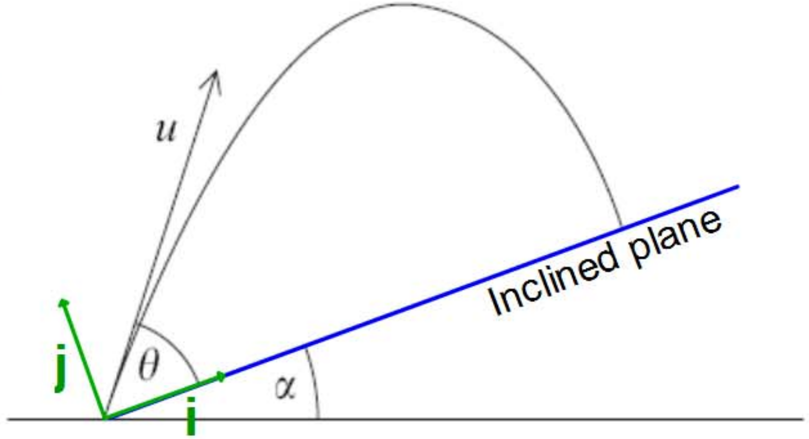
Projectile - part 2

Specifications

Projectiles on Inclined Planes

Projectiles launched onto inclined planes.	Problems will be set on projectiles that are launched and land on an inclined plane. Candidates may approach these problems by resolving the acceleration parallel and perpendicular to the plane. Questions will be set which require the use of trigonometric identities, but any identities which are needed for questions on this topic will be quoted if they are not included in Core 2 or the preamble to the specification. Candidates will be expected to find the maximum range for a given slope and speed of projection. Candidates may be expected to determine whether a projectile lands at a higher or lower point on the plane after a bounce.

This chapter is an extension of the previous chapter.



(The projectile is a particule of mass m. The resistance of the air is negligeable. The only force apllied on the particule is the gravity)

The projectile is launched with an angle θ and an initial speed u from an inclined plane (angle α with the horizontal)

To establish the equation of the trajectory, we introduce a set of unit vectors (**i** , **j**)

i is parallel to the inclined plane.

j is perpendicular to **i**

The relationships between the vectors **a**, **v** and **s** established in part 1 are still valid in this situation.

BUT the component of the vector **a** are different.

According to Newton's law : $\mathbf{F} = m\mathbf{a}$

The only force applied to the particle is gravity

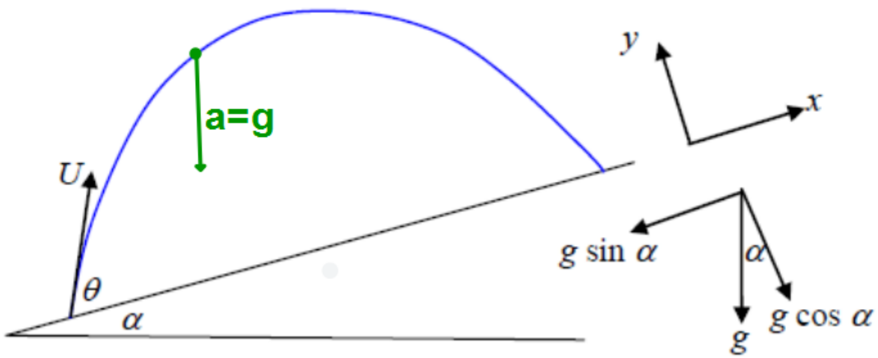
so $\quad \mathbf{F} = m\mathbf{g} = m\mathbf{a}$ *therefore* $\quad \mathbf{a} = \mathbf{g}$

We know that $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$ *therefore* $\quad \mathbf{v} = \mathbf{a}t + \mathbf{u}$

and $\mathbf{v} = \dfrac{d\mathbf{s}}{dt}$ *therefore* $\quad \mathbf{s} = \dfrac{1}{2}\mathbf{a}t^2 + \mathbf{u}t + \mathbf{s}_0$

