some simply assumed the result, stating that since the gradient of *AB* was -1/3 then *BC* had gradient 3. It was necessary to show, by considering the differences of the coordinates that *BC* had gradient 3. Far too many simply found the two gradients and wrote “therefore the lines *BC* and *AB* are perpendicular”. Since this was a proof, it was expected that the product of the two gradients would be shown to equal −1 before a statement was made about angle *ABC* being a right angle. Some were successful in proving the result using Pythagoras’ Theorem, but many attempts were incomplete with several candidates writing $\sqrt{40}+\sqrt{40}=\sqrt{80}$ or other inaccurate statements. Others used the cosine rule, and one or two used the scalar product of two vectors in order to prove the result. A surprising number confused “isosceles” with “right-angled” and, having found two equal sides, stated that the result was proved.

Question 2:	Exam report
$y=x^4-32x+5$ $a)\,\frac{dy}{dx}=4x^3-32$ $b)\,M$ is a stationary point. Let's solve $\frac{dy}{dx}=0$ $4x^3-32=0$ $x^3=8\qquad x=2$ $c)\,i)\,\frac{d^2y}{dx^2}=12x^2$ $ii)\,\frac{d^2y}{dx^2}(x=2)=12\times 2^2=48>0$ M is a minimum $d)\,\frac{dy}{dx}(x=0)=-32<0$ The curve is decreasing.	In part (a), most candidates were able to find the correct expression for $\frac{dy}{dx}$, although there were some who left + 5 in their answer or added +C. In part (b), It had been expected that candidates would solve the equation $\frac{dy}{dx}=0$ and obtain the equation $x^3 = 8$ and hence deduce that x = 2 . It seemed, however, that many were unable to formulate an appropriate equation, but merely spotted the correct answer: x = 2. This was not penalised on this occasion, provided that the candidate stated clearly that the x-coordinate of M was equal to 2. In part (c)(i), the expression for $\frac{d^2y}{dx^2}$ was usually correct. In part (c)(ii), although the method was left open, most candidates found the value of the second derivative when x = 2 and correctly concluded that M was a minimum point. In part (c)(iii), some candidates were not aware of the need to find the value of

	$\frac{dy}{dx}$ when x = 0 in order to ascertain whether the curve was increasing or decreasing at that point.

Question 3:	**Exam report**
$a)\,5\sqrt{8}+\frac{6}{\sqrt{2}}=5\sqrt{4\times2}+\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=5\times2\sqrt{2}+\frac{6\sqrt{2}}{2}$ $=10\sqrt{2}+3\sqrt{2}=13\sqrt{2}$ $b)\,\frac{\sqrt{2}+2}{3\sqrt{2}-4}=\frac{\sqrt{2}+2}{3\sqrt{2}-4}\times\frac{3\sqrt{2}+4}{3\sqrt{2}+4}=\frac{6+4\sqrt{2}+6\sqrt{2}+8}{9\times2-16}$ $=\frac{14+10\sqrt{2}}{2}=7+5\sqrt{2}$	Candidates did not always approach part (a) of the question with confidence. Several wrote $5\sqrt{8}=5\sqrt{4\times2}=7\sqrt{2}\ or\ 5+2\sqrt{2}$, others tried to rationalise $\frac{6}{\sqrt{2}}$ by simply multiplying the denominator by $\sqrt{2}$. Consequently, it was quite common to see only one of the two terms expressed correctly in the form $k\sqrt{2}$. It was quite strange, though, to see many obtaining an answer of $13\sqrt{2}$ from completely wrong working; clearly this was not given any credit. Some combined the two terms with a common denominator but often with an incorrect numerator. In part (b), it was not uncommon to see the denominator and numerator multiplied by different surds and the usual errors occurred as candidates tried to multiply out brackets. A few multiplied top and bottom by the conjugate of the numerator. Nevertheless, this part of the question seemed to be answered much better than similar questions in previous years, despite the fairly difficult denominator.

Question 4:	**Exam report**
$x^2+y^2-10y+20=0$ $a)\,x^2+(y-5)^2-25+20=0$ $x^2+(y-5)^2=5$ $b)\,i)\,C(0,5)$ $ii)\,r=\sqrt{5}$ $c)\,y=2x$ $i)\,\text{solve simultaneously}\begin{cases}y=2x\\x^2+y^2-10y+20=0\end{cases}$ Substitute y by $2x$: $x^2+(2x)^2-10\times(2x)+20=0$ $5x^2-20x+20=0$ $x^2-4x+4=0$ $ii)\,x^2-4x+4=(x-2)^2=0$ $x=2$ is a repeated root The line $y=2x$ is tangent to the circle. $d)\,Q(-1,4)$ $x^2+y^2-10y+20=(-1)^2+4^2-10\times4+20$ $=1+16-40+20=-3<5$ Q is inside the circle	In part (a), it was only necessary to complete the square for the y-terms. As a result, there were probably fewer errors this year expressing the left-hand side of the equation of the circle as $(y-5)^2$. However, the right hand side was often written as $\sqrt{5}$, –5 or – 45 instead of 5. In part (b), quite a number who had the correct circle equation in part (a) wrote the coordinates of the centre as (5, 0) or (0, –5). Generous follow through marks were awarded for the radius provided the right-hand side of the equation had a positive value. The wording in the question reassured most, though, that the radius was $\sqrt{5}$. In part (c)(i), those with poor algebraic skills, often writing $2x^2$ instead of $(2x)^2$, struggled to establish the given quadratic equation. Also, quite a few made errors in their working but miraculously wrote down the given equation on their final line. A surprising number derived an equation in y. Quite a few simply solved the given quadratic equation in this part and thus failed to show an understanding of what was required. In part (c)(ii), it was necessary to state that the equation had a repeated root of x = 2, or to use the zero value of the discriminant to show that the equation had equal roots, and hence to conclude that the line was a tangent to the circle. In part (d), far too many simply substituted the coordinates of the point Q into the equation of the circle obtaining a nonsensical statement such as "–3 = 0 so the point lies inside the circle". It was necessary to see that the distance CQ was being calculated and then concluded that this distance was less than the radius of the circle, and hence the point Q must lie inside the circle.

Question 5:	**Exam report**
$a)\ 9-8x-x^2=(9+x)(1-x)$ $b)\ 25-(x+4)^2=25-(x^2+8x+16)$ $=-x^2-8x+9$ $c)\ y=25-(x+4)^2$ $i)$ Line of symmetry : $x=-4$ $ii)$ *Vertex* $(-4,25)$ $iii)$ The graph crosses the x-axis at $(-9,0)$ and $(1,0)$ the y-axis at $(0,9)$	Candidates did not seem confident working with a quadratic expression where the coefficient of x^2 was negative. Throughout this question, candidates chose instead to work with the expression x^2+8x-9, or the equation $x^2+8x-9=0$, and lost quite a lot of marks. In part (a), a large number of candidates could not factorise the given quadratic correctly, a few clearly not even recognising what was required. In part (b), those who kept brackets in their working were usually successful in proving the identity. Some able candidates started with $9-8x-x^2$ and showed their skill in completing the square. In part (c), quite a large number of candidates seemed unfamiliar with the terms “line of symmetry” and “vertex” and certainly failed to see the link with part (b) of the question. Some stated that the coordinates of the maximum point were (–4, 25) and then wrote the coordinates of the vertex as something entirely different. The sketches were somewhat varied: some found the wrong x-intercepts and drew a curve through these points; those who had completely changed the question into $y=x^2+8x-9$ had a U-shaped graph. Those who drew a graph with the vertex in the correct position and with the correct shape usually had the y-intercept marked correctly as 9. However some drew their curve with a maximum point on the y-axis.

Question 6:	**Exam report**
$a)\ p(x)=x^3-7x-6$ $i)$ *Work out* $p(-1)$: $(-1)^3-7\times(-1)-6=-1+7-6=0$ -1 is a root of p, so $(x+1)$ is a factor of p $ii)\ p(x)=(x+1)(x^2-x-6)=(x+1)(x-3)(x+2)$ $b)\ i)\ A(-2,0)\quad (root\ of\ \ p)$ $ii)\ \int_{-1}^{3}(x^3-7x-6)dx=\left[\frac{1}{4}x^4-\frac{7}{2}x^2-6x\right]_{-1}^{3}$ $=\left(\frac{81}{4}-\frac{63}{2}-18\right)-\left(\frac{1}{4}-\frac{7}{2}+6\right)$ $=(20\tfrac{1}{4}-31\tfrac{1}{2}-18)-\left(\frac{1}{4}-3\tfrac{1}{2}+6\right)$ $=20-28-24=-32$ $iii)$ *Area shaded* $=32$ $iv)$ The gradient of the curve at B is $\frac{dy}{dx}(x=-1)$ $\frac{dy}{dx}=3x^2-7\ \ and\ \ for\ \ x=-1,\ \frac{dy}{dx}=3-7=-4$ $v)$ The normal has gradient $-\frac{1}{m_B}=-\frac{1}{-4}=\frac{1}{4}$ The equation of the normal at B: $y-0=\frac{1}{4}(x+1)$ $y=\frac{1}{4}(x+1)$	In part (a)(i), a few candidates ignored the request to use the factor theorem and scored no marks for using long division. It was necessary to make a statement that “ x + 1 is a factor”, after showing that f(–1) = 0, in order to score full marks. Part (a)(ii) was not answered as well as similar questions in previous years. Perhaps the sketch lured some into trying to write down three factors without any further working, rather than using the intermediate step of showing that $p(x)=(x+1)(x^2-x-6)$ before writing p(x) as a product of three factors. Many who tried long division were flummoxed by there being no x^2 term. In part (b)(i), those who had the correct linear factors in part (a)(ii) usually wrote down correctly that A had coordinates (–2, 0), although some carelessly wrote the point as (0,–2). Many candidates simply found an indefinite integral in part (b)(ii) and then a definite integral in part (b)(iii). The two parts were generously treated holistically when candidates did this. The fractions once again caused problems to most candidates who are so used to having a calculator to do this work for them. It was very rare to see the correct answer of – 32 for the definite integral. In part (b)(iii), many lost out on an easy mark because they rolled their two sections into one: those who wrote “integral = –32 =32” gained full credit for part (b)(ii) but did not score the mark in part (b)(iii). It was necessary to give a positive value for the area of the region and to make this explicit. In anticipation of a lot of wrong answers in part (b)(ii), a follow through mark was awarded in part (b)(iii): for example, if a candidate’s answer in part (b)(ii) was –20 and they concluded that the area was 20 in part (b)(iii), they scored the mark. In part (b)(iv), most candidates differentiated correctly, but quite a few thought that $3(-1)^2-7$ was equal to –10 and thus obtained the wrong gradient of the curve. In part (b)(v), a large number of candidates found the correct equation of the normal but some still confused tangents and normals and consequently thought that the gradient of the normal was equal to –4. It was quite common for weaker candidates to either negate their gradient or take the reciprocal but to fail to do both.

Question 7:	Exam report
$Curve\ C: y = x^2 + 7$ $Line\ L: y = k(3x+1)$ $a)$ By identification, $(y =)$ $x^2 + 7 = k(3x+1)$ $x^2 - 3kx + 7 - k = 0$ $b)$ There are two points of intersection which means that discriminant >0 $(-3k)^2 - 4 \times 1 \times (7-k) > 0$ $9k^2 + 4k - 28 > 0$ $c) 9k^2 + 4k - 28 = (9k-14)(k+2) > 0$ $critical\ values: \frac{14}{9}\ and\ -2$ $solutions: \ k < -2\ or\ k > \frac{14}{9}$	In part (a), some weaker candidates did not realise how to derive the given equation, and others made algebraic slips when proving the printed result, or failed to write “= 0”. In part (b), the condition for two distinct points of intersection required candidates to use the condition that $b^2 - 4ac > 0$ at any early stage of their argument. Those who simply wrote “> 0” on their final line of working, without any previous reference to the discriminant being positive, failed to convince the examiners that they deserved full marks. In part (c), quite a number were unable to factorise the quadratic correctly and many resorted to using the quadratic equation formula to find the critical values. Where this was done correctly but left in surd form, it was given due credit except for the final mark. Very able candidates can write down the answer to the inequality once they have factorised the quadratic but far too many guessed at answers and an approach using a sign diagram or sketch is recommended. Candidates also need to realise that the final form of the answer cannot be written as $\frac{14}{9} < k < -2$.

GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	59	51	43	36	29

MY SCORE FOR THIS PAPER IS ….. / 75

MY GRADE : …….

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

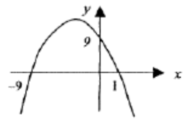
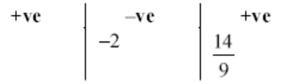
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	Mid-point of $BC = (3, -2)$	B1		Either coordinate correct
		B1	2	Both cords correct. Accept $x = 3, y = -2$
(b)(i)	$\frac{\Delta y}{\Delta x} = \frac{3-1}{-2-4}$	M1		$\pm\frac{2}{6}$ OE implies M1
	$= -\frac{1}{3}$	A1	2	
(ii)	$y - 3 =$ "their grad"$(x + 2)$ or $y - 1 =$ "their grad"$(x - 4)$	M1		Or $y = mx + c$ and correct attempt to find c
	Hence $x + 3y = 7$	A1	2	
(iii)	$y + 5 =$ "their grad AB"$(x - 2)$	M1		Or "their $x + qy = c$" and attempt to find c
	$y+5=-\frac{1}{3}(x-2)$ or $x + 3y + 13 = 0$	A1	2	OE
(c)	Grad $BC = 3$ (from $\frac{\Delta y}{\Delta x}=\frac{1+5}{4-2}$ OE)	B1		Or 2 lengths correct: $AB=\sqrt{40}; BC=\sqrt{40}; AC=\sqrt{80}$
	$m_1m_2 = -1$ stated **or** grad $BC = 3$ and grad $AB = -\frac{1}{3}$ **or** grad BC × grad AB $(= 3\times -\frac{1}{3})$	M1		Or attempt at Pythagoras or Cosine Rule
	Product of gradients $= -1$ Hence AB and BC are perpendicular	A1 CSO	3	$AC^2 = AB^2 + BC^2 \Rightarrow \angle ABC = 90°$ Completing proof and statement
	Total		**11**	
2(a)	$\frac{dy}{dx} = 4x^3 - 32$	M1 A1 A1	3	Reduce one power by 1 One term correct All correct (no $+ c$ etc)
(b)	Stationary point $\Rightarrow \frac{dy}{dx} = 0$	M1		
	$\Rightarrow x^3 = 8$	A1√		$x^n = k$ following from their $\frac{dy}{dx}$
	$\Rightarrow x = 2$	A1	3	CSO
(c)(i)	$\frac{d^2y}{dx^2} = 12x^2$	B1√	1	FT their $\frac{dy}{dx}$
(ii)	When $x = 2$, $\frac{d^2y}{dx^2}$ considered	M1		Or complete test with $2 \pm \varepsilon$ using $\frac{dy}{dx}$
	$\Rightarrow$ minimum point	E1√	2	
(d)	Putting $x = 0$ into their $\frac{dy}{dx}$ $(= -32)$	M1		
	$\frac{dy}{dx} < 0 \Rightarrow$ decreasing	A1√	2	Allow "increasing" if their $\frac{dy}{dx} > 0$
	Total		**11**	

Q	Solution	Marks	Total	Comments
3(a)	$5\sqrt{8}=10\sqrt{2}$	B1		Or $\dfrac{5\sqrt{16}+6}{\sqrt{2}}$ gets B1
	$\dfrac{6}{\sqrt{2}}=\dfrac{6\sqrt{2}}{2}$ $(=3\sqrt{2})$	M1		then M1 for rationalising; and A1 answer
	Answer = $13\sqrt{2}$	A1	3	$n=13$
(b)	$\dfrac{\sqrt{2}+2}{3\sqrt{2}-4}\times\dfrac{3\sqrt{2}+4}{3\sqrt{2}+4}$	M1		Multiplying top & bottom by $\pm\left(3\sqrt{2}+4\right)$
	Numerator = $6+6\sqrt{2}+4\sqrt{2}+8$	m1		Multiplying out (condone one slip)
	Denominator = 18 – 16 (= 2)	B1		
	Final answer = $5\sqrt{2}+7$	A1	4	
	Total		**7**	
4(a)	$x^2+(y-5)^2$	B1		$b=5$
	RHS = 5	B1	2	$k=5$
(b)(i)	Centre (0, 5)	B1√	1	FT their b from part (a)
(ii)	Radius = $\sqrt{5}$	B1√	1	FT their k from part (a); RHS must be > 0
(c)(i)	$x^2+4x^2-20x+20=0$	M1		May substitute into original or “their (a)”
	$\Rightarrow x^2-4x+4=0$	A1	2	CSO; AG
(ii)	$(x-2)^2=0$ or $x=2$	M1		
	Repeated root implies tangent	E1		Or b^2-4ac shown = 0 plus statement
	Point of contact is $P(2, 4)$	A1	3	
(d)	$\left(CQ^2=\right)1^2+1^2$	M1		FT their C
	$\sqrt{2}<\sqrt{5}\Rightarrow Q$ lies inside circle	A1 CSO	2	CQ or CQ^2 OE must appear for A1
	Total		**11**	
5(a)	$(9+x)(1-x)$	M1		$\pm(9\pm x)(1\pm x)$
		A1	2	Correct factors
(b)	$25-\left(x^2+8x+16\right)=9-8x-x^2$	B1	1	AG
(c)(i)	$x=-4$ is line of symmetry	B1	1	
(ii)	Vertex is (–4, 25)	B1,B1	2	
(iii)		M1		General ⋂ shape
		B1		–9 and 1 marked on x-axis or stated
		A1	3	9 marked on y-axis and maximum to the left of y-axis Must continue below x-axis at both ends
	Total		**9**	
6(a)(i)	p(–1) = – 1 + 7 – 6	M1		Finding p(–1)
	= 0 therefore x + 1 is a factor	A1	2	Shown to = 0 **plus statement**
(ii)	$\mathrm{p}(x)=(x+1)(x^2-x-6)$	M1		Long division/inspection (2 terms correct)
		A1		Quadratic factor correct
	$\mathrm{p}(x)=(x+1)(x+2)(x-3)$	A1	3	May earn M1,A1 for correct second factor then A1 for $(x+1)(x+2)(x-3)$
(b)(i)	$A(-2,0)$	B1	1	Condone $x=-2$
(ii)	$\dfrac{x^4}{4}-\dfrac{7x^2}{2}-6x\quad(+c)$	M1		One term correct
		A1		Another term correct
	(may have + c or not)	A1		All correct unsimplified
	$\left[\dfrac{81}{4}-\dfrac{63}{2}-18\right]-\left[\dfrac{1}{4}-\dfrac{7}{2}+6\right]$	m1		F(3) – F(–1) attempted in correct order
	$=-32$	A1	5	CSO; OE
(iii)	Area of shaded region = 32	B1√	1	FT their (b)(ii) but positive value needed
(iv)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x^2-7$	M1		One term correct
		A1		All correct (no + c etc)
	When $x=-1$, gradient = – 4	A1	3	CSO
(v)	Gradient of normal = $\dfrac{1}{4}$	B1√		
	$y=$ “their gradient” $(x\pm1)$	M1		Must be finding **normal**, not tangent
	$y=\dfrac{1}{4}(x+1)$	A1	3	CSO; any correct form eg $4y-x=1$
	Total		**18**	
7(a)	$x^2+7=k(3x+1)\Rightarrow x^2-3kx+7-k=0$	B1	1	AG
(b)	$b^2-4ac=(-3k)^2-4(7-k)$	M1		Clear attempt at b^2-4ac Condone slip in one term of expression
	(2 distinct roots when) $b^2-4ac>0$	B1		Must involve k
	$9k^2+4k-28>0$	A1	3	CSO; AG
(c)	$(9k-14)(k+2)$	M1		Factors or formula correct unsimplified
	Critical points –2 and $\dfrac{14}{9}$	A1		
	Sketch ∪ or sign diagram correct	M1		
	$k<-2,\ k>\dfrac{14}{9}$	A1	4	
	Total		**8**	
	TOTAL		**75**	

