Quadratic functions

VOCABULARY

- A quadratic function can be written $f(x) = ax^2 + bx + c$, where $a, b,$ *and* c *are* 3 *real numbers.*
- The graph of a quadratic function is called a PARABOLA

  if $a > 0$, the parabola is $\cup$-shaped

  if $a < 0$, the parabola is $\cap$-shaped
- The maximum or minimum point of the parabola is called the VERTEX.
- The parabola is symmetrical around the 'vertical' line going through the vertex.

COMPLETED SQUARE FORM

- $ax^2 + bx + c$ can be re-arrange into $a(x + p)^2 + q$.

  This is the completed square form
- The vertex of the parabola is $V(-p, q)$.
- The axis of symmetry of the parabola has equation $x = -p$.
- Transformation: $y = x^2$ *is mapped onto* $y = (x + p)^2 + q$ *by a* translation with vector $\begin{bmatrix} -p \\ q \end{bmatrix}$

QUADRATIC EQUATIONS

A quadratic equation can be written $ax^2 + bx + c = 0$

- The discriminant is the value of the expression $b^2 - 4ac$.

if $b^2 - 4ac < 0$, there is no solution.

if $b^2 - 4ac = 0$, there is a repeated/double root.

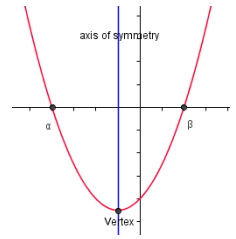
if $b^2 - 4ac > 0$, there are two solutions/roots: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- The roots are the values of x, where the parabola crosses the $x-axis$.

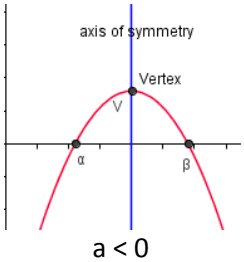
$b^2 - 4ac > 0$

$f(x) = ax^2 + bx + c$

$f(x) = a(x - \alpha)(x - \beta)$



a > 0



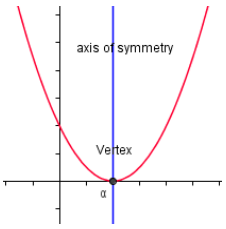
a < 0

$b^2 - 4ac = 0$

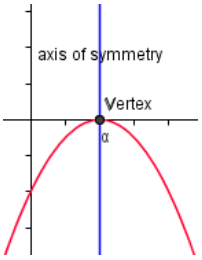
$f(x) = ax^2 + bx + c$

$f(x) = a(x - \alpha)^2$

The parabola is TANGENT to the x-*axis*



a > 0



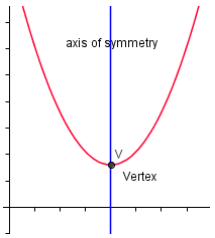
a < 0

$b^2 - 4ac < 0$

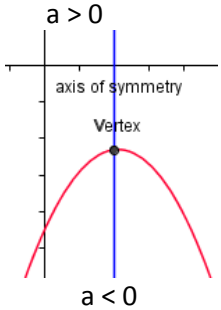
$f(x) = ax^2 + bx + c$

$f(x)$ *can't be factorised*

The parabola does not cross the x-*axis*.



a > 0



a < 0