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS MPC1
Unit Pure Core 1

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x+3)(x-1)$$

(a) Sketch on the same axes the line L and the curve C, showing the values of the intercepts on the x-axis and the y-axis. *(5 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

(a) xy ; *(1 mark)*

(b) $\dfrac{y}{x}$; *(2 marks)*

(c) $(x+y)^2$. *(3 marks)*

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\dfrac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

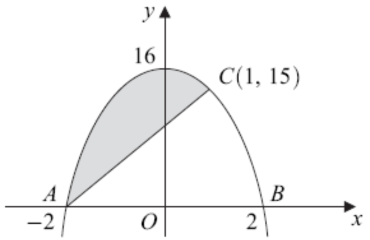
(iii) Find the maximum value of V. *(1 mark)*

4 (a) Express $x^2 - 3x + 4$ in the form $(x - p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)*

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\, dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

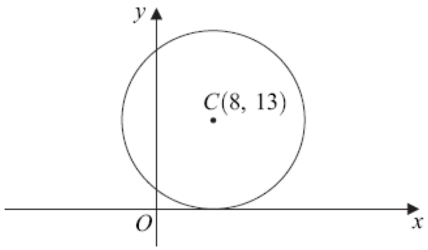
(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

7 The circle S has centre $C(8, 13)$ and touches the x-axis, as shown in the diagram.



(a) Write down an equation for S, giving your answer in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(2 marks)*

(b) The point P with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through P and C. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle S at the point P, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(4 marks)*

(iii) The point Q also lies on the circle S, and the length of PQ is 10. Calculate the shortest distance from C to the chord PQ. *(3 marks)*

8 The quadratic equation $(k + 1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 – June 2008 – Answers

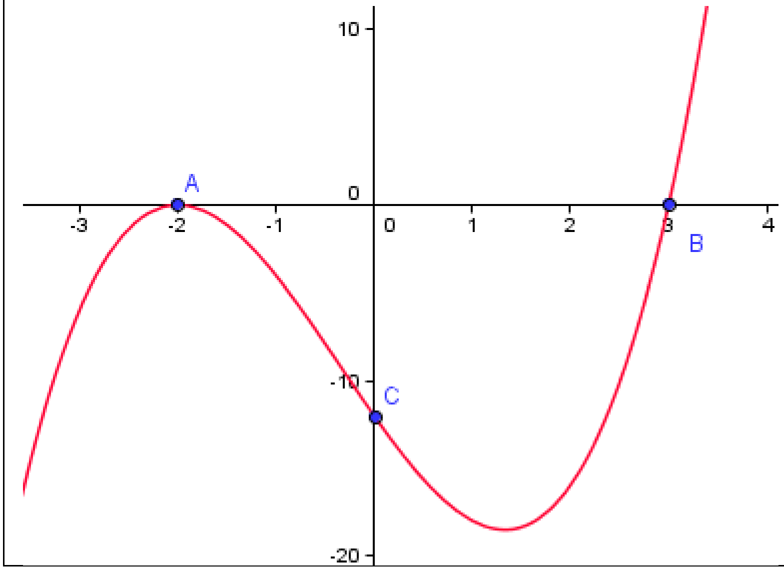
Question 1:	**Exam report**
$Line\ L: y=3x-1$ $Curve\ C: y=(x+3)(x-1)=x^2+2x-3$ $a)$ The line crosses the y-axis at $(0,-1)$ and the x-axis at $(\frac{1}{3},0)$ The curve crosses the y-axis at $(0,-3)$ and the x-axis at $(-3,0)$ and $(1,0)$ $b)$ Solve simultaneously $\begin{cases} y=3x-1 \\ y=x^2+2x-3 \end{cases}$ $by\ identification$ $(y=)x^2+2x-3=3x-1$ $x^2-x-2=0$ $c)\quad x^2-x-2=(x-2)(x+1)=0$ $x=2\ \ or\ \ x=-1$ $and\ y=3x-1 \qquad y=5\ or\ \ y=-4$ The line and the curve cross at $(2,5)$ and $(-1,-4)$	

Question 2:	**Exam report**
$x=\sqrt{3}\ \ and\ \ y=\sqrt{12}$ $a)\ xy=\sqrt{3}\times\sqrt{12}=\sqrt{36}=6$ $b)\ \frac{y}{x}=\frac{\sqrt{12}}{\sqrt{3}}=\frac{2\sqrt{3}}{\sqrt{3}}=2$ $c)\ (x+y)^2=\left(\sqrt{3}+\sqrt{12}\right)^2=3+12+2\sqrt{36}=27$	

Question 3:	**Exam report**
$3x+y=9 \quad x\geq 0, y\geq 0$ $V=xy^2$ $a)\ y=9-3x \quad so\ \ V=xy^2=x(9-3x)^2$ $V=x\left(81+9x^2-54x\right)$ $V=9x^3-54x^2+81x$ $b)i)\ \frac{dV}{dx}=9\times 3x^2-54\times 2x+81=27x^2-108x+81$ $\frac{dV}{dx}=27\left(x^2-4x+3\right)$ $ii)\ \frac{dV}{dx}=0\ \ when\ \ x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=3\ or\ \ x=1$ $c)\ \frac{d^2V}{dx^2}=27(2x-4)=54x-108$ $d)i)\ \frac{d^2V}{dx^2}(x=3)=27(2\times 3-4)=27\times 2=54>0$ $\frac{d^2V}{dx^2}(x=1)=27(2\times 1-4)=27\times -2=-54<0$ $ii)$ There is a maximum for $x=1$ $iii)\ For\ \ x=1,\ V=81-54+9=36$	

Question 4:	**Exam report**
$a)\ x^2-3x+4=\left(x-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+4=\left(x-\frac{3}{2}\right)^2+\frac{7}{4}$ $b)$ For all x, $\left(x-\frac{3}{2}\right)^2\geq 0\ \ so\ \ \left(x-\frac{3}{2}\right)^2+\frac{7}{4}\geq\frac{7}{4}$ The minimum value is $\frac{7}{4}$ $c)\ x\xrightarrow{\substack{translation\\ 1.5 units\, in\, x-dir}}x-\frac{3}{2}\xrightarrow{f}\left(x-\frac{3}{2}\right)^2\xrightarrow{\substack{translation\\ \frac{7}{4} units\, in\, y-dir}}\left(x-\frac{3}{2}\right)^2+\frac{7}{4}$ $y=x^2$ *is mapped* into $y=x^2-3x+4$ by the translation vector $\begin{bmatrix}\frac{3}{2}\\ \frac{7}{4}\end{bmatrix}$	

Question 5:	**Exam report**
$y=16-x^4$ $A(-2,0),\ B(2,0)\ and\ C(1,15)$ $a)\ Gradient\ of\ \ AC=m_{AC}=\frac{15-0}{1+2}=5$ $Equation\ of\ \ AC: y-0=5(x+2)$ $y=5x+10$ $b)\, i)\int_{-2}^{1}\left(16-x^4\right)dx=\left[16x-\frac{1}{5}x^5\right]_{-2}^{1}=\left(16-\frac{1}{5}\right)-\left(-32+\frac{32}{5}\right)$ $=48-\frac{33}{5}=48-6\tfrac{3}{5}=41\tfrac{2}{5}$ $ii)$ Call the point $H(1,0)$ the area of the shaded region is $(area\ beneath\ the\ curve)-(area\ of\ \ triangle\ AHC)$ $41\tfrac{2}{5}-\frac{1}{2}\times3\times15=41\tfrac{2}{5}-22\tfrac{1}{2}=18\tfrac{9}{10}$	

Question 6:	**Exam report**
$p(x) = x^3 + x^2 - 8x - 12$ $a)$ The remainder of the division by $(x-1)$ *is* $p(1)$ $p(1) = 1+1-8-12 = -18$ $b)\,i)\; p(-2) = (-2)^3 + (-2)^2 - 8\times(-2) - 12$ $= -8+4+16-12 = 0$ -2 is a root of p, so $(x+2)$ is a factor of p. $ii)\; x^3 + x^2 - 8x - 12 = (x+2)(x^2 - x - 6)$ $= (x+2)(x-3)(x+2)$ $= (x+2)^2(x-3)$ $c)\,i)$ By substituting x by 0, we have $p(0) = -12$ The curve passes through the point $(0,-12)$ $ii)$ The curve crosses the $x-axis$ at $(3,0)$ and it is tangent to it at $(-2,0)$.  	

Question 7:

Exam report

$Circle\ S\ has\ centre\ C(8,13)$ and touches the x-axis

The radius is 13 (y-coordinate of C)

$a)$ Equation of S: $(x-8)^2+(y-13)^2=13^2$

$b)\ P(3,1)$ lies on the circle

$$i)\ gradient = m_{PC} = \frac{13-1}{8-3} = \frac{12}{5}$$

$ii)$ The tangent at P is perpendicular to the radius PC

$$\text{the gradient of the tangent is } -\frac{1}{m_{PC}} = -\frac{5}{12}$$

$$\text{The equation of the tangent is : } y-1=-\frac{5}{12}(x-3)$$

$$12y-12=-5x+15$$

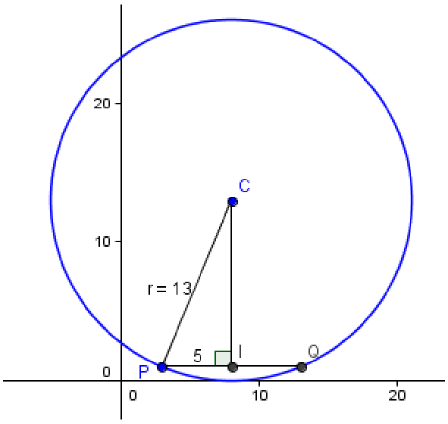
$$5x+12y=27$$

$iii)$ Call I the midpoint of PQ. The triangle PIC is a right-angled triangle and the shortest distance from C to the chord PQ is the distance CI.

$$\text{Using pythagoras' theorem: } CI^2 = CP^2 - PI^2$$

$$=13^2-5^2=169-25=144$$

$$CI=\sqrt{144}=12$$



Question 8:

Exam report

$(k+1)x^2+4kx+9=0\ has\ real\ roots$

which means that the discriminant ≥ 0.

$$a)\ (4k)^2-4\times(k+1)\times 9\geq 0$$

$$16k^2-36k-36\geq 0 \qquad (\div 4)$$

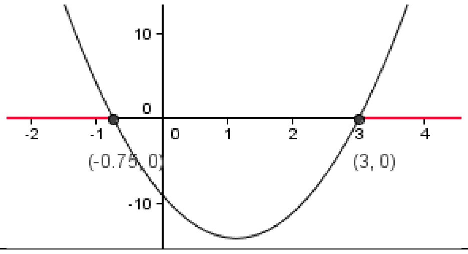
$$4k^2-9k-9\geq 0$$

$$b)\ 4k^2-9k-9\geq 0$$

$$(4k+3)(k-3)\geq 0$$

$$critical\ values\ -\frac{3}{4}\ and\ 3$$

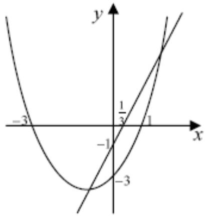
$$(4k+3)(k-3)\geq 0\quad for\ \ k\leq -\frac{3}{4}\ \ or\ \ k\geq 3$$



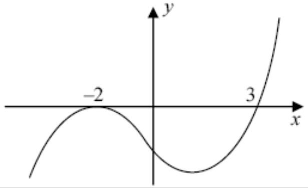
MY SCORE FOR THIS PAPER IS ..... / 75

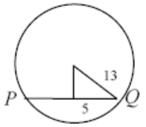
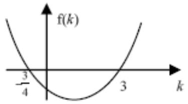
MY GRADE : ....

GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	59	51	43	35	28

Q	Solution	Marks	Total	Comments
1(a)	L: straight line with positive gradient and negative intercept on y-axis	B1		Line must cross both axes but need not reach the curve
	cutting at $\left(\frac{1}{3},0\right)$ and $(0,-1)$ (intercepts **stated** or marked on sketch)	B1		Condone 0.33 or better for $\frac{1}{3}$
	C: **attempt at parabola** $\cup$ or $\cap$ through $(-3,0)$ and $(1,0)$ or values –3 and 1 stated as intercepts on x-axis	B1		
	$\cup$ shaped graph – vertex below x-axis and cutting x-axis twice	M1		
	through $(0,-3)$ and minimum point to left of y-axis	A1	5	(y-intercept or coordinates marked)
(b)	$(x+3)(x-1)=3x-1$	M1		
	$x^2+3x-x-3-3x+1=0$ $\Rightarrow x^2-x-2=0$	A1	2	AG; must have "= 0" and no errors
(c)	$(x-2)(x+1)=0$	M1		$(x\pm1)(x\pm2)$ or use of formula (one slip)
	$\Rightarrow x=2,\ -1$	A1		correct values imply M1A1
	Substitute one value of x to find y	m1		
	Points of intersection (2, 5) and (–1,–4)	A1	4	May say $x=2, y=5$ etc SC: (2, 5) $\Rightarrow$ B2 (–1,–4) $\Rightarrow$ B2 without working
	Total		**11**	
2(a)	$xy=6$	B1	1	B0 for $\sqrt{36}$ or ±6
(b)	$\frac{y}{x}=\frac{2\sqrt{3}}{\sqrt{3}}$ or $\sqrt{\frac{12}{3}}$ or $\sqrt{\frac{4}{1}}$ or $\frac{\sqrt{12}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$	M1		Allow M1 for ±2
	$=2$	A1	2	
(c)	$x^2+2xy+y^2$ or $(\sqrt{3}+2\sqrt{3})^2$ correct	M1		or $\left(\sqrt{3}+\sqrt{12}\right)\left(\sqrt{3}+\sqrt{12}\right)$ expanded as 4 terms – no more than one slip
	Correct with 2 of $x^2, y^2, 2xy$ simplified	A1		Correct but unsimplified – one more step
	$3+2\sqrt{36}+12$ or $3^2\times3$ or $\left(3\sqrt{3}\right)^2$ $=27$	A1	3	
	Total		**6**	

Q	Solution	Marks	Total	Comments
3(a)	$V=x(9-3x)^2$	M1		Attempt at V in terms of x (condone slip when rearranging formula for $y = 9 - 3x$) or $(9-3x)^2=81-54x+9x^2$
	$V=x(81-54x+9x^2)$ $=81x-54x^2+9x^3$	A1	2	AG; no errors in algebra
(b)(i)	$\frac{dV}{dx}=81-108x+27x^2$	M1 A1 A1		One term correct Another correct All correct (no + c etc)
	$=27(x^2-4x+3)$	A1	4	CSO; all algebra and differentiation correct
(ii)	$(x-3)(x-1)$ or $(27x-81)(x-1)$ etc	M1		"Correct" factors or correct use of formula
	$\Rightarrow x=1, 3$	A1	2	SC: B1,B1 for $x = 1$, $x = 3$ found by inspection (provided no other values)
(c)	$\frac{d^2V}{dx^2}=-108+54x$ (condone one slip)	M1		ft their $\frac{dV}{dx}$ (may have cancelled 27 etc)
		A1	2	CSO; all differentiation correct
(d)(i)	$x=3\Rightarrow\frac{d^2V}{dx^2}=54;\quad x=1\Rightarrow\frac{d^2V}{dx^2}=-54$	B1√	1	ft their $\frac{d^2V}{dx^2}$ and their **two** x-values
(ii)	$(x=)$ 1 (gives maximum value)	E1	1	Provided their $\frac{d^2V}{dx^2}<0$
(iii)	$V_{\max}=36$	B1	1	CAO
	Total		**13**	
4(a)	$\left(x-\frac{3}{2}\right)^2$	B1		Must have $(\ )^2$ $p=1.5$
	$+\frac{7}{4}$	B1	2	$q=1.75$
(b)	Minimum value is $\frac{7}{4}$	B1√	1	ft their q or correct value
(c)	**Translation** (and **no other** transformation stated)	E1		(**not** shift, move, transformation etc)
	through $\begin{bmatrix}\frac{3}{2}\\ \frac{7}{4}\end{bmatrix}$ (or equivalent in words)	M1		M1 for one component correct or ft their p or q values
		A1	3	CSO; condone 1.5 right and 1.75 up etc
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)	Grad $AC = \frac{15}{3} = 5$	B1		OE
	Equation of AC: $y = m(x+2)$ or $(y-15) = m(x-1)$	M1		Or use of $y = mx + c$ with (–2, 0) or (1, 15) correctly substituted for x and y
	$y = 5x + 10$	A1	3	OE eg $y - 15 = 5(x-1)$, $y = 5(x+2)$
(b)(i)	$\left[16x - \frac{x^5}{5}\right]$	M1 A1 A1		Raise one power by 1 One term correct All correct
	$\left(16 - \frac{1}{5}\right) - \left(-32 + \frac{32}{5}\right)$	m1		$F(1) - F(-2)$ attempted
	$= 41\frac{2}{5}$ (or 41.4, $\frac{207}{5}$ etc)	A1	5	CSO; withhold if $+ c$ added
(ii)	Area $\Delta = \frac{1}{2} \times 3 \times 15$ or $22\frac{1}{2}$ or 22.5	B1		Or $\int_{-2}^{1}(5x+10)\,dx = 22.5$
	Shaded area = "their (b)(i) answer" – correct triangle	M1		Condone "difference" if $\Delta > \int$
	$\Rightarrow$ shaded area $= 18\frac{9}{10}$	A1	3	CSO; OE (18.9 etc)
	Total		**11**	
6(a)	Remainder $= p(1) = 1 + 1 - 8 - 12$ $= -18$	M1 A1	2	Use of p(1) NOT long division
(b)(i)	$p(-2) = -8 + 4 + 16 - 12$ $= 0 \Rightarrow (x+2)$ is factor	M1 A1	2	NOT long division p(–2) shown = 0 **and** statement
(ii)	Quad factor by comparing coefficients or $\left(x^2 + kx \pm 6\right)$ by inspection	M1		Or full long division or attempt at Factor Theorem using $f(\pm 3)$
	$p(x) = (x+2)(x^2 - x - 6)$	A1		Correct quadratic factor or $(x-3)$ shown to be factor by Factor Theorem
	$p(x) = (x+2)^2(x-3)$ or $(x+2)(x+2)(x-3)$	A1	3	CSO; SC: B1 for $(x+2)(x***)(x-3)$ by inspection or without working
(c)(i)	$(k =)$ -12	B1	1	Condone $y = -12$ or $(0, -12)$
(ii)		M1 A1 A1	3	Cubic shape (one max and one min) Maximum at (–2,0) and through (3, 0) – at least one of these values marked "correct" graph as shown (touching smoothly at –2, 3 marked and minimum to right of y-axis)
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$(x-8)^2 + (y-13)^2$ $= 13^2$	B1 B1	2	**Exactly this** with + and squares Condone 169
(b)(i)	grad $PC = \frac{12}{5}$	B1	1	Must simplify $\frac{-12}{-5}$
(ii)	grad of tangent $= \frac{-1}{\text{grad } PC} = -\frac{5}{12}$	B1√		Condone $-\frac{1}{2.4}$ etc
	tangent has equation $y - 1 = -\frac{5}{12}(x-3)$	M1 A1		ft gradient but M0 if using grad PC Correct – but not in required final form
	$5x + 12y = 27$ OE	A1	4	MUST have integer coefficients
(iii)	half chord = 5	B1		Seen or stated
	 $d^2 = (\text{their } r)^2 - 5^2$ (provided $r > 5$)	M1		Pythagoras used correctly $d^2 = 13^2 - 5^2$
	Distance = 12	A1	3	CSO
	Total		**10**	
8(a)	$b^2 - 4ac = 16k^2 - 36(k+1)$	M1		Condone one slip
	Real roots: discriminant $\geqslant 0$	B1		
	$\Rightarrow 16k^2 - 36k - 36 \geqslant 0$ $\Rightarrow 4k^2 - 9k - 9 \geqslant 0$	A1	3	AG (watch signs)
(b)	$(4k+3)(k-3)$	M1		Or correct use of formula (unsimplified)
	critical points $(k =)$ $-\frac{3}{4}$, 3	A1		Not in a form involving surds Values may be seen in inequalities etc
	 sketch	M1		Or sign diagram
	$k \geqslant 3$, $k \leqslant -\frac{3}{4}$	A1	4	NMS full marks Condone use of word "and" but final answer in a form such as $3 \leqslant k \leqslant -\frac{3}{4}$ scores A0
	Total		**7**	
	TOTAL		**75**	

General Certificate of Education
January 2009
Advanced Subsidiary Examination

AQA

MATHEMATICS **MPC1**
Unit Pure Core 1

Friday 9 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The points A and B have coordinates $(1, 6)$ and $(5, -2)$ respectively. The mid-point of AB is M.

(a) Find the coordinates of M. *(2 marks)*

(b) Find the gradient of AB, giving your answer in its simplest form. *(2 marks)*

(c) A straight line passes through M and is perpendicular to AB.

(i) Show that this line has equation $x - 2y + 1 = 0$. *(3 marks)*

(ii) Given that this line passes through the point $(k, k+5)$, find the value of the constant k. *(2 marks)*

2 (a) Factorise $2x^2 - 5x + 3$. *(1 mark)*

(b) Hence, or otherwise, solve the inequality $2x^2 - 5x + 3 < 0$. *(3 marks)*

3 (a) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

(b) Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. *(3 marks)*

4 (a) (i) Express $x^2 + 2x + 5$ in the form $(x+p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence show that $x^2 + 2x + 5$ is always positive. *(1 mark)*

(b) A curve has equation $y = x^2 + 2x + 5$.

(i) Write down the coordinates of the minimum point of the curve. *(2 marks)*

(ii) Sketch the curve, showing the value of the intercept on the y-axis. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 2x + 5$. *(3 marks)*

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dx}{dt}$; *(3 marks)*

(ii) $\frac{d^2x}{dt^2}$. *(2 marks)*

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of x with respect to t when $t = 1$. *(2 marks)*

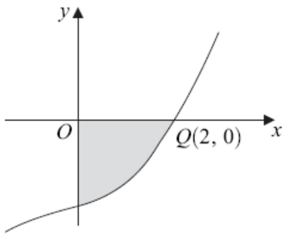
(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. *(2 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x-2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\,dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

7 A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) The point D has coordinates $(7, -2)$.

(i) Verify that the point D lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point D, giving your answer in the form $mx + ny = p$, where m, n and p are integers. *(3 marks)*

(d) (i) A line has equation $y = kx$. Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$ *(2 marks)*

(ii) Find the values of k for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

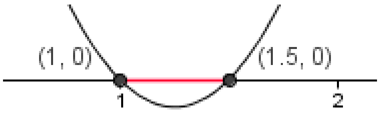
has equal roots. *(5 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii). *(1 mark)*

END OF QUESTIONS

AQA – Core 1 - Jan 2009 – Answers

Question 1:	**Exam report**
a) $A(1,6)\ \ B(5,-2)$ The mid-point $M\left(\frac{5+1}{2};\frac{-2+6}{2}\right)=M\ (3,2)$ b) Gradient of $AB=m_{AB}=\frac{-2-6}{5-1}=-2$ c) i) The gradient of the perpendicular to AB is $-\frac{1}{m_{AB}}=\frac{1}{2}$ The equation of the perpendicular bisector is : $y-2=\frac{1}{2}(x-3)$ $2y-4=x-3$ $x-2y+1=0$ ii) Substitute x by k and y by $k+5$ in the equation: $k-2(k+5)+1=0$ $k-2k-10+1=0$ $k=-9$	In part (a) most candidates were able to find the correct coordinates of the mid point, although a few transposed the coordinates and others subtracted rather than adding the coordinates before halving the results. Full marks were only awarded in part (b) for a gradient of –2 and quite a few candidates did not give their answer in this simplest form. In part (c)(i) most candidates realised that the product of the gradients should be –1. However, not all were able to calculate the negative reciprocal. Others used an incorrect point such as *A* or *B* and therefore found an equation of the wrong line. The most successful used an equation of the form $y - y_1=m(x - x_1)$ as flagged above. The printed answer helped most candidates to be successful in finding the correct equation of the line. In part (c)(ii) most candidates made an attempt at this part of the question, but the failure to use brackets for the second term caused the majority to find an incorrect value for *k*. Others foolishly tried to substitute $x = k$ and $y = k + 5$ into their own incorrect line equation rather than using the printed answer from part (c)(i).

Question 2:	**Exam report**
a) $2x^2-5x+3=(2x-3)(x-1)$ b) Critical values $\frac{3}{2}$ and 1 $2x^2-5x+3<0\ \ for\ 1<x<\frac{3}{2}$ 	In part (a) it was quite alarming to see the number of candidates who were unable to factorise this quadratic. Most candidates scored only a single mark in part (b) for attempting to find the critical values. Many would benefit from practising the solution of inequalities of this type by drawing a suitable sketch or by familiarising themselves with the technique of using a sign diagram as indicated in previous mark schemes

Question 3:	**Exam report**
a) $\frac{7+\sqrt{5}}{3+\sqrt{5}}=\frac{7+\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}=\frac{21-7\sqrt{5}+3\sqrt{5}-5}{9-5}$ $=\frac{16-4\sqrt{5}}{4}=4-\sqrt{5}$ b) $\sqrt{45}+\frac{20}{\sqrt{5}}=\sqrt{9\times5}+\frac{20\sqrt{5}}{5}=3\sqrt{5}+4\sqrt{5}=7\sqrt{5}$	In part (a) it was pleasing to see that most candidates were familiar with the technique for rationalising the denominator in this type of problem and, although there were some who made slips when multiplying out the two brackets in the numerator, most obtained the correct answer in the given form. In part (b) the term $\sqrt{45}$ was usually expressed as $3\sqrt{5}$, but the term $\frac{20}{\sqrt{5}}$ caused far more difficulties than expected. Consequently, the final correct answer was only obtained by the better candidates.

Question 4:

$$a)\,i)\; y = x^2 + 2x + 5 = (x+1)^2 - 1 + 5 = (x+1)^2 + 4$$

$$ii)\;\text{For all } x,\ (x+1)^2 \geq 0 \;\; so\; (x+1)^2 + 4 \geq 4$$

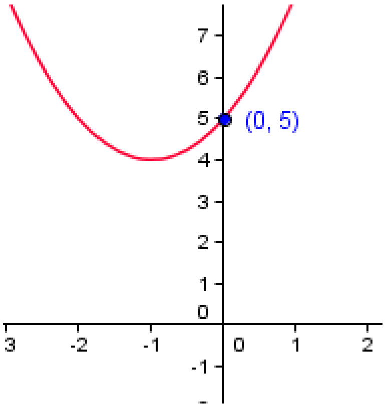
$$y \text{ is always positive.}$$

$$b)\,i)\;\text{The minimum point is } (-1, 4)$$

$$ii)\;\text{The curve crosses the y-axis at } (0,5)$$

$$c)\; x \xrightarrow[-1unit\,in\,x-dir]{translation} x+1 \xrightarrow{f} (x+1)^2 \xrightarrow[4units\,in\,y-dir]{translation} (x+1)^2 + 4$$

$$summary{:}Translation\ vector \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$



Exam report

In part (a)(i) the completion of the square was done successfully by most candidates, although occasionally the value 6 was seen instead of 4 for q.

In part (a)(ii) it was necessary to comment on both parts of the expression; $(x+1)^2 \geq 0$ and hence adding 4 implies that $(x+1)^2 + 4 > 0$ for all values of x. Because of the word "hence", an argument based on algebra rather than the features of a curve was required; for instance, an answer explaining that $(x+1)^2 + 4$ has a minimum value of 4 was acceptable, but a statement about the curve having a minimum point at (–1, 4) was not.

In part (b)(i) most candidates were able to write down the correct minimum point. A few chose to use differentiation but sometimes made arithmetic slips in finding the coordinates of the stationary point.

In part (b)(ii) those with the correct minimum point were usually able to produce a correct sketch, although the value of the y-intercept was sometimes missing. Some credit was given to candidates with an incorrect minimum point, usually (1, 4), provided their graph was consistent with this minimum point.

The more able candidates earned full marks in part (c). The term translation was required but generally the wrong word was used or it was accompanied by another transformation such as a stretch. A very common (but incorrect) vector stated was $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

Question 5:

$$x = \frac{1}{2}t^4 - 20t^2 + 66t \qquad 0 \leq t \leq 4$$

$$a)\,i)\; \frac{dx}{dt} = \frac{1}{2} \times 4t^3 - 20 \times 2t + 66$$

$$\frac{dx}{dt} = 2t^3 - 40t + 66$$

$$ii)\; \frac{d^2x}{dt^2} = 6t^2 - 40$$

$$b)\;\text{Verify that } \frac{dx}{dt}(t=3) = 0.$$

$$\frac{dx}{dt}(t=3) = 2 \times 3^3 - 40 \times 3 + 66$$

$$= 54 - 120 + 66 = 0$$

$$\text{There is a stationary point when } t = 3.$$

$$\frac{d^2x}{dt^2}(t=3) = 6 \times 3^2 - 40 = 54 - 40 = 14 > 0$$

$$\text{This point is a MINIMUM.}$$

$$c)\;\text{The rate of change is } \frac{dx}{dt}(t=1) = 2 - 40 + 66 = 28$$

$$d)\; \frac{dx}{dt}(t=2) = 2 \times 2^3 - 40 \times 2 + 66 = 16 - 80 + 66 = 2 > 0$$

$$\text{The distance is INCREASING when } t = 2.$$

Exam report

In part (a) almost all candidates were able to find the first and second derivative correctly, although there was an occasional arithmetic slip and some could not cope with the fraction term.

Those who substituted t = 3 into $\frac{dx}{dt}$ in part (b) did not always explain that $\frac{dx}{dt}$ is the condition for a stationary point. Some assumed that a stationary point occurred when t = 3 and went straight to the test for maximum or minimum and only scored half the marks. It was advisable to use the second derivative test here; those who considered values of $\frac{dx}{dt}$ on either side of t = 3 usually reached an incorrect conclusion because of the proximity of another stationary point.

In part (c) the concept of "rate of change" was not understood by many. Approximately equal numbers of candidates substituted t = 1 into the expression for $\frac{dx}{dt}$ or $\frac{d^2x}{dt^2}$ and so only about half of the candidates were able to score any marks on this part. Those who used $\frac{dx}{dt}$ often made careless arithmetic errors when adding three numbers.

In part (d), as in part (c), candidates did not realise which expression to use and many wrongly selected the second derivative. It is a general weakness that candidates do not realise that the sign of the first derivative indicates whether a function is increasing or decreasing at a particular point.

Question 6:	**Exam report**
$a)\ p(x)=x^3+x-10$ $i)\ p(2)=2^3+2-10=8+2-10=\ p(2)=0$ 2 is a root of p, so $(x-2)$ is a factor of p. $ii)\ x^3+x-10=(x-2)(x^2+2x+5)$ $b)\ i)$ The gradient of the curve at Q is $\frac{dy}{dx}(x=2)$ $\frac{dy}{dx}=3x^2+1$ *and* *for* $x=2,\ \frac{dy}{dx}=m_Q=13$ $ii)$ The equation of the tangent at Q is : $y-0=13(x-2)$ $y=13x-26$ $iii)\ \int(x^3+x-10)dx=\frac{1}{4}x^4+\frac{1}{2}x^2-10x+c$ $iv)$ The curve is below the x-axis, so the area of the shaded part is $-\int_0^2(x^3+x-10)dx=\left[-\frac{1}{4}x^4-\frac{1}{2}x^2+10x\right]_0^2$ $=(-4-2+20)-(0)=14$ $Area=14$	In part (a)(i) the majority of candidates realised the need to find the value of f(x) when x = 2. However, it was also necessary, after showing that f (2) = 0, to write a statement that the zero value implied that x – 2 was a factor. It was good to see more candidates being aware of this. In part (a)(ii), those who used inspection were the most successful here. Methods involving long division or equating coefficients usually contained algebraic errors. In part (b)(i) a surprising number of candidates failed to see the need to differentiate in order to find the gradient at Q. Those who attempted to find $\frac{dy}{dx}$ sometimes wrote it as $3x^2 + x$, but usually were aware of the need to substitute x = 2. In part (b)(ii) those who had the correct gradient in part (b)(i) were usually successful in finding the correct equation of the tangent, and most obtained at least a method mark here. In part (b)(iii) most were well drilled in integration and earned full marks, although some wrote $\frac{x^4}{4}+\frac{x^2}{2}-10$ and others gave $\frac{x^4}{4}+\frac{x^2}{2}-\frac{10^2}{2}$ as their answer. For part (b)(iv) the correct limits were usually used, although many sign/arithmetic slips occurred after substitution of the numbers 0 and 2 and it was incredible how many could not evaluate 6 – 20 without a calculator. Very few candidates realised the need to show clearly that, although the integral from 0 to 2 gave a value of –14, the area of the shaded region was 14. A separate statement was needed and those who simply wrote 4 + 2 – 20 = –14 = 14 did not score full marks. Those more able candidates who made a statement about the region being entirely below the x-axis and who subsequently evaluated the integral from 2 to 0 correctly scored full marks.

Question 7:

$$x^2+y^2-6x+10y+9=0$$

$$a)\,(x-3)^2-9+\left(y+5\right)^2-25+9=0$$

$$\left(x-3\right)^2+\left(y+5\right)^2=5^2$$

$b)\,i)$ The centre $C(3,-5)$

$ii)\; radius\;\; r=5$

$c)\,D(7,-2)$

$i)$ Substitute x and y by 7 and -2:

$$\left(x-3\right)^2+\left(y+5\right)^2$$

$$=\left(7-3\right)^2+\left(-2+5\right)^2=16+9=25$$

D belongs to the circle.

$ii)$ The normal to to circle at D is the line CD

$$\mathrm{m}_{CD}=\frac{-2+5}{7-3}=\frac{3}{4}$$

The equation of the normal: $y+2=\frac{3}{4}\left(x-7\right)$

$$4y+8=3x-21$$

$$3x-4y=29$$

$d)\,i)$ Solve simultaneously $\begin{cases} y=kx \\ \left(x-3\right)^2+\left(y+5\right)^2=25 \end{cases}$

by substitution, we have $\left(x-3\right)^2+\left(kx+5\right)^2=25$

$$x^2+9-6x+k^2x^2+25+10kx=25$$

$$(1+k^2)x^2+(10k-6)x+9=0$$

$$(1+k^2)x^2+2(5k-3)x+9=0$$

$ii)$ This equation has equal roots

if the discriminant $=0.$

$$(10k-6)^2-4\times\left(1+k^2\right)\times 9=0$$

$$100k^2+36-120k-36k^2-36=0$$

$$64k^2-120k=0$$

$$k\left(64k-120\right)=0$$

$$k=0\;\; or\;\; k=\frac{120}{64}=\frac{15}{8}$$

$iii)$ When $k=0\; or\; \frac{15}{8}$, the line is tangent to the circle.

Exam report

In part (a) most candidates found the correct values of a and b, but correct values for r^2 were not so common. Some sloppiness was again evident with candidates failing to write squared outside the brackets or omitting the plus sign between the terms on the left hand side. It was common to see things such as 25 = 25 = $\sqrt{25}$ = 5^2 and this could be penalised in the future.

In part (b) the coordinates of the centre, C and the radius r, although not always correct, usually gained full credit when following through from part (a).

In part (c)(i) most candidates attempted to verify that the point D was on the circle, although some, who had obviously worked a previous examination question, were keen to show that the distance from C to D was less than the radius and that D lay inside the circle. This verification was marked fairly strictly and the argument had to be correct including a final concluding statement. Those who simply wrote $4^2 + 3^2 = 25$, for example, did not earn the mark.

In part (c)(ii) many candidates found the gradient of CD and then assumed they had to find the negative reciprocal of this since the question asked for the normal at D. Reference to a sketch might have prevented this incorrect assumption.

In part (d)(i) most candidates made errors by not using brackets; the expression kx^2 was seen almost as often as the correct form k^2x^2 after substituting y = kx into their circle equation.

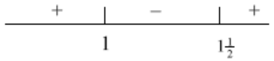
In part (d)(ii), although there were some correct solutions seen, the discriminant often contained algebraic slips and the condition for equal roots was rarely stated. Often it was several lines into the working before an "= 0" appeared and many times this was omitted entirely. The value k = 0 was often ignored in otherwise correct solutions, but it was more common to see a three term quadratic because of previous algebraic errors.

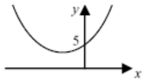
In part (d)(iii) several candidates realised that the line would be a tangent for each of the two values of k, but many completely missed the point and talked about transformations, often giving vectors in their answer.

MY SCORE FOR THIS PAPER IS ….. / 75

MY GRADE : …….

GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	62	54	46	39	32

Q	Solution	Marks	Total	Comments
1(a)	$M\,(3, 2)$	B1 B1	2	B1 for each coordinate
(b)	Gradient $AB = \frac{-2-6}{5-1}=\left(\frac{-8}{4}\right)$	M1		May use coords of M instead of A or B - condone one slip
	$= -2$	A1	2	CSO Answer must be simplified to –2
(c) (i)	Gradient of perpendicular = $\frac{1}{2}$	B1√		ft "their" −1/gradient AB
	$\Rightarrow\ y-2=\frac{1}{2}(x-3)$	M1		attempt at perp to AB; ft their M coords
	$\Rightarrow\ 2y-4=x-3\ \ \Rightarrow\ x-2y+1=0$ **AG**	A1	3	CSO Must write down the printed answer
(ii)	$k-2(k+5)+1=0$ or $\frac{(k+5)-2}{k-3}=\frac{1}{2}$	M1		Sub into given line equation or correct expression involving gradients Condone omission of brackets or use of x
	$\Rightarrow k=-9$	A1	2	Condone $x=-9$ *(Full marks for correct answer without working)*
	Total		**9**	
2(a)	$(x-1)(2x-3)$	B1	1	$(1-x)(3-2x)$ or $2(x-1)(x-1.5)$ etc
(b)	Critical values are 1, $1\frac{1}{2}$	B1√		Correct or ft their factors from (a)
	Sign diagram or sketch	M1		
	$\Rightarrow 1<x<1\frac{1}{2}$	A1	3	*Full marks for correct inequality without working*
	Total		**4**	
3(a)	$\frac{7+\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$	M1		Multiply by $\frac{3-\sqrt{5}}{3-\sqrt{5}}$ or $\frac{\sqrt{5}-3}{\sqrt{5}-3}$
	Numerator $= 21+3\sqrt{5}-7\sqrt{5}-\left(\sqrt{5}\right)^2$	m1		Condone one slip $16-4\sqrt{5}$
	Denominator = 9 – 5 = 4	B1		(Or 5 – 9 = – 4 from other conjugate)
	Answer = $4-\sqrt{5}$	A1	4	CSO
(b)	$\sqrt{45}=3\sqrt{5}$	B1		
	$\frac{20}{\sqrt{5}}=\frac{20\sqrt{5}}{5}$	M1		May score if combined as one expression Must have 5 in denominator
	Sum $= 7\sqrt{5}$	A1	3	
	Total		**7**	

Q	Solution	Marks	Total	Comments
4(a)(i)	$(x+1)^2$	B1		$p=1$
	$+\ 4$	B1	2	$q=4$
(ii)	$(x+1)^2\geqslant 0\Rightarrow(x+1)^2+4>0$ $(\Rightarrow x^2+2x+5>0$ for all values of $x)$	E1	1	Condone if they say $(x+1)^2$ *positive* and adding 4 so always positive
(b)(i)	$x=-1$ **or** $y=4$	M1		ft their $x=-p$ or $y=q$
	Minimum point is (–1, 4)	A1	2	
(ii)		B1		Sketch roughly as shown
		B1	2	y-intercept 5 or (0, 5) marked or stated
(c)	Translation (not shift, move etc)	E1		and NO other transformation stated
	through $\begin{bmatrix}-1\\4\end{bmatrix}$ (or 1 left, 4 up etc)	M1		either component correct or ft their $-p$, q
		A1	3	correct translation M1, A1 independent of E mark
	Total		**10**	
5(a)(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=2t^3-40t+66$	M1		one term correct
		A1		another term correct
		A1	3	all correct unsimplified (no + c etc)
(ii)	$\frac{\mathrm{d}^2x}{\mathrm{d}\,t^2}=6t^2-40$	M1		ft one term correct
		A1√	2	ft all "correct", 2 terms equivalent
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=54-120+66$	M1		substitute $t=3$ into their $\frac{\mathrm{d}x}{\mathrm{d}t}$
	$=0\Rightarrow$ stationary value	A1		CSO shown = 0 (54 or 2×27 seen) and statement
	Substitute $t=3$ into $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ (= 14)	M1		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}>0\ \Rightarrow$ minimum value	A1	4	CSO; all values (if stated) must be correct
(c)	Substitute $t=1$ into their $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1		must be their $\frac{\mathrm{d}x}{\mathrm{d}t}$ NOT $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ etc
	$\frac{\mathrm{d}x}{\mathrm{d}t}=28$	A1√	2	ft their $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $t=1$
(d)	Substitute $t=2$ into their $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1		must be their $\frac{\mathrm{d}x}{\mathrm{d}t}$ NOT $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ or x
	$=16-80+66=2\ \ (>0)$			Interpreting their value of $\frac{\mathrm{d}x}{\mathrm{d}t}$
	$\Rightarrow$ increasing when $t=2$	E1√	2	Allow decreasing if their $\frac{\mathrm{d}x}{\mathrm{d}t}<0$
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{p}(2) = 8 + 2 - 10$	M1		Must find p(2) NOT long division
	$\Rightarrow \mathrm{p}(2) = 0 \Rightarrow (x - 2)$ is factor	A1	2	Shown = 0 plus a statement
(ii)	Attempt at long division (generous)	M1		Obtaining a quotient $x^2 + cx + d$ or equating coefficients (full method)
	$\mathrm{p}(x) = (x - 2)(x^2 + 2x + 5)$	A1	2	$a = 2,\ b = 5$ by inspection B1, B1
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 1$	M1 A1		One term correct All correct – no $+c$ etc
	When $x = 2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times 4 + 1$	m1		Sub $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Therefore gradient at Q is 13	A1	4	CSO
(ii)	$y = 13(x - 2)$	M1		Tangent (NOT normal) attempted ft their gradient answer from (b)(i)
		A1	2	CSO; correct in any form
(iii)	$\int \dots\, \mathrm{d}x = \frac{x^4}{4} + \frac{x^2}{2} - 10x\ (+c)$	M1 A1 A1	3	one term correct second term correct all correct (condone no +c)
(iv)	$[4 + 2 - 20] - [0] = -14$	M1		$\mathrm{F}(2)$ attempted and possibly F(0) Must have earned M1 in (b)(iii)
	Area of shaded region = 14	A1	2	CSO; separate statement following correct evaluation of limits
	Total		**15**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$(x - 3)^2 + (y + 5)^2$	B1 B1		One term correct LHS correct with + and squares
	$= 25 - 9 + 9 = 25 \quad (= 5^2)$	B1	3	Condone RHS = 25
(b)(i)	$C\ (3, -5)$	B1√		
(ii)	Radius = 5	B1√	2	Correct or ft their RHS provided > 0
(c)(i)	$(7 - 3)^2 + (-2 + 5)^2 = 16 + 9 = 25$ $\Rightarrow D$ lies on circle *Must see statement*	B1	1	Or sub'n of $(7, -2)$ in original equation $7^2 + (-2)^2 - 42 - 20 + 9 = 0$ Or sub x=7 into eqn & showing $y = -2$ etc
(ii)	Attempt at gradient of CD as normal	M1		withhold if subsequently uses $m_1 m_2 = -1$
	grad $CD = \frac{-2 - (-5)}{7 - 3} = \frac{3}{4}$			$\frac{\Delta y}{\Delta x}$ (condone one slip) FT their centre C
	$y + 2 = \frac{3}{4}(x - 7)$ or $y + 5 = \frac{3}{4}(x - 3)$	A1		Correct equation in any form $y = \frac{3}{4}x - \frac{29}{4}$
	$\Rightarrow 3x - 4y = 29$	A1	3	CSO ***Integer*** coefficients Condone $4y - 3x + 29 = 0$ etc
(d)(i)	$y = kx$ sub'd into original circle equation $x^2 + (kx)^2 - 6x + 10kx + 9 = 0$	M1		or using their completed square form and multiplying out
	$\Rightarrow (k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$ **AG**	A1	2	CSO must see at least previous line for A1 any error such as $kx^2 = \dots = k^2 x^2$ gets A0
(ii)	$4(5k - 3)^2 - 36(k^2 + 1)$	M1		Discriminant in k (can be seen in quad formula) Condone one slip
	$= 64k^2 - 120k$	A1		or $8k^2 - 15k = 0$ OE
	Equal roots: $4(5k - 3)^2 - 36(k^2 + 1) = 0$	B1		$b^2 - 4ac = 0$ clearly stated or evident by an equation in k with at most 2 slips.
	$8k^2 - 15k = 0$			
		m1		Attempt to solve ***their*** quadratic or linear equation if k has been cancelled
	$\Rightarrow k = 0, \quad k = \frac{15}{8}$	A1	5	OE but must have k=0 If "=0" is not seen but correct values of k are found, candidate will lose B1 mark but may earn all other marks
(iii)	(Line is a) **tangent** (to the circle)	E1	1	Line **touches** circle at one point
	Total		**17**	
	TOTAL		**75**	

June 2009

1 The line AB has equation $3x + 5y = 11$.

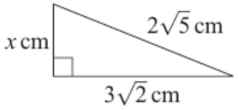
(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

2 (a) Express $\dfrac{5+\sqrt{7}}{3-\sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers. *(4 marks)*

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x. *(3 marks)*

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Verify that the point P is a stationary point of the curve. *(2 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

(d) Find an equation of the tangent to the curve at the point where $x = 1$. *(4 marks)*

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

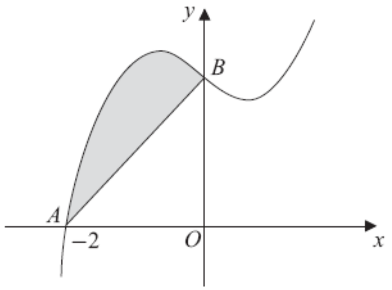
(i) Find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x) = x^3 - x + 6$ in the form $(x+2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $p(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A(-2, 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\displaystyle\int_{-2}^{0} (x^3 - x + 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

5 A circle with centre C has equation

$$(x-5)^2+(y+12)^2=169$$

(a) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the circle passes through the origin O. *(1 mark)*

(ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p. *(3 marks)*

(c) The point $A\,(-7, -7)$ lies on the circle.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the tangent to the circle at the point A, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. *(3 marks)*

6 (a) (i) Express $x^2-8x+17$ in the form $(x-p)^2+q$, where p and q are integers. *(2 marks)*

(ii) Hence write down the minimum value of $x^2-8x+17$. *(1 mark)*

(iii) State the value of x for which the minimum value of $x^2-8x+17$ occurs. *(1 mark)*

(b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7-x)$.

(i) Expand $(x-5)^2$. *(1 mark)*

(ii) Show that $AB^2=2(x^2-8x+17)$. *(3 marks)*

(iii) Use your results from part (a) to find the minimum value of the distance AB as x varies. *(2 marks)*

7 The curve C has equation $y=k(x^2+3)$, where k is a constant.

The line L has equation $y=2x+2$.

(a) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$kx^2-2x+3k-2=0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points.

(i) Show that

$$3k^2-2k-1<0$$ *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - June 2009 – Answers

Question 1:

$$Line\ \ AB: 3x+5y=11$$

$$a)\,i)\,Make\ \ y\ \ the\ subject: 5y=-3x+11$$

$$y=-\frac{3}{5}x+\frac{11}{5}$$

$$\text{The gradient of } AB = m_{AB} = -\frac{3}{5}$$

$$ii)\ A(2,1).$$

The gradient of the line perpendicular to AB is

$$-\frac{1}{m_{AB}}=\frac{5}{3}$$

$$\text{The equation of the line is : } y-1=\frac{5}{3}\left(x-2\right)$$

$$3y-3=5x-10$$

$$5x-3y=7$$

$$b)\,\text{Solve simultaneously } \begin{cases} 3x+5y=11 & (\times 2) \\ 2x+3y=8 & (\times -3) \end{cases} \ gives$$

$$\begin{cases} 6x+10y=22 \\ -6x-9y=-24 \end{cases} \ and\ \ by\ \ adding$$

$$y=-2$$

$$and\ \ \ 3x+5y=11 \qquad 3x-10=11 \qquad\qquad x=7$$

$$\text{The lines intersect at } \left(7,-2\right)$$

Exam report

In part (a)(i) many candidates were unable to make *y* the subject of the equation 3*x* + 5*y* = 11 and, as a result, many incorrect answers for the gradient were seen. Those who tried to use two points on the line to find the gradient were rarely successful.

In part (a)(ii) most candidates realised that the product of the gradients of perpendicular lines should be –1 and credit was given for using this result together with their answer from part(a)(i). Although many correct answers for the coordinates of *C* were seen in part (b)(i), the simultaneous equations defeated a large number of candidates. No credit was given for mistakenly using their equation from part (a)(ii) instead of the correct equation for *AB*.

Question 2:

$$a)\,\frac{5+\sqrt{7}}{3-\sqrt{7}}=\frac{5+\sqrt{7}}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}=\frac{15+5\sqrt{7}+3\sqrt{7}+7}{9-7}$$

$$=\frac{22+8\sqrt{7}}{2}=11+4\sqrt{7}$$

$$b)\,\text{Using pythagoras' theorem, we have}$$

$$x^2=\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2$$

$$=4\times5-9\times2=2$$

$$x=\sqrt{2}\ \ or\ \ x=-\sqrt{2}$$

$$\text{but } x \text{ is a length, } x>0,\ \ so\ \ x=\sqrt{2} \text{ is the answer.}$$

Exam report

In part (a) most candidates recognised the first crucial step of multiplying the numerator and denominator by $3+\sqrt{7}$ and many obtained $\frac{22+8\sqrt{7}}{2}$, but then poor cancellation led to a very common incorrect answer of $11+8\sqrt{7}$.

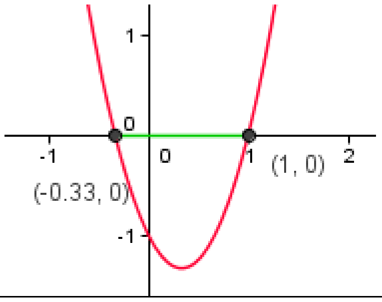
Candidates found part (b) more difficult than part (a) and revealed a lack of understanding of surds. Most candidates realised the need to use Pythagoras' Theorem but many could not square $2\sqrt{5}\ \ and\ \ 3\sqrt{2}$ correctly. Little credit was given for those who wrote things such as $x=\sqrt{20}-\sqrt{18}=\sqrt{2}$ and candidates need to realise that "getting the right answer" is not always rewarded with full marks. Although the equation $x^2 = 2$ has the solution $x=\pm\sqrt{2}$, it was necessary to consider the context and to give the value of x as $\sqrt{2}$.

Question 3:	**Exam report**
$y=x^5+20x^2-8 \qquad P(-2,y_P)$ a) $\frac{dy}{dx}=5x^4+40x$ b) for $x=-2, \frac{dy}{dx}=5\times(-2)^4+40\times(-2)$ $\frac{dy}{dx}=5\times16-80=0$ P is a stationary point. c) i) $\frac{d^2y}{dx^2}=20x^3+40$ and for $x=-2$ $=20\times(-2)^3+40=-160+40=-120<0$ ii) P is a MAXIMUM point. d) when $x=1,\ y=1^5+20\times1^2-8=1+20-8=13$ $\frac{dy}{dx}(x=1)=5+40=45$ the equation of the tangent at (1,13) is $y-13=45(x-1)$ $y=45x-32$	In part (a) almost everyone obtained the correct expression for $\frac{dy}{dx}$, although a few spoiled their solution by dividing each term by 5 or adding "+ c" to their answer. In part (b) most candidates substituted x = –2 into their expression for $\frac{dy}{dx}$, but, in order to score full marks, it was necessary to show $(-2)^4$ written as 16 or to show that $\frac{dy}{dx}$ = 80 – 80 = 0 and then to write an appropriate conclusion about P being a stationary point. For part (c) many candidates simply wrote down an expression for $\frac{d^2y}{dx^2}$ in terms of x when answering part (i) and only evaluated the second derivative when determining the nature of the stationary point in part (ii). On this occasion full credit was given, but candidates need to realise what is meant by the demand to "find the value of" since this may be penalized in future examinations. In part (d) some candidates failed to find the y-coordinate of P, which was necessary in order to find the equation of the tangent. It was pleasing to see most candidates using the value of $\frac{dy}{dx}$ when x = 1, but unfortunately many tried to find the equation of the normal instead of the tangent to the curve.

Question 4:	**Exam report**
a) $p(x)=x^3-x+6$ i) The remainder is $p(3)$ $p(3)=3^3-3+6=27-3+6=30$ ii) $p(-2)=(-2)^3-(-2)+6$ $=-8+2+6=0$ -2 is a root of p, so $(x+2)$ is a factor of p. iii) $p(x)=(x+2)(x^2-2x+3)$ iv) $p(x)=0$ means $(x-2)(x^2-2x+3)=0$ so $x-2=0$ or $x^2-2x+3=0$ $x=2$ the discriminant $=(-2)^2-4\times1\times3=-8<0$ no solution. b) i) $B(0,6) \qquad (p(0)=0-0+6=6)$ ii) $\int_{-2}^{0}(x^3-x+6)\,dx=\left[\frac{1}{4}x^4-\frac{1}{2}x^2+6x\right]_{-2}^{0}$ $=(0)-(4-2-12)=10$ iii) The shaded area = area beneath the curve – area of triangle ABO $=10-\frac{1}{2}\times2\times6=4$	Those candidates who used the remainder theorem in part (a)(i) were usually successful in finding the correct remainder. Those who tried to use long division were usually confused by the lack of an x^2 term and were rarely successful in showing that the remainder was 30. Those who used long division in part (a)(ii) scored no marks. Most candidates realised the need to show that p(–2) = 0, but quite a few omitted sufficient working such as p(–2) = –8 + 2 + 6 = 0 together with a concluding statement about x + 2 being a factor and therefore failed to score full marks. Many candidates have become quite skilled at writing down the correct product of a linear and quadratic factor and these scored full marks in part (a)(iii). Others used long division effectively but lost a mark for failing to write p(x) in the required form. Others tried methods involving comparing coefficients, but often after several lines of working were unable to find the correct values of b and c because of poor algebraic manipulation. In part (a)(iv), although many candidates tried to consider the value of the discriminant of their quadratic factor, quite a few used a =1, b = –1 and c = 6 (from the cubic equation) and scored no marks for this part of the question. Others drew a correct conclusion using the quadratic equation formula, indicating that it was not possible to find the square root of –8 and others, after completing the square showed that the equation $(x-1)^2=-2$ has no real solutions. Some wrongly concluded that because it was not possible to factorise their quadratic then the corresponding quadratic equation had no real roots. Most obtained the correct y-coordinate of B in part (b)(i). In part (b)(ii) it was pleasing to see most candidates being able to integrate correctly but a large number did not answer the question set and simply found the indefinite integral in this part. Many candidates use poor techniques when finding a definite integral and it was often difficult to see the evaluation of F(0) – F(-2) in their solution. Many obtained an answer of –10 which was miraculously converted into +10 with some comment about an area being positive. This and similar dubious working was penalized. In part (b)(iii) some obtained an answer of –6 for the area of the triangle by using –2 as the base. Credit was given to candidates who later realised that the area of the triangle was actually 6. Unless candidates had scored full marks in part(ii) they were not able to score full marks in this part either, even if they obtained a correct value of 4 for the shaded area.

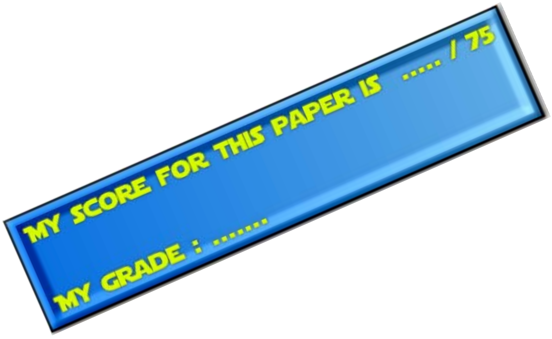
Question 5:	Exam report
$(x-5)^2+(y+12)^2=169$ $a)\,i)\,C(5,-12)$ $ii)\,r=\sqrt{169}=13$ $b)\,i)\,O(0,0)$ $(0-5)^2+(0+12)^2=25+144=169$ O belongs to the circle. $ii)\,(0,p):(0-5)^2+(p+12)^2=169$ $25+(p+12)^2=169$ $(p+12)^2=144$ $p+12=\pm 12$ $p=0\ \ or\ \ p=-24$ $(0,-24)$ belongs to the circle. $c)\,A(-7,-7)$ *lies on the circle* $i)$ *gradient of* $AC=m_{AC}=\frac{-12+7}{5+7}=-\frac{5}{12}$ $ii)$ The tangent is perpendicular to the radius AC The gradient of the tangent is $\frac{12}{5}$ The equation of the tangent is $y+7=\frac{12}{5}(x+7)$ $5y+35=12x+94$ $12x-5y+49=0$	In part (a)(i) most candidates realised what the correct coordinates of the centre were, although some wrote these as (–5, 12) instead of (5, –12). Some gave the radius as 169 and others evaluated $\sqrt{169}$ incorrectly in part (a)(ii). The majority of candidates obtained the correct value of the radius. In part (b)(i) most were able to verify that the circle passed through the origin, although some neglected to make a statement as a conclusion to their calculation and so failed to earn this mark. A surprisingly large number made no attempt at this part. Most sketches were correct in part (b)(ii), though some were very untidy with some making several attempts at the circle so the diagram resembled the chaotic orbit of a planet. In spite of being asked to verify that the circle passed through the origin many sketches did not do so. Credit was given for freehand circles with the centre in the correct quadrant and which passed through the origin, although it was good to see some circles drawn using compasses. Many used algebraic methods, putting x = 0, but often their poor algebra prevented them from finding the value of p. Those using the symmetry, doubling the y- coordinate, were usually more successful, although an answer of –25 (from –12–13) was common. In part (c)(i) the majority of candidates tried to find the gradient of AC but careless arithmetic meant that far fewer actually succeeded in finding its correct simplified value. In part (c)(ii), in order to find the tangent, it was necessary to use the negative reciprocal of the answer from part (c)(i) in order to find the gradient. Although some did, many chose to use the same gradient obtained in the previous part of the question and scored no marks at all.

Question 6:	Exam report
$a)\,i)\,x^2-8x+17=(x-4)^2-16+17=(x-4)^2+1$ $ii)$ For all x, $(x-4)^2\geq 0\ so\ (x-4)^2+1\geq 1$ The minimum value is 1 $iii)$ the minimum occurs when $(x-4)^2=0$ *i.e.* $x=4$ $b)\,A(5,4)\quad B(x,7-x)$ $i)\,(x-5)^2=x^2+25-10x$ $ii)\,AB^2=(x_B-x_A)^2+(y_B-y_A)^2$ $=(x-5)^2+(3-x)^2$ $=x^2+25-10x+9+x^2-6x$ $=2x^2-16x+34$ $AB^2=2(x^2+8x+17)$ $iii)$ The minimum value of $x^2+8x+17$ *is* 1 *so* the minimum value of AB^2 *is* 2 the minimum value of AB *is* $\sqrt{2}$	Completing the square was done well by most candidates in part (a)(i), although quite a few wrote q as 17 instead of 1. Part (a)(ii) of this question was answered very badly with many giving their answer as coordinates. Candidates were either “hedging their bets” or were simply presenting the coordinates of a minimum point of a curve as their answer. In part (a)(iii) many candidates obtained the correct value for x in this part, but there was confusion with many about how to answer parts (i) and (ii). The question was deliberately designed to test the understanding of the minimum value of a quadratic expression and when this occurred. Those who wrote “(ii) 4 and (iii)1” scored no marks at all for these two parts of the question. In part (b)(i) practically everyone scored a mark for multiplying out $(x - 5)^2$ correctly. In part (b)(ii) only the best candidates obtained a correct expression for AB^2 and then completed the resulting algebra to obtain the printed answer. It was good to see that many saw the link between the various parts in part (b)(iii). Many more able candidates substituted x = 4 into the expression and obtained $AB^2 = 2$, but they then failed to take the positive square root in order to find the minimum distance.

Question 7:	**Exam report**
curve $C: y = k\left(x^2+3\right)$ *line* $L: y = 2x+2$ $a)$ By indentifying the y's: $k\left(x^2+3\right) = 2x+2$ $kx^2-2x+3k-2=0$ $b)$ The curve and the line have two distinct points of intersection, this means that the discriminant >0. $(-2)^2-4\times k\times(3k-2)>0$ $4-12k^2+8k>0 \qquad (\div -4)$ $3k^2-2k-1<0$ $iii) \quad (3k+1)(k-1)<0$ *critical values* $-\frac{1}{3}$ *and* 1 $(3k+1)(k-1)<0$ *for* $-\frac{1}{3}<k<1$  	Most candidates scored the mark for the correct printed equation in part (a), but some omitted "= 0" and others made algebraic slips when taking terms from one side of their equation to the other. In part (b)(i) only the more able candidates were able to obtain the printed inequality using correct algebraic steps. Many began by stating that the discriminant was less than 0, clearly being influenced by the answer. Not all assigned the correct terms to a, b and c in the expression $b^2 - 4ac$ and others made sign errors when removing brackets. The factorisation was usually correct in part (b)(ii), but many wrote down one of the critical values as 1/3. Most found critical values and either stopped or immediately tried to write down a solution without any working. Candidates are strongly advised to use a sign diagram or a sketch showing their critical values when solving a quadratic inequality.

GRADE BOUNDARIES

Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	63	55	48	41	34



AQA

General Certificate of Education

Mathematics 6360

MPC1 Pure Core 1

Mark Scheme

2009 examination - June series

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

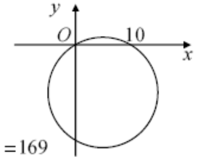
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

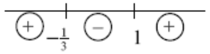
Q	Solution	Marks	Total	Comments
1(a)(i)	$y=-\frac{3}{5}x+\frac{11}{5}$	M1		Attempt at $y=\mathrm{f}(x)$
	Or correct expression for gradient using two correct points			**Or** answer $=\frac{3}{5}$ or $-\frac{3}{5}x$ gets M1 But answer of $\frac{3}{5}x$ gets M0
	(Gradient of AB =) $-\frac{3}{5}$	A1	2	Correct answer scores 2 marks . Condone error in rearranging formula if answer for gradient is correct.
(ii)	$m_1m_2=-1$	M1		Used or stated
	Gradient of perpendicular = $\frac{5}{3}$	A1√		ft their answer from (a)(i) or correct
	$y-1=\frac{5}{3}(x-2)$ OE	A1	3	$5x-3y=7$; or $y=\frac{5}{3}x+c$, $c=-\frac{7}{3}$ etc CSO
(b)	Eliminating x or y but must use $3x+5y=11$ & $2x+3y=8$	M1		An equation in x only or y only
	$x = 7$	A1		
	$y=-2$	A1	3	Answer only of $(7, -2)$ scores 3 marks
	Total		**8**	
2(a)	$\frac{5+\sqrt{7}}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}$	M1		
	Numerator = $15+5\sqrt{7}+3\sqrt{7}+7$	m1		Condone one error or omission
	Denominator = $9-7$ (= 2)	B1		Must be seen as the denominator
	(Answer =) $11+4\sqrt{7}$	A1	4	
(b)	$\left(2\sqrt{5}\right)^2=20$ *or* $\left(3\sqrt{2}\right)^2=18$	B1		Either correct
	their $\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2$	M1		Condone missing brackets and x^2
	($x^2=20-18$)			$x^2=2\Rightarrow$ Bl, M1
	$(\Rightarrow x=)\ \sqrt{2}$	A1	3	$\pm\sqrt{2}$ scores A0 Answer only of 2 scores B0, M0 Answer only of $\sqrt{2}$ scores 3 marks
	Total		7	

Q	Solution	Marks	Total	Comments
		M1		One of these powers correct
3(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=5x^4+40x$	A1		One of these terms correct
		A1	3	All correct (no + c etc)
(b)	$x=-2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=5\times(-2)^4+(40\times-2)$	M1		Substitute $x=-2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=5\times16+(40\times-2)=0$			
	$\Rightarrow P$ is stationary point	A1		CSO Shown = 0 plus statement eg “st pt”, “as required”, “grad = 0”etc
	Or *their* $\frac{\mathrm{d}y}{\mathrm{d}x}=0\ \Rightarrow x^n=k$	(M1)		
	$x^3=-8\ \Rightarrow x=-2$	(A1)	2	CSO $x=0$ need not be considered
(c)(i)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=20x^3+40$	B1√		Correct ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$=20\times(-2)^3+40$	M1		Subst $x=-2$ into their second derivative
	$(=-160+40)\quad=-120$	A1	3	CSO
(ii)	Maximum (value) their c(i) answer must be < 0 Other valid methods acceptable provided “maximum” is the conclusion	E1√	1	Accept minimum if their $c(i)$ *answer* > 0 and correctly interpreted Parts (i) and (ii) may be combined by candidate but –120 must be seen to award A1 in part (c)(i)
(d)	(When $x=1$) $y=13$	B1		
	When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=5+40$	M1		Sub $x=1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$y=$ (their 45)$x+k$ OE	m1		ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Tangent has equation $y-13=45(x-1)$	A1	4	CSO OE $y=45x+c$, $c=-32$
	Total		**13**	

Q	Solution	Marks	Total	Comments
4(a)(i)	$\mathrm{p}(3)=27-3+6$	M1		p(3) attempted
	(Remainder) = 30	A1		
	Or long division up to remainder	(M1)		
	Quotient= x^2+3x+8 and remainder = 30 clearly stated or indicated	(A1)	2	
(ii)	$\mathrm{p}(-2)=-8+2+6$	M1		p(–2) attempted **:** **NOT** long division
	$\mathrm{p}(-2)=0 \Rightarrow x+2$ is factor Minimum statement required "factor"	A1	2	Shown = 0 plus statement May make statement *first* such as "x+2 is a factor if p(–2) = 0"
(iii)	$b=-2$	B1		No working required for B1 + B1
	$c=3$	B1		Try to mark first using B marks
	or long division/comparing coefficients	(M1)		Award M1 if B0 earned and a clear method is used
	$\mathrm{p}(x)=(x+2)(x^2-2x+3)$	(A1)	2	Must write final answer in this form if long division has been used to get A1
(iv)	$b^2-4ac=(-2)^2-4\times 3$	M1		Discriminant correct from their quadratic M0 if $b=-1$, $c=6$ used (using cubic eqn)
	$b^2-4ac=-8$ (or < 0) $\Rightarrow$ no (other) real roots	A1		CSO All values must be correct plus statement
	Or $(x-1)^2+2$	(M1)		Completion of square for their quadratic
	$(x-1)^2+2>0$ therefore no real roots Or $(x-1)^2=-2$ has no real roots	(A1)	2	Shown to be positive plus statement regarding no real roots
(b)(i)	$(y_B=)\,6$	B1	1	Condone (0, 6)
(ii)	$\frac{x^4}{4}-\frac{x^2}{2}+6x$	M1 A1 A1		One term correct Another term correct All correct (ignore + c or limits)
	$\Big[\quad\Big]_{-2}^{0}=0-(4-2-12)$	m1		$\mathrm{F}(-2)$ attempted
	= 10	A1	5	CSO Clearly from F(0) – F(–2)
(iii)	Area of $\Delta=\frac{1}{2}\times 2\times 6$	M1		Condone – 2 and ft their y_B value **Or** $\int_{-2}^{0}(3x+6)\mathrm{d}x$ and attempt to integrate
	= 6	A1		Must be positive allow –6 converted to +6
	Shaded region area = 10 – 6 = 4	A1	3	CSO 10 must come from correct working
	Total		**17**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$C\ (5,-12)$	B1	1	
(ii)	Radius = 13 (or $\sqrt{169}$)	B1	1	$\pm\sqrt{169}$ or ± 13 as final answer scores B0
(b)(i)	$(-5)^2+12^2$ *or* $25+144$ $=169 \Rightarrow$ circle passes through O	B1	1	Correct arithmetic plus statement Eg "O lies on circle", "as required" etc
(ii)	Sketch 	B1		Freehand circle through origin and cutting positive x-axis with centre in 4th quadrant Condone value 10 missing or incorrect
	$25+(p+12)^2=169$	M1		Or doubling their y_C-coordinate
	$(p+12)=\pm 12 \qquad p=-24$	A1	3	Condone use of y instead of p SC B2 for correct value of p stated or marked on diagram
(c)(i)	grad $AC=\frac{-12+7}{5+7}$	M1		correct expression, but ft their C
	$=-\frac{5}{12}$	A1	2	Condone $\frac{5}{-12}$
(ii)	grad tangent = $\frac{12}{5}$	B1 √		$\frac{-1}{\text{their grad } AC}$
	$y+7=\frac{12}{5}(x+7)$	M1		ft "their $\frac{12}{5}$" must be **tangent** and not AC
	$\Rightarrow 12x-5y+49=0$	A1	3	OE with **integer** coefficients with all terms on one side of the equation
	Total		**11**	

Q	**Solution**	**Marks**	**Total**	**Comments**
6(a)(i)	$(x-4)^2$ *or* $p=4$	B1		ISW for $p=-4$ if $(x-4)^2$ seen
	$+1$ *or* $q=1$	B1	2	
(ii)	(Minimum value is) 1	B1√	1	Correct or FT "their q" (NOT coords)
(iii)	(Minimum occurs when $x=$) 4	B1√	1	Correct or FT "their p" – may use calculus Condone (p, **) for this B1 mark
(b)(i)	$(x-5)^2 = x^2-10x+25$	B1	1	
(ii)	$(x-5)^2+(7-x-4)^2$ $=(x-5)^2+(3-x)^2$	M1		Condone one slip in one bracket May be seen under $\sqrt{\ }$ sign
	$=x^2-10x+25+9-6x+x^2$	A1		From a fully correct expression
	$AB^2=2x^2-16x+34$ $=2(x^2-8x+17)$	A1	3	AG CSO
(iii)	Minimum $AB^2 = 2\times$ "their (a)(ii)"	M1		Or use of their $x=4$ in expression Or use of their $B(4, 3)$ and $A(5, 4)$ in distance formula M0 if calculus used Answer only of $2\times$ "their (a)(ii)" scores M1, A0
	Minimum $AB=\sqrt{2}$	A1	2	
	Total		**10**	

Q	**Solution**	**Marks**	**Total**	**Comments**
7(a)	$k(x^2+3)=2x+2$ $\Rightarrow kx^2-2x+3k-2=0$	B1	1	AG OE all terms on one side and = 0
(b)(i)	Discriminant $=(-2)^2-4k(3k-2)$	M1		Condone one slip (including x is one slip) Condone 2^2 or 4 as first term
	$=4-12k^2+8k$	A1		condone recovery from missing brackets
	Two distinct real roots $\Rightarrow b^2-4ac>0$ $4-12k^2+8k>0$ $\Rightarrow 12k^2-8k-4<0$	B1√		"their discriminant in terms of k" >0 Not simply the statement $b^2-4ac>0$ Change from >0 to <0 and divide by 4
	$\Rightarrow 3k^2-2k-1<0$	A1	4	AG CSO
(ii)	$(3k+1)(k-1)$	M1		**Correct** factors or **correct** use of formula
	Critical values 1 and $-\frac{1}{3}$	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working
	Use of sign diagram or sketch   	M1		If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise, M1 may be earned for an attempt at the sketch or sign diagram using their critical values.
	$\Rightarrow -\frac{1}{3}<k<1$ or $1>k>-\frac{1}{3}$ condone $-\frac{1}{3}<k$ AND $k<1$ for full marks but not OR or "," instead of AND	A1	4	Full marks for correct final answer with or without working $\leqslant$ loses final A mark *Answer only* of $1<k<-\frac{1}{3}$ or $k<-\frac{1}{3}; k<1$ *etc* scores M1,A1,M0 since the correct critical values are evident *Answer only* of $\frac{1}{3}<k<1$ etc where critical values are not both correct gets M0,M0
	Total		**9**	
	TOTAL		**75**	

General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MPC1

Unit Pure Core 1

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(b) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

2 The triangle ABC has vertices $A(1,\ 3)$, $B(3,\ 7)$ and $C(-1,\ 9)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence show that angle ABC is a right angle. *(2 marks)*

(b) (i) Find the coordinates of M, the mid-point of AC. *(2 marks)*

(ii) Show that the lengths of AB and BC are equal. *(3 marks)*

(iii) Hence find an equation of the line of symmetry of the triangle ABC. *(3 marks)*

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \tfrac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

4 (a) Show that $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}$ is an integer and find its value. *(3 marks)*

(b) Express $\dfrac{2\sqrt{7}-1}{2\sqrt{7}+5}$ in the form $m+n\sqrt{7}$, where m and n are integers. *(4 marks)*

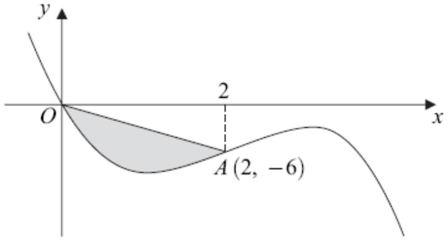
5 (a) Express $(x-5)(x-3)+2$ in the form $(x-p)^2+q$, where p and q are integers. *(3 marks)*

(b) (i) Sketch the graph of $y=(x-5)(x-3)+2$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(ii) Write down an equation of the tangent to the graph of $y=(x-5)(x-3)+2$ at its vertex. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y=x^2$ onto the graph of $y=(x-5)(x-3)+2$. *(3 marks)*

6 The curve with equation $y=12x^2-19x-2x^3$ is sketched below.



The curve crosses the x-axis at the origin O, and the point $A\,(2,\ -6)$ lies on the curve.

(a) (i) Find the gradient of the curve with equation $y=12x^2-19x-2x^3$ at the point A. *(4 marks)*

(ii) Hence find the equation of the normal to the curve at the point A, giving your answer in the form $x+py+q=0$, where p and q are integers. *(3 marks)*

(b) (i) Find the value of $\int_0^2 (12x^2-19x-2x^3)\,dx$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

7 A circle with centre C has equation $x^2+y^2-4x+12y+15=0$.

(a) Find:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the x-axis. *(2 marks)*

(c) The point P with coordinates $(5,\ k)$ lies outside the circle.

(i) Show that $PC^2=k^2+12k+45$. *(2 marks)*

(ii) Hence show that $k^2+12k+20>0$. *(1 mark)*

(iii) Find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - Jan 2010 – Answers

Question 1:	**Exam report**
$p(x)=x^3-13x-12$ $a)\ p(-3)=(-3)^3-13\times(-3)-12$ $p(-3)=-27+39-12=0$ -3 is a root of p, so $(x+3)$ is a factor of p $b)\ p(x)=x^3-13x-12=(x+3)(x^2-3x-4)$ $=(x+3)(x-4)(x+1)$	In part (a), most candidates realised the need to find the value of f(*x*) when *x* = –3. However, it was also necessary, after showing that f(–3) = 0, to write a statement that the zero value implied that *x* + 3 was a factor. It was good to see quite a large number of candidates being aware of this but others lost a valuable mark. In part (b), some candidates used long division effectively to find the quadratic factor and, although this was the most successful method, some were confused by the lack of an x^2 term; others used the method of comparing coefficients or found the terms of the quadratic by inspection; a number used the Factor Theorem to find another linear factor, but seldom found both of the remaining factors. Very able candidates were able to write down the correct product of three linear factors but many more were unsuccessful when they tried to do this without any discernible method.

Question 2:	**Exam report**
$A(1,3)\ ,\ B(3,7)\ ,\ C(-1,9)$ $a)\,i)\,\text{Gradient of } AB = m_{AB} = \dfrac{7-3}{3-1} = 2$ $ii)\,\text{Gradient of BC} = m_{BC} = \dfrac{9-7}{-1-3} = -\dfrac{1}{2}$ $m_{BC}\times m_{AB} = 2\times -\dfrac{1}{2} = -1$ $BC\ and\ AB$ are perpendicular so the traingle ABC is a right-angled triangle $b)\,i)\,\text{Mid-point of AC } = M\left(\dfrac{1-1}{2},\dfrac{3+9}{2}\right) = M\,(0,6)$ $ii)\ AB=\sqrt{(3-1)^2+(7-3)^2}=\sqrt{4+16}=\sqrt{20}$ $BC=\sqrt{(-1-3)^2+(9-7)^2}=\sqrt{16+4}=\sqrt{20}$ $length\ \ AB = length\ \ BC$ $iii)$ The triangle ABC is an isosceles right-angled triangle The line of symmetry is the line BM. $\text{gradient of } BM = m_{BM} = \dfrac{7-6}{3-0} = \dfrac{1}{3}$ $Equation\ of\ \ BM:\ y-7=\dfrac{1}{3}(x-3)$ $3y-21=x-3$ $x-3y+18=0$	In part (a)(i), although some made arithmetic errors when finding the gradient of AB, the majority of answers were correct. It was necessary to reduce fractions such as 4/2 in order to score full marks. In part (a)(ii), those who chose to use Pythagoras' Theorem, calculating lengths of sides to prove that the triangle was right angled, scored no marks here. The word "hence" indicated that the gradient of AB needed to be used in the proof that angle ABC was a right angle. A large number of those using gradients failed to score full marks on this part of the question. It was not sufficient to show that the gradient of BC was –1/2 and then to simply say "therefore ABC is a right angle"; an explanation that the product of the gradients was equal to – 1 was required. In part (b)(i), most candidates were able to find the correct coordinates of the mid point, although a few transposed the coordinates and others subtracted, rather than added, the coordinates before halving the results. In part (b)(ii), it was rare to see a solution with all mathematical statements correct. Too often candidates wrote things like AB = $2^2 + 4^2 = 20 = \sqrt{20}$ and, although this was not penalised on this occasion, examiners in the future might not be quite so generous. It was surprising how many candidates did not know the distance formula. Some wrote down vectors but, unless their lengths were calculated, no marks were scored. In part (b)(iii), many candidates found an equation of the wrong line. The line of symmetry was actually BM, although some chose an equivalent method using the gradient of a line perpendicular to AC. The most successful candidates often used an equation of the form $y - y_1 = m(x - x_1)$; far too often those using y = mx + c were unable to find the correct value of c, usually because of poor arithmetic.

Question 3:	**Exam report**
$y=\frac{1}{8}t^4-2t^2+4t \qquad 0\le t\le 4$ $a)\,i)\,\frac{dy}{dt}=\frac{1}{2}t^3-4t+4$ $ii)\,\frac{d^2y}{dt^2}=\frac{3}{2}t^2-4$ $b)\,\frac{dy}{dt}(t=2)=\frac{1}{2}\times(2)^3-4\times(2)+4$ $=4-8+4=0$ At $t=2$, $\frac{dy}{dx}=0$, there is a stationary point. $\frac{d^2y}{dt^2}(t=2)=\frac{3}{2}\times(2)^2-4=6-4=2>0$ The stationary point is a MINIMUM. $c)\,i)$ The rate of change is $\frac{dy}{dt}(t=1)=\frac{1}{2}-4+4=\frac{1}{2}=0.5\,m/s$ $ii)\,\frac{dy}{dt}(t=1)>0$, so the depth is INCREASING at $t=1$.	In part (a), almost all candidates were able to find the first and second derivative correctly, although there was an occasional arithmetic slip and some could not cope with the fraction term. In part (b), those who substituted t = 2 into $\frac{dy}{dx}$ did not always explain that $\frac{dy}{dx}$ = 0 is the condition for a stationary point. Some assumed that a stationary point occurred when t = 2, went straight to the test for maximum or minimum and only scored half the marks. It was advisable to use the second derivative test here; those who considered values of $\frac{dy}{dx}$ on either side of t = 2 usually reached an incorrect conclusion because of the proximity of another stationary point. In part (c)(i), the concept of ‘rate of change’ was not understood by many who failed to realise the need to substitute t = 1 into $\frac{dy}{dx}$. Some candidates wrongly substituted t = 1 into the initial expression for y or into their expression for $\frac{d^2y}{dx^2}$ and these candidates were unable to score any marks at all on this part. Even those who used $\frac{dy}{dx}$ sometimes made careless arithmetic errors when adding three numbers. In part (c)(ii), it was not enough to simply write the word “increasing”: some explanation about $\frac{dy}{dx}$ being positive was also required. Some candidates erroneously found the value of the second derivative when t = 1 or calculated the value of y on either side of t = 1.

Question 4:	**Exam report**
$a)\,\frac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}=\frac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}}=\frac{8\sqrt{2}}{2\sqrt{2}}=\frac{8}{2}=4$ $b)\,\frac{2\sqrt{7}-1}{2\sqrt{7}+5}=\frac{2\sqrt{7}-1}{2\sqrt{7}+5}\times\frac{2\sqrt{7}-5}{2\sqrt{7}-5}=\frac{28-10\sqrt{7}-2\sqrt{7}+5}{28-25}$ $=\frac{33-12\sqrt{7}}{3}=11-4\sqrt{7}$	In part (a), there were far more mistakes than had been anticipated; for example, $\sqrt{50}=2\sqrt{5}$ and $\sqrt{18}=2\sqrt{3}$. It was also common to see poor cancelling such as $\frac{8\sqrt{2}}{2\sqrt{2}}=4\sqrt{2}$. Examiners had to take care that totally incorrect work leading to the correct answer was not rewarded. For instance, $\frac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}=\frac{2\sqrt{5}+2\sqrt{9}}{2\sqrt{4}}=\frac{10+6}{4}=4$ was seen on a number of occasions. In part (b), it was pleasing to see that most candidates were familiar with the technique for rationalising the denominator in this type of problem and, although there were some who made slips when multiplying out the two brackets in the numerator, particularly when trying to calculate $2\sqrt{7}\times2\sqrt{7}$, many obtained the correct answer in the given form and it was good to see most getting the final step correct by dividing **both** terms by 3.

Question 5:

$$a)\left(x-5\right)\left(x-3\right)+2=x^2-8x+15+2=x^2-8x+17$$

$$x^2-8x+17=\left(x-4\right)^2-16+17=\left(x-4\right)^2+1$$

$b)\,i)$ The minimum point/vertex is (4,1)

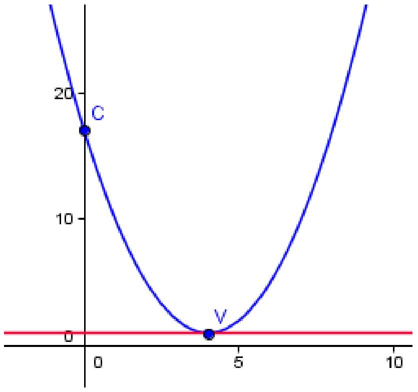
The curve crosses the y-axis at (0,17)

$ii)$ At the vertex, the tangent to the curve is "horizontal"

$$y=1$$

$$c)\;x\xrightarrow[4\,units\,in\,x-dir]{translation}x-4\xrightarrow{f}\left(x-4\right)^2\xrightarrow[1\,unit\,in\,y-dir]{trnslation}\left(x-4\right)^2+1$$

The transformation is a translation vector $\begin{bmatrix}4\\1\end{bmatrix}$.



Exam report

In part (a), the slightly unusual form of the initial quadratic caused problems to those who forgot to add 2 after multiplying out the brackets. The first term in the completion of the square was done successfully by most candidates, although the answer $(x-4)^2-3$ was seen almost as often as the correct form. It was unfortunate that an error at this stage was seldom picked up by candidates when they went on to sketch the curve.

In part (b)(i), the sketch was usually of the correct shape but many drew a parabola with the vertex in the wrong quadrant, or the y-intercept was incorrect. The question did ask for the coordinates of the minimum point and this was not always stated by candidates.

In part (b)(ii), a few candidates immediately wrote down the correct equation for the tangent, whereas many felt the need to differentiate in order to find the gradient of the curve at the vertex. Many candidates seemed unaware that the vertex was actually the minimum point and that the tangent at the vertex would be parallel to the line y = 0.

In part (c), the more able candidates earned full marks. The term translation was required and, although there seemed to be an improvement in candidates' using the correct word, it was still common to see words such as "shift" or "move" being used instead. Others thought that simply writing down a vector was enough. A very commonly stated, but incorrect, vector was $\begin{bmatrix}-4\\1\end{bmatrix}$

Question 6:

$$y=12x^2-19x-2x^3\qquad O(0,0)\;and\;\;A(2,-6)$$

$$a)i)\frac{dy}{dx}=24x-19-6x^2$$

$$\frac{dy}{dx}\left(x=2\right)=m_A=24\times2-19-6\times2^2=5$$

The gradient of the curve at A is 5.

$ii)$ The gradient of the normal at A is $-\dfrac{1}{m_A}=-\dfrac{1}{5}$

The eqaution of the normal: $y+6=-\dfrac{1}{5}\left(x-2\right)$

$$5y+30=-x+2$$

$$x+5y+28=0$$

$$b)i)\int_0^2(12x^2-19x-2x^3)dx=\left[4x^3-\frac{19}{2}x^2-\frac{1}{2}x^4\right]_0^2$$

$$=\left(32-38-8\right)-\left(0\right)=-14$$

$ii)$ The area comprised between the curve and the x-axis is 14.

The area of the triangle is $\dfrac{1}{2}\times2\times6=6.$

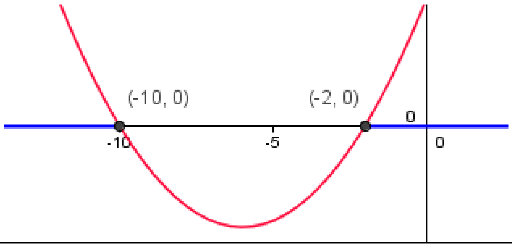
The area of the shaded region is $14-6\;\;=8$

Exam report

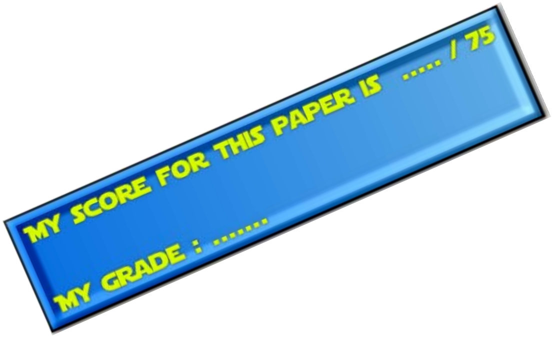
In part (a)(i), many candidates did not realise that differentiation was required in order to find the gradient of the curve, but instead erroneously used the coordinates of O and A. Some tried to rearrange the terms but usually made sign errors in doing so. In part (a)(ii), those who had the correct gradient in part (a)(i) were usually successful in finding the correct equation of the normal, though not everyone followed through to the required form, and sign errors were common. However, most obtained at least a method mark here, unless they found the equation of the tangent. The main casualties were once again those who always use the y = mx + c form for the equation of a straight line.

In part (b)(i), most candidates were well drilled in integration and scored full marks, although some wrote down terms with incorrect denominators. The limits 2 and 0 were usually substituted correctly, but it was incredible how many could not evaluate 32 – 38 – 8 without a calculator. The correct value of the integral was –14, but far too many thought that they had to change their answer to +14 and so lost a mark. A small number of candidates differentiated or substituted into the expression for y rather than the integrated function.

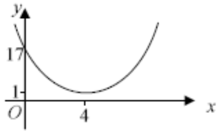
In part (b)(ii), there was still some apparent confusion about area when a region lies below the x-axis. The area of the triangle was 6 units and hence the area of the shaded region was 14 – 6 = 8 but, not surprisingly, there were all kinds of combinations of positive and negative quantities seen here. It was worrying to see so many candidates failing to calculate the triangle area, with several finding the length of OA instead. Some able candidates found the equation of OA and the area under it by integration, but this was not the expected method.

Question 7:	**Exam report**
$x^2+y^2-4x+12y+15=0$ $(x-2)^2-4+(y+6)^2-36+15=0$ $(x-2)^2+(y+6)^2=25$ $a)i)\ Centre\ C(2,-6)$ $ii)\ radius\ r=\sqrt{25}=5$ $b)$ The distance from C to the x-axis is 6, which is more that the radius 5. c) $P(5,k)$ *lies outside the circle* $i)\ PC^2=(2-5)^2+(-6-k)^2=9+36+k^2+12k$ $PC^2=k^2+12k+45$ $ii)$ P lies outside the circle so $PC>5$ *or* $PC^2>25$ *we have then* $k^2+12k+45>25$ $k^2+12k+20>0$ $iii)\ k^2+12k+20>0$ $(k+2)(k+10)>0$ *critical values*$:-10$ *and* -2 $(k+2)(k+10)>0$ *for* $k<-10$ *or* $k>-2$  	In part (a), most candidates found at least one of the correct coordinates for the centre C, with the most common error being at least one sign error or writing the centre as (4, –12). However, the correct value for the radius was not so common, with $\sqrt{15}$ being frequently seen. In part (b), most explanations involving a comparison of the y-coordinate of the centre and the radius of the circle earned at least one mark. In order to score the second mark, some of the better answers explained why the y-coordinate of the “highest” point of the circle was –1, but most comments were insufficient. Those who proved that the circle did not intersect the x-axis needed to state that the y-coordinate of the centre was negative in order to score full marks. Part (c)(i) was very poorly done. Many candidates tried to use their circle equation and essentially faked the given result instead of correctly using the distance formula. Once again, if candidates are asked to “show that ...” then the full equation needs to be seen in the final line of the proof and an essential part of the working was to see a statement such as $PC^2 = 3^2 + (k + 6)^2$, leading to the printed answer. In part (c)(ii), the word “hence” indicated that the expression for PC^2 in part (c)(i) needed to be used. Candidates needed to realise that the point P lies outside the circle when PC > r and that using this result immediately leads to the given inequality. The inequality sign here caused confusion with many looking for a discriminant. In part (c)(iii), many candidates scored only the two marks available for finding the critical values –2 and –10. No doubt some would have benefited from practising the solution of inequalities of this type by drawing a suitable sketch, or by familiarising themselves with the technique of using a sign diagram as indicated in previous mark schemes.

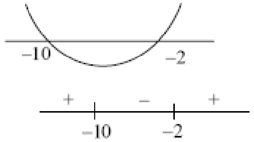
GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	62	54	47	40	33



Q	Solution	Marks	Total	Comments
1(a)	$p(-3)=(-3)^3-13(-3)-12$	M1		must attempt $p(-3)$ NOT long division
	$=-27+39-12$ $=0 \quad \Rightarrow x+3$ is factor	A1	2	shown $=0$ plus statement
(b)	$(x+3)(x^2+bx+c)$	M1		Full long division, comparing coefficients or by inspection either $b=-3$ or $c=-4$
	(x^2-3x-4) obtained	A1		or M1A1 for either $(x-4)$ or $(x+1)$ clearly found using factor theorem
	$(x+3)(x-4)(x+1)$	A1	3	CSO; must be seen as a product of 3 factors NMS full marks for correct product SC B1 for $(x+3)(x-4)(\ )$ or $(x+3)(x+1)(\ )$ or $(x+3)(x+4)(x-1)$ NMS
	Total		**5**	
2(a)(i)	grad $AB=\frac{7-3}{3-1}$	M1		$\frac{\Delta y}{\Delta x}$ correct expression, possibly implied
	$=2$ (must simplify 4/2)	A1	2	
(ii)	grad $BC=\frac{7-9}{3+1}=-\frac{2}{4}$	M1		Condone one slip
	grad $AB\times$ grad $BC=-1$ $\Rightarrow \angle ABC=90^\circ$ or AB & BC perpendicular	A1	2	**NOT** Pythagoras or cosine rule etc convincingly proved plus statement SC B1 for $-1/$(their grad AB) or statement that $m_1m_2=-1$ for perpendicular lines if M0 scored
(b)(i)	$M\ (0,6)$	B2	2	B1 + B1 each coordinate correct
(ii)	$(AB^2=)\quad (3-1)^2+(7-3)^2$ $(BC^2=)\quad (3+1)^2+(7-9)^2$	M1		either expression correct, simplified or unsimplified
	$AB^2=2^2+4^2$ or $BC^2=4^2+2^2$ or $\sqrt{20}$ found as a length	A1		Must see either $AB^2=$.., or $BC^2=$...,
	$AB^2=BC^2 \Rightarrow AB=BC$ or $AB=\sqrt{20}$ and $BC=\sqrt{20}$	A1	3	
(iii)	grad $BM=\frac{7-6}{3-0}$ or $-1/$(grad AC) attempted	M1		ft their M coordinates
	$=\frac{1}{3}$	A1		correct gradient of line of symmetry
	BM has equation $y=\frac{1}{3}x+6$	A1	3	CSO, any correct form
	Total		**12**	
3(a)(i)	$\frac{dy}{dt}=\frac{4t^3}{8}-4t+4$	M1 A1 A1	3	one term correct another term correct all correct (no $+c$ etc) unsimplified
(ii)	$\frac{d^2y}{dt^2}=\frac{12t^2}{8}-4$	M1		ft one term "correct"
		A1	2	correct unsimplified (penalise inclusion of $+c$ once only in question)
(b)	$t=2$; $\frac{dy}{dt}=4-8+4$	M1		Substitute $t=2$ into their $\frac{dy}{dt}$
	$\frac{dy}{dt}=0 \Rightarrow$ stationary value	A1		CSO; shown = 0 plus statement
	$t=2;\ \frac{d^2y}{dt^2}=6-4=2$	M1		Sub $t=2$ into their $\frac{d^2y}{dt^2}$
	$\Rightarrow y$ has MINIMUM value	A1	4	CSO
(c)(i)	$t=1;\ \frac{dy}{dt}=\frac{1}{2}-4+4$	M1		Substitute $t=1$ into their $\frac{dy}{dt}$
	$=\frac{1}{2}$	A1	2	OE; CSO NMS full marks if $\frac{dy}{dt}$ correct
(ii)	$\frac{dy}{dt}>0 \Rightarrow$ (depth is) INCREASING	E1√	1	allow decreasing if states that their $\frac{dy}{dt}<0$ **Reason must be given not just the word increasing or decreasing**
	Total		**12**	
4(a)	$\sqrt{50}=5\sqrt{2}$; $\sqrt{18}=3\sqrt{2}$; $\sqrt{8}=2\sqrt{2}$ At least two of these correct	M1		or $\times\frac{\sqrt{8}}{\sqrt{8}}$ or $\left(\times\frac{\sqrt{2}}{\sqrt{2}}\right)$ or $\sqrt{\frac{25}{4}}+\sqrt{\frac{9}{4}}$
	$\frac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}}$	A1		any **correct** expression all in terms of $\sqrt{2}$ or with denominator of 8, 4 or 2 simplifying numerator eg $\frac{\sqrt{400}+\sqrt{144}}{8}$
	Answer $=4$	A1	3	CSO
(b)	$\frac{(2\sqrt{7}-1)(2\sqrt{7}-5)}{(2\sqrt{7}+5)(2\sqrt{7}-5)}$	M1		OE
	numerator = $4\times7-2\sqrt{7}-10\sqrt{7}+5$	m1		expanding numerator (condone one error or omission)
	denominator = 3	B1		(seen as denominator)
	Answer $=11-4\sqrt{7}$	A1	4	
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)	$x^2-8x+15+2$	B1		Terms in x must be collected, PI
	their $(x-4)^2 \quad (+k)$	M1		ft $(x-p)^2$ for their quadratic
	$=(x-4)^2+1$	A1	3	ISW for stating $p=-4$ if correct expression seen
(b)(i)		M1		∪ shape in any quadrant (generous)
		A1		correct with min at (4, 1) stated or 4 and 1 marked on axes condone within first quadrant only
		B1	3	crosses y-axis at (0, 17) stated or 17 marked on y-axis
(ii)	$y=k$	M1		$y=$ constant
	$y=1$	A1	2	Condone $y=0x+1$
(c)	Translation (*not shift, move etc*)	E1		and no other transformation
	with vector $\begin{bmatrix}4\\1\end{bmatrix}$	M1		One component correct or ft either their p or q
		A1	3	CSO; condone 4 across, 1 up; or two separate vectors etc
	Total		11	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\frac{dy}{dx}=24x-19-6x^2$	M1 A1		2 terms correct all correct (no + c etc)
	when $x=2$, $\frac{dy}{dx}=48-19-24$	m1		
	$\Rightarrow$ gradient $=5$	A1	4	CSO
(ii)	grad of normal $=-\frac{1}{5}$	B1√		ft their answer from (a)(i)
	$y+6=\left(\textit{their}-\frac{1}{5}\right)(x-2)$ or $y=\left(\textit{their}-\frac{1}{5}\right)x+c$ and c evaluated using $x=2$ and $y=-6$	M1		ft grad of their normal using **correct** coordinates BUT must not be tangent condone omission of brackets
	$x+5y+28=0$	A1	3	CSO; condone all on one side in different order
(b)(i)	$\frac{12}{3}x^3-\frac{19}{2}x^2-\frac{2}{4}x^4$	M1 A1 A1		one term correct another term correct all correct (ignore $+c$ or limits)
	$=32-38-8$	m1		$F(2)$ attempted
	$=-14$	A1	5	CSO; withhold A1 if changed to +14 here
(ii)	Area $\Delta=\frac{1}{2}\times2\times6=6$	B1		condone -6
	Shaded region area $=14-6$	M1		difference of $\pm\left\|\int\right\|\ \pm\left\|\Delta\right\|$
	$=8$	A1	3	CSO
	Total		15	

Q	Solution	Marks	Total	Comments
7(a)(i)	$x=\pm2$ or $y=\pm6$ or $(x-2)^2+(y+6)^2$	M1		
	$C(2,-6)$	A1	2	correct
(ii)	$(r^2=)4+36-15$	M1		(RHS =) *their* $(-2)^2+$ *their* $(6)^2-15$
	$\Rightarrow r=5$	A1	2	Not ±5 *or* $\sqrt{25}$
(b)	explaining why $\|y_C\|>r$; $6>5$	E1		Comparison of y_C and r, eg $-6+5=-1$ or indicated on diagram
	full convincing argument, but must have correct y_C and r	E1	2	Eg "highest point is at $y=-1$" scores E2 E1: showing no real solutions when $y=0$ +E1 stating centre or any point below x-axis
(c)(i)	$(PC^2=)\quad(5-2)^2+(k+6)^2$			ft their C coords
	$=9+k^2+12k+36$	M1		and attempt to multiply out
	$PC^2=k^2+12k+45$	A1	2	**AG** CSO (must see $PC^2=$ at least once)
(ii)	$\left.\begin{array}{l}PC>r\Rightarrow PC^2>25\\ \Rightarrow k^2+12k+20>0\end{array}\right\}$	B1	1	**AG** Condone $\left.\begin{array}{l}k^2+12k+45>25\\ \Rightarrow k^2+12k+20>0\end{array}\right\}$
(iii)	$(k+2)(k+10)$	M1		**Correct** factors or **correct** use of formula
	$k=-2,\ k=-10$ are critical values	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working.
	Use of sketch or sign diagram: 	M1		If previous A1 earned, sign diagram or sketch must be correct for M1, otherwise M1 may be earned for an attempt at the sketch or sign diagram using their critical values.
	$\Rightarrow k>-2,\ k<-10$	A1	4	$k\geqslant-2,\ k\leqslant-10$ loses final A mark
	Condone $k>-2$ OR $k<-10$ for full marks but not AND instead of OR Take final line as their answer			*Answer only* of $k>-2,\ k>-10$ *etc* scores M1, A1, M0 since the critical values are evident. *Answer only* of $k>2,\ k<-10$ *etc* scores M0, M0 since the critical values are not both correct.
	Total		13	
	TOTAL		75	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MPC1

Unit Pure Core 1

Monday 24 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

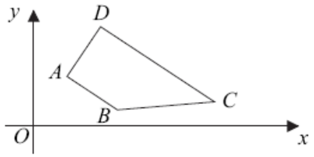
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The trapezium $ABCD$ is shown below.



The line AB has equation $2x + 3y = 14$ and DC is parallel to AB.

(a) Find the gradient of AB. *(2 marks)*

(b) The point D has coordinates $(3, 7)$.

(i) Find an equation of the line DC. *(2 marks)*

(ii) The angle BAD is a right angle. Find an equation of the line AD, giving your answer in the form $mx + ny + p = 0$, where m, n and p are integers. *(4 marks)*

(c) The line BC has equation $5y - x = 6$. Find the coordinates of B. *(3 marks)*

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2 (a) Express $\left(3 - \sqrt{5}\right)^2$ in the form $m + n\sqrt{5}$, where m and n are integers. *(2 marks)*

(b) Hence express $\dfrac{\left(3 - \sqrt{5}\right)^2}{1 + \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers. *(4 marks)*

---

3 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

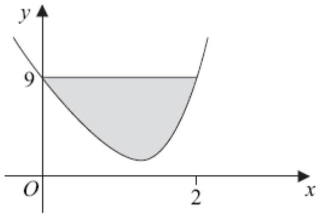
(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 2$. *(2 marks)*

(c) (i) Verify that $\mathrm{p}(-1) < \mathrm{p}(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

4 The curve with equation $y = x^4 - 8x + 9$ is sketched below.



The point (2, 9) lies on the curve.

(a) (i) Find $\int_0^2 (x^4 - 8x + 9)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

(b) The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i) Find the gradient of the curve at the point A. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

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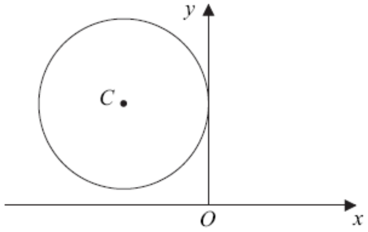
5 A circle with centre $C(-5, 6)$ touches the y-axis, as shown in the diagram.



(a) Find the equation of the circle in the form

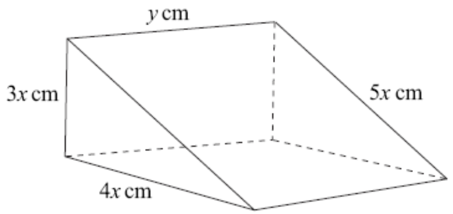
$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) (i) Verify that the point $P(-2, 2)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point P. *(3 marks)*

(iii) The mid-point of PC is M. Determine whether the point P is closer to the point M or to the origin O. *(4 marks)*

6 The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is 144 cm^2.

(a) (i) Show that $xy + x^2 = 12$. *(3 marks)*

(ii) Hence show that the volume of the block, V cm^3, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Show that V has a stationary value when $x = 2$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

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7 (a) (i) Express $2x^2 - 20x + 53$ in the form $2(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. *(2 marks)*

(b) The quadratic equation $(2k - 1)x^2 + (k + 1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leqslant 0$. *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - June 2010 – Answers

Question 1:	**Exam report**
$AB: 2x+3y=14$ a) $3y=-2x+14$ $\quad y=-\frac{2}{3}x+\frac{14}{3}$ The gradient of AB is $-\frac{2}{3}$. b) i) $D(3,7)$ and Dc is paralle to AB so $m_{DC}=m_{AB}=-\frac{2}{3}$ The equation of DC: $y-7=-\frac{2}{3}(x-3)$ $3y-21=-2x+6$ $2x+3y=27$ ii) The line AD is perpendicular to AB so $m_{AD}=-\frac{1}{m_{AB}}=\frac{3}{2}$ The eqation of AD: $y-7=\frac{3}{2}(x-3)$ $2y-14=3x-9$ $3x-2y+5=0$ c) The point B is the intersection of AB and BC. Solve simultaneously $\begin{cases}2x+3y=14\\-x+5y=6\end{cases}$ $\quad(\times 2)\quad$ $\begin{cases}2x+3y=14\\-2x+10y=12\end{cases}$ $13y=26$ so $y=2$ and $x=5y-6=10-6=4$ The coordinates of B are $(4,2)$.	Part (a) Many candidates were unable to make *y* the subject of the equation 2*x*+3*y* =14 and, as a result, many incorrect answers for the gradient were seen. Those who tried to use two points on the line to find the gradient were rarely successful. Part (b)(i) Those candidates who obtained a value for the gradient in part (a) were usually aware that the line *DC* had the same gradient. Those using *y=mx+c* often made errors when finding the value of *c*, whereas those writing down an equation of the form $y - y_1 = m(x - x_1)$ usually scored full marks. Part (b)(ii) Most candidates realised that the product of the gradients of perpendicular lines should be –1 and credit was given for using this result together with their answer from part (a). Careless arithmetic prevented many from obtaining the final equation in the given form with integer coefficients. Part (c) Although many correct answers for the coordinates of *B* were seen, the simultaneous equations defeated a large number of candidates. No credit was given for mistakenly using their equation from part (b)(i) or part (b)(ii) instead of the correct equation for *AB,* clearly printed below the diagram. Many did not recognize the need to use the equation of *AB* at all. It was common to see *x* = 0 or *y* = 0 substituted into the equation for *BC* and then solved to obtain the other coordinate.

Question 2:	**Exam report**
a) $\left(3-\sqrt{5}\right)^2=9+5-6\sqrt{5}=14-6\sqrt{5}$ b) $\frac{(3-\sqrt{5})^2}{1+\sqrt{5}}=\frac{(3-\sqrt{5})^2}{1+\sqrt{5}}\times\frac{1-\sqrt{5}}{1-\sqrt{5}}=\frac{\left(14-6\sqrt{5}\right)\left(1-\sqrt{5}\right)}{1-5}$ $=\frac{14-14\sqrt{5}-6\sqrt{5}+30}{-4}=\frac{44-20\sqrt{5}}{-4}$ $=-11+5\sqrt{5}$	Part (a) Many candidates were successful with this part, although sign errors and arithmetic slips were common. Part (b) Most candidates recognised the first crucial step of multiplying the numerator and denominator by $1-\sqrt{5}$ and many obtained $\frac{44-20\sqrt{5}}{-4}$, but then inaccurate evaluation of the numerator or poor cancellation led to many failing to obtain the correct final answer.

Question 3:

$p(x) = x^3 + 7x^2 + 7x - 15$

$a)\,i)\; p(-3) = (-3)^3 + 7\times(-3)^2 + 7\times(-3) - 15$

$= -27 + 63 - 21 - 15 = 63 - 63 = 0$

-3 is a root of p, so $(x+3)$ is a foacto of p

$ii)\; x^3 + 7x^2 + 7x - 15 = (x+3)(x^2+4x-5)$

$= (x+3)(x+5)(x-1)$

$b)$ The remainder of the division by $(x-2)$ is p(2)

$p(2) = 2^3 + 7\times 2^2 + 7\times 2 - 15$

$p(2) = 8 + 28 + 14 - 15 = 35$

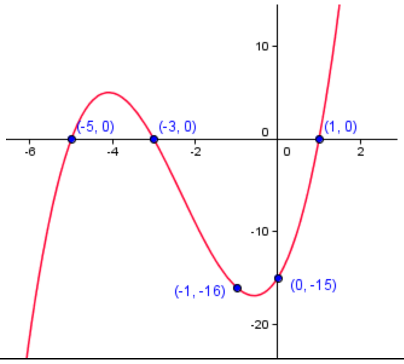
$c)\,i)\; p(-1) = -1 + 7 - 7 - 15 = -16$

$p(0) = -15$

$p(-1) < p(0)$

$ii)$ The curve crosses the $x - axis$ at $(-3,0), (-5,0), (1,0)$

The curve crosses the $y - axis$ at $(0,-15)$



Exam report

Part (a)(i) Those who used long division instead of the Factor Theorem scored no marks. Most candidates realised the need to show that p(–3) = 0. However quite a few omitted sufficient working such as p(–3) = –27 + 63 –21 –15 = 0, together with a concluding statement about x + 3 being a factor, and therefore failed to score full marks.
Part (a)(ii) Many candidates have become quite skilled at writing down the correct product of a linear and quadratic factor and then writing p(x) as the product of three linear factors and these scored full marks. Others used long division or the Factor Theorem effectively but lost a mark for failing to write p(x) as a product of linear factors. Others tried methods involving comparing coefficients, but often after several lines of working were unable to find the correct values of the coefficients because of poor algebraic manipulation. Speculative attempts to write down p(x) immediately as the product of three factors were rarely successful.
Part (b) Those candidates who used the Remainder Theorem were usually able to find the correct remainder, though once again arithmetic errors abounded. Those who used long division, synthetic division or other algebraic methods again scored no marks since the question specifically asked candidates to use the Remainder Theorem.
Part (c)(i) This part of the question was intended to help candidates when sketching the curve. Those who found the correct values of p(–1) and p(0) usually scored the mark but in future a carefully written proof may be called for. Here again many arithmetic errors were seen.
Part (c)(ii) The sketch was intended to bring various parts of the question together but even very good candidates ignored the hint from part (c)(i) and showed a minimum point on the y-axis. A few lost the final mark when their curve stopped on the x-axis. Confusion between roots and factors spoiled many sketches and several showed the y-intercept of –15 on the positive y-axis. It was disappointing that many candidates did not recognize the shape of a cubic curve at all.

Question 4:

$y = x^4 - 8x + 9$

$a)\,i)\; \int_0^2 (x^4 - 8x + 9)\,dx = \left[\frac{1}{5}x^5 - 4x^2 + 9x\right]_0^2$

$= \left(\frac{32}{5} - 16 + 18\right) - (0) = 8\tfrac{2}{5}$

$ii)$ The area of the shaded region is

area rectangle – area beneath the curve

$9\times 2 - 8\tfrac{2}{5} = 9\tfrac{3}{5}$

$b)\; A(1,2)$ lies on the curve.

$i)$ The gradient of the curve at a is $\frac{dy}{dx}(x=1)$.

$\frac{dy}{dx} = 4x^3 - 8 \;\; and$

$for\; x = 1, m_A = 4\times 1^3 - 8 = -4$

$ii)$ The equation of the tangent at A is :

$y - 2 = -4(x-1)$

$y = -4x + 6$

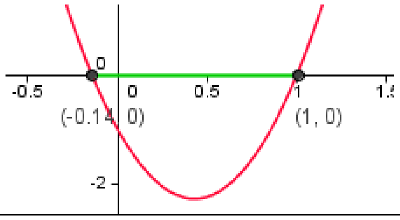
Exam report

Part (a)(i) Most candidates were able to integrate the expression with only the weakest candidates unable to do this basic integration. Poor notation was used with many including the integral sign after integrating. It would have been thought that this bad habit would have been corrected by the time of the examination. Many candidates did not find the actual value of the definite integral until part (a)(ii) and on this occasion full credit was given. It was alarming that many candidates who had correct fractions were unable to combine these to give a value of 8.4 or equivalent. Weaker candidates were seen substituting values into the expression for y or $\frac{dy}{dx}$ showing a complete lack of understanding.
Part (a)(ii) It was necessary to consider a rectangle of area 18 and then to subtract their answer from part (a)(i) in order to obtain the area of the shaded region. Many believed that the area of the rectangle was 9 and others failed to do this basic subtraction correctly, even when their answer to part (a)(i) was correct.

Part (b)(i) Many candidates did not realise the need to find $\frac{dy}{dx}$ before substituting the value x = 1 and thus failed to score some easy marks for finding the gradient of the curve. A substantial number of candidates tried to calculate the gradient of the straight line between two points on the curve and scored no marks for this.
Part (b)(ii) Unfortunately many candidates tried to find the equation of the normal instead of the tangent to the curve. Otherwise, since there was a generous follow through in this part of the question, most were able to score this final mark. The only exceptions were those who insisted on using y = mx + c where poor arithmetic often prevented them from finding a value for c.

Question 5:	Exam report
$a)$ Centre of the circle $C(-5,6)$ the circle touches the y-axis, the radius is 5. The equation of the circle is $(x+5)^2+(y-6)^2=5^2$ $b)\,i)$ $P(-2,2)$ $(-2+5)^2+(2-6)^2$ $=3^2+(-4)^2=9+16=25=5^2$ P lies on the circle. $ii)$ The normal to the circle at P is the line CP Gradient of $CP=m_{CP}=\frac{2-6}{-2+5}=\frac{-4}{3}$ The equation of the normal is : $y-2=-\frac{4}{3}(x+2)$ $3y-6=-4x-8$ $4x+3y=-2$ $iii)$ The distance $PM=\frac{1}{2}r=2.5$ The distance $PO=\sqrt{(-2)^2+(2)^2}=\sqrt{8}=2\sqrt{2}>2.5$ $(\sqrt{2}\approx 1.4)$ The point P is closer to M.	Part (a) Most candidates obtained the correct left hand side of the circle equation but many failed to recognize that the radius was 5. Alarmingly many thought that r was equal to –5 or wrote down the right hand side of the equation as -5^2 , thus displaying a fundamental misunderstanding of the idea of radius as a length. Part (b)(i) Most who had the correct circle equation were able to verify that the circle passed through the point P, although those who neglected to make a statement as a conclusion to their calculation failed to earn this mark. Part (b)(ii) The negative signs caused problems for many when finding the gradient of PC and only the better candidates obtained the correct value. Many candidates then found the negative reciprocal of this fraction instead of using the gradient of PC to find the normal to the circle at the point P. Part (b)(iii) There were basically two approaches to this question, although some candidates were merely guessing and no credit was given for a correct answer without supporting working. The most common method involved distances or squares of distances; many made errors in finding the coordinates of M and then struggled with the fractions when squaring and adding to find the length of PM; whereas others noted that the length of PM was simply half the radius. A simple comparison with the length of PO led to the correct conclusion. The second approach was essentially one using vectors or the differences of coordinates, but this method was not always explained correctly and left examiners in some doubt as to whether candidates really understood what they were doing. The best candidates wrote down the correct vectors PM and OP and reasoned that these vectors had the same y-component but different x-components and it was then easy to deduce that P was closer to the point M.

Question 6:	Exam report
$a)\,i)$ The surface area is $S=\frac{1}{2}\times 3x\times 4x+\frac{1}{2}\times 3x\times 4x+5x\times y+4x\times y+3x\times y$ $S=12x^2+12xy=144cm^2 \quad (\div 12)$ $x^2+xy=12 \qquad (1)$ $ii)$ The volume $V=\left(\frac{1}{2}\times 3x\times 4x\right)\times y=6x^2y$ Making y the subject in (1): $y=\frac{12-x^2}{x}$ $Now,$ substituting y in V, we have $V=6x^2y=6x^2\times\frac{12-x^2}{x}=72x-6x^3$ $b)\,i)\ \frac{dV}{dx}=72-18x^2$ $ii)$ $When\ x=2,\ \frac{dV}{dx}=72-18\times 2^2=72-72=0$ There is a stationary point when $x=2$. $c)\ \frac{d^2V}{dx^2}=-36x\ \ and\ \ when\ x=2,\ \frac{d^2V}{dx^2}=-72<0$ $For\ \ x=2,$ The value of V is a MAXIMUM.	Part (a)(i) Usually after a few abortive attempts many candidates realised that they had to add together the areas of the various faces. Once they had the correct expression for the total surface area most candidates were able to obtain the printed result. There was clearly some fudging on the part of weaker candidates and they could earn little more than a single method mark. Part (a)(ii) This was surprisingly one of the biggest discriminators on the paper with only the best candidates being able to obtain the correct expression for V. Trying to make y the subject of the equation from part (a)(i) caused problems for many who took this approach; others substituted for xy in the formula V = 6x(xy) and were often more successful. Once again it was quite common to see totally incorrect expressions being miraculously transformed into the printed answer. Part (b)(i) Most candidates scored full marks for this basic differentiation although it was not always clearly identified as $\frac{dV}{dx}$ in their working. Part (b)(ii) Most substituted x = 2 into their expression for $\frac{dV}{dx}$ and found the value to be zero. It was then necessary to make a statement about the implication of there being a stationary value in order to score full marks. Part (c) Some made a sign error when finding the second derivative, but the majority of candidates scored full marks in this part. Credit was given if the correct conclusion was drawn from the sign of their second derivative, provided no further arithmetic errors occurred.

Question 7:	**Exam report**
a) i) $2x^2-20x+53=2\left(x^2-10x\right)+53$ $=2\left[(x-5)^2-25\right]+53$ $=2(x-5)^2-50+53$ $=2(x-5)^2+3$ ii) $2x^2-20x+53=0$ is equivalent to $2(x-5)^2+3=0$ or $\left(x-5\right)^2=-\frac{3}{2}$ For all x, $(x-5)^2\geq 0$ so there is no solution. b) $(2k-1)x^2+(k+1)x+k=0$ has real roots, This means that the discriminant ≥ 0 i.e $\left(k+1\right)^2-4\times(2k-1)\times(k)\geq 0$ $k^2+2k+1-8k^2+4k\geq 0$ $-7k^2+6k+1\geq 0 \qquad (\times -1)$ $7k^2-6k-1\leq 0$ iii) $7k^2-6k-1\leq 0$ $\left(7k+1\right)\left(k-1\right)\leq 0$ critical values $-\frac{1}{7}$ and 1 $\left(7k+1\right)\left(k-1\right)\leq 0$ for $-\frac{1}{7}\leq k\leq 1$ 	Part (a)(i) Candidates did not seem well drilled in completing the square when the coefficient of x^2 is not equal to 1. It was very rare to see a correct answer here although a few did realise that p = 5. Clearly further practice is required at this type of question. Part (a)(ii) Only the more able candidates were able to reason sufficiently well using the result from part (a)(i) as well as providing a concluding statement. No credit was given for using the discriminant to show that the equation had no real roots since the wording of the question excluded this approach. Part (b)(i) This kind of question has been set several times before and the usual errors were seen. Candidates should state the condition for real roots (b^2 - 4ac ≥ 0) and find an expression in terms of k for the discriminant using the correct inequality throughout. The inequality is then reversed when multiplying by a negative number. Again, many candidates would benefit from practising this technique, using brackets where appropriate to avoid algebraic errors. Part (b)(ii) The factorisation of the quadratic was usually correct, but several candidates wrote down one of the critical values as 1/7. Most found critical values and either stopped or immediately tried to write down a solution without any working. Candidates are strongly advised to use a sign diagram or a sketch graph showing their critical values when solving a quadratic inequality.

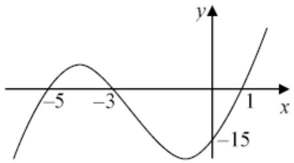
GRADE BOUNDARIES

Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	63	55	47	40	33

MY SCORE FOR THIS PAPER IS ..... / 75

MY GRADE : ....

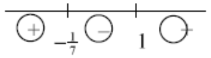
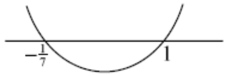
Q	Solution	Marks	Total	Comments
1(a)	$y=\frac{14}{3}-\frac{2}{3}x$	M1		Attempt at $y=\ldots$
	Gradient $AB=-\frac{2}{3}$	A1	2	Condone error in rearranging equation
(b)(i)	$y-7=\text{"}\textit{their grad } AB\text{"}(x-3)$	M1		or $2x+3y=k$ and sub $x=3, y=7$ or $y=mx+c$, $m=$ *their grad AB* and attempt to find c using $x=3, y=7$
	$y-7=-\frac{2}{3}(x-3)$ OE	A1	2	$2x+3y=27$, $y=-\frac{2}{3}x+9$ etc
(ii)	$m_1m_2=-1$	M1		or *negative reciprocal* (stated or used PI)
	$\Rightarrow$ grad $AD=\frac{3}{2}$	A1$\sqrt{}$		FT their grad AB
	$y-7=\frac{3}{2}(x-3)$	A1		Any correct equation unsimplified
	$\Rightarrow 3x-2y+5=0$	A1	4	Integer coefficients; all terms on one side, condone different order or multiples. eg $0=4y-6x-10$
(c)	$2x+3y=14$ and $5y-x=6$ used with x or y eliminated (generous)	M1		$2(5y-6)+3y=14$ etc
	$x=4$,	A1		
	$y=2$	A1	3	$B(4,2)$ full marks NMS
	Total		**11**	
2(a)	$\left(3-\sqrt{5}\right)^2=9-6\sqrt{5}+\left(\sqrt{5}\right)^2$	M1		Allow one slip in one of these terms M0 if middle term is omitted
	$=14-6\sqrt{5}$	A1	2	
(b)	$\frac{\left(3-\sqrt{5}\right)^2}{1+\sqrt{5}}\times\frac{1-\sqrt{5}}{1-\sqrt{5}}$	M1		or $\ldots\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$
	$14+6\sqrt{5}\sqrt{5}-6\sqrt{5}-14\sqrt{5}$ $(=44-20\sqrt{5})$	m1		Expanding ***their*** numerator (condone one error or omission)
	(Denominator) $=-4$	B1		Must be seen as denominator
	(Answer) $=-11+5\sqrt{5}$	A1	4	Accept "answer $=5\sqrt{5}-11$"
	Total		**6**	

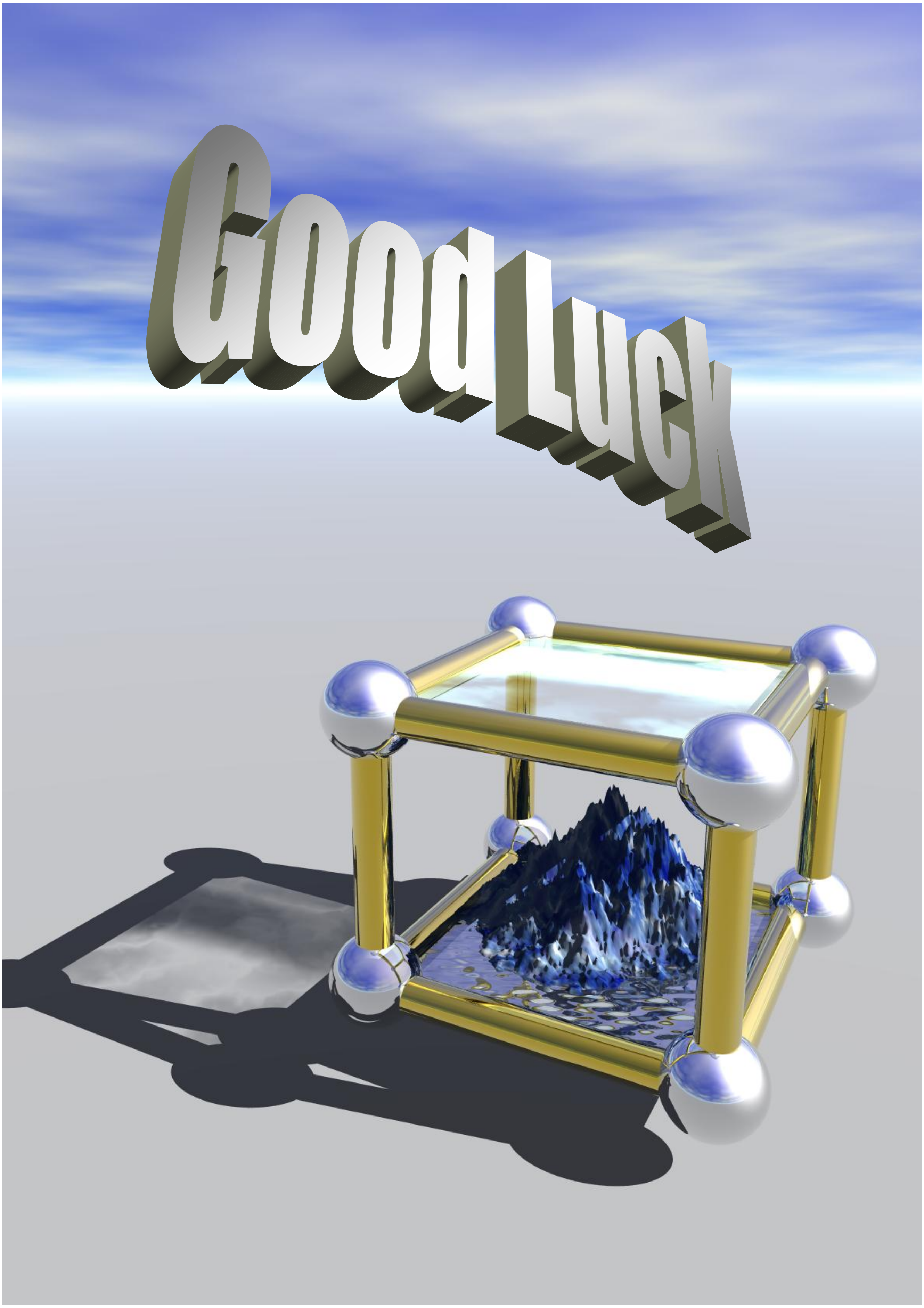
Q	Solution	Marks	Total	Comments
3(a)(i)	$\mathrm{p}(-3)=(-3)^3+7(-3)^2+7(-3)-15$ $=-27+63-21-15$	M1		$\mathrm{p}(-3)$ attempted; NOT long division This line alone implies M1
	$\mathrm{p}(-3)=0 \Rightarrow (x+3$ is) factor	A1	2	$\mathrm{p}(-3)$ shown $=0$ plus statement
(ii)	$\mathrm{p}(x)=(x+3)\left(x^2+px+q\right)$	M1		Full long division, comparing coefficients or by inspection either $p=4$ or $q=-5$
	(Quadratic factor) $\left(x^2+4x-5\right)$	A1		or M1 A1 for either $x-1$ or $x+5$ ***clearly*** found using Factor Theorem
	$(\mathrm{p}(x)=)$ $(x+3)(x-1)(x+5)$	A1	3	Must be seen as a product of 3 factors NMS full marks for correct product SC B2 for 3 correct factors listed NMS SC B1 for $(x+3)(x-1)(\ )$ or $(x+3)(x+5)(\ )$ or $(x+3)(x+1)(x-5)$
(b)	$\mathrm{p}(2)=2^3+7\times2^2+7\times2-15$ or $(2+3)(2-1)(2+5)$	M1		NOT long division; must be $\mathrm{p}(2)$ May use "their" product of factors
	(Remainder) $=35$	A1cso	2	
(c)(i)	$\mathrm{p}(-1)=-16$; $\mathrm{p}(0)=-15$ $\Rightarrow \mathrm{p}(-1)<\mathrm{p}(0)$	B1	1	Values must be evaluated correctly
(ii)		B1		y- intercept -15 marked or $(0,-15)$ stated
		M1		Cubic graph – 1 max, 1 min
		A1		∿ shape with –5, –3, 1 marked
	Cannot score M1A0A1 but can score B0M1A1A1	A1	4	Graph correct with minimum point to left of y-axis and going beyond both –5 and 1 Previous A1 must be scored
	Total		**12**	

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{x^5}{5}-\frac{8}{2}x^2+9x$	M1 A1 A1		One term correct Another term correct All correct (may have + c)
	$\frac{32}{5}-16+18$	m1		$\mathrm{F}(2)$ attempted
	$=8\frac{2}{5}$	A1	5	$\frac{42}{5}$, 8.4
(ii)	Shaded area = 18 – '*their integral*' $= 9\frac{3}{5}$	M1 A1	2	PI by 18 – (a)(i) NMS $\frac{48}{5}$, 9.6 NMS full marks
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=4x^3-8$	M1 A1		One term correct All correct (no + c etc)
	$x=1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=4-8$	m1		sub $x=1$ into *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$
	(Gradient of curve) = -4	A1cso	4	No ISW
(ii)	$y-2=-4(x-1)$; $y=-4x+c,\ c=6$	B1√	1	any correct form ; FT *their* answer from (b)(i) but must use $x = 1$ and $y = 2$
	Total		**12**	

Q	Solution	Marks	Total	Comments
5(a)	$(x+5)^2+(y-6)^2=5^2$	M1 A1 B1	3	One term correct LHS LHS all correct RHS correct: condone = 25
(b)(i)	sub $x=-2,\ y=2$ into circle equation $3^2+(-4)^2=25$ $\Rightarrow$ lies on circle	B1	1	Circle equation must be correct Must have concluding statement
(ii)	Grad PC $=-\frac{4}{3}$	B1		Condone $\frac{4}{-3}$
	Normal to circle has equation $y-6=$ '*their gradient PC*'$(x+5)$ or $y-2=$ '*their gradient PC*'$(x+2)$	M1		M0 if tangent attempted or incorrect coordinates used
	$y-6=-\frac{4}{3}(x+5)$ or $y-2=-\frac{4}{3}(x+2)$	A1cso	3	Any correct form eg $4x+3y+2=0$ $y=-\frac{4}{3}x+c,\quad c=-\frac{2}{3}$
				Alternative 1
(iii)	$PM=\frac{1}{2}\times$radius	M1		Attempt at $M\left(-\frac{7}{2},4\right)$ with at least one correct coordinate **and** PM^2 attempted
	$=2.5$	A1cso		$PM^2=\frac{9}{4}+4=\frac{25}{4}$
	$PO=\sqrt{8}$	B1		$PO^2=4+4=8$
	P is closer to the point M	E1cso	4	Statement following correct values
				Alternative 2
		(M1)		Attempt at $M\left(-\frac{7}{2},4\right)$ with at least one correct coordinate **and** attempt at vectors or difference of coordinates
		(A1cso)		$\overrightarrow{PM}=\begin{pmatrix}-1.5\\2\end{pmatrix}$ OE
		(E1cso)		P is closer to the point M
		(E1)	(4)	Components of their $\overrightarrow{PM}$ and $\overrightarrow{OP}$ considered – ***totally independent*** of M1
	Total		**11**	

Q	Solution	Marks	Total	Comments
6(a)(i)	S.A. $= 4xy + 5xy + 3xy + 6x^2 + 6x^2$ OE	M1		Condone one slip or omission
	$= 12xy + 12x^2$	A1		
	$144 = 12xy + 12x^2$			Must see this line
	$\Rightarrow xy + x^2 = 12$	A1cso	3	AG
(ii)	(Volume =) $\frac{1}{2} \times 3x \times 4x \times y$ OE	M1		
	$= 6x^2 \times \frac{(12 - x^2)}{x}$			Must see $(y =)\frac{(12-x^2)}{x}$ or $xy = 12 - x^2$ for A1
	$(V =) 72x - 6x^3$	A1	2	AG must be convinced not working back from answer
(b)(i)	$\frac{dV}{dx} = 72 - 18x^2$	M1 A1	2	One term correct All correct (no + c etc)
(ii)	$x = 2 \Rightarrow \frac{dV}{dx} = 72 - 18 \times 2^2$	M1		Substitute $x = 2$ into their $\frac{dV}{dx}$
	$\Rightarrow \frac{dV}{dx} = 72 - 72 = 0$			
	$\Rightarrow$ stationary (value when $x = 2$)	A1	2	Shown = 0 plus statement Statement may appear first
(c)	$\frac{d^2V}{dx^2} = -36x$	B1√		FT their $\frac{dV}{dx}$
	$\frac{d^2V}{dx^2} = -72$ *or* when $x = 2 \Rightarrow \frac{d^2V}{dx^2} < 0$			
	$\Rightarrow$ maximum	E1√	2	FT their $\frac{d^2V}{dx^2}$ value when $x = 2$ with appropriate conclusion
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$2(x-5)^2$	B1		$p = 5$
	$+ 3$	B1	2	$q = 3$
(ii)	Stating both $(x-5)^2 \geqslant 0$ and $3 > 0$	M1		FT their p & q, but must have $q > 0$
	$\Rightarrow 2x^2 - 20x + 53 > 0$ *or* $2(x-5)^2 + 3 > 0$			
	$\Rightarrow 2x^2 - 20x + 53 = 0$ has no real roots	A1cso	2	Must have statement and correct p & q.
(b)(i)	$b^2 - 4ac = (k+1)^2 - 4k(2k-1)$	M1		Condone one slip (including x is one slip)
	$= -7k^2 + 6k + 1$	A1		Condone recovery from missing brackets
	real roots $\Rightarrow b^2 - 4ac \geqslant 0$			Their discriminant $\geqslant 0$ (in terms of k)
	$-7k^2 + 6k + 1 \geqslant 0$	B1√		Need not be simplified & may earn earlier
	$\Rightarrow 7k^2 - 6k - 1 \leqslant 0$	A1cso	4	AG (must see sign change)
(ii)	$(7k+1)(k-1)$	M1		**Correct** factors or **correct** use of formula
	Critical values $k = 1, -\frac{1}{7}$	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working.
	Use of sign diagram or sketch   	M1		If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise M1 may be earned for an attempt at the sketch or sign diagram using ***their*** critical values.
	$-\frac{1}{7} \leqslant k \leqslant 1$	A1	4	$\left(-\frac{1}{7} < k < 1\right)$, $\left(k \geqslant -\frac{1}{7} \text{ OR } k \leqslant 1\right)$, $\left(k \geqslant -\frac{1}{7}, k \leqslant 1\right)$ score M1A1M1A0
	Full marks for correct answer NMS Condone $-\frac{2}{14}$ throughout Condone $k \geqslant -\frac{1}{7}$ AND $k \leqslant 1$ for full marks ***Take their final line as their answer.***			*Answer only of* $k < -\frac{1}{7}$, $k < 1$ etc scores M1, A1, M0 since the critical values are evident. *Answer only of* $\frac{1}{7} \leqslant k \leqslant 1$ etc scores M0, M0 since the critical values are not both correct.
	Total		**12**	
	TOTAL		**75**	


Good Luck