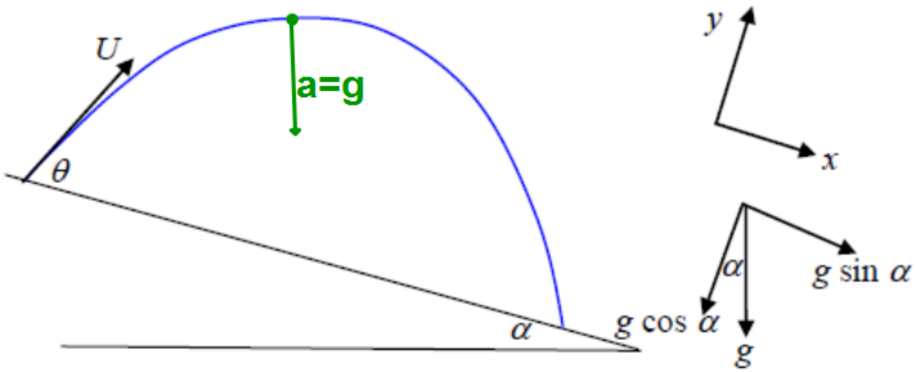
Components of the vectors

Upward planes



$$\mathbf{a} = \begin{pmatrix} -gSin(\alpha) \\ -gCos(\alpha) \end{pmatrix}$$

Downward planes



$$\mathbf{a} = \begin{pmatrix} gSin(\alpha) \\ -gCos(\alpha) \end{pmatrix}$$

But integrating, work out the components of **v** and **s**:

Upward planes:

$$\mathbf{v} = \mathbf{a}t + \mathbf{u}$$

$$\mathbf{v} = \begin{pmatrix} -gSin(\alpha)t + uCos(\theta) \\ -gCos(\alpha)t + uSin(\theta) \end{pmatrix}$$

$$\mathbf{s} = \frac{1}{2}\mathbf{a}t^2 + \mathbf{u}t \ (+\mathbf{s}_0)$$

$$\mathbf{s} = \begin{pmatrix} -\frac{1}{2}gSin(\alpha)t^2 + uCos(\theta)t \ \ (+x_0) \\ -\frac{1}{2}gCos(\alpha)t^2 + uSin(\theta)t \ \ (+y_0) \end{pmatrix}$$

Downward planes:

$$\mathbf{v} = \mathbf{a}t + \mathbf{u}$$

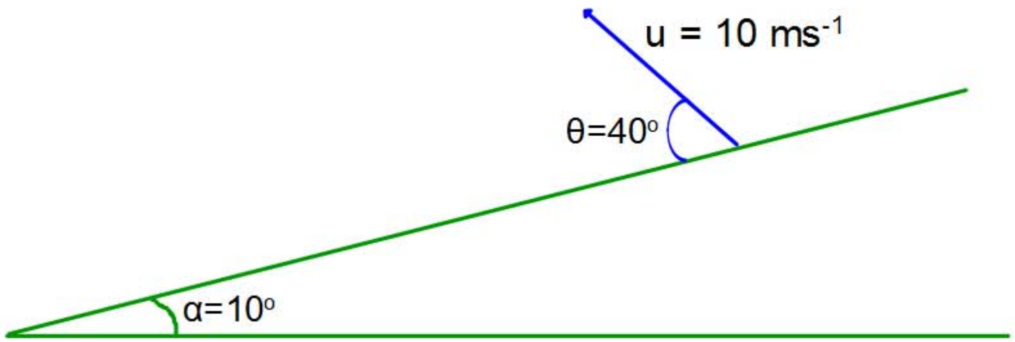
$$\mathbf{v} = \begin{pmatrix} gSin(\alpha)t + uCos(\theta) \\ -gCos(\alpha)t + uSin(\theta) \end{pmatrix}$$

$$\mathbf{s} = \frac{1}{2}\mathbf{a}t^2 + \mathbf{u}t \ (+\mathbf{s}_0)$$

$$\mathbf{s} = \begin{pmatrix} \frac{1}{2}gSin(\alpha)t^2 + uCos(\theta)t \ \ (+x_0) \\ -\frac{1}{2}gCos(\alpha)t^2 + uSin(\theta)t \ \ (+y_0) \end{pmatrix}$$

Also to consider...

A particle is launched at a 40° angle with an initial speed of $10ms^{-1}$ on a slope (10° above the horizontal) as shown on the diagram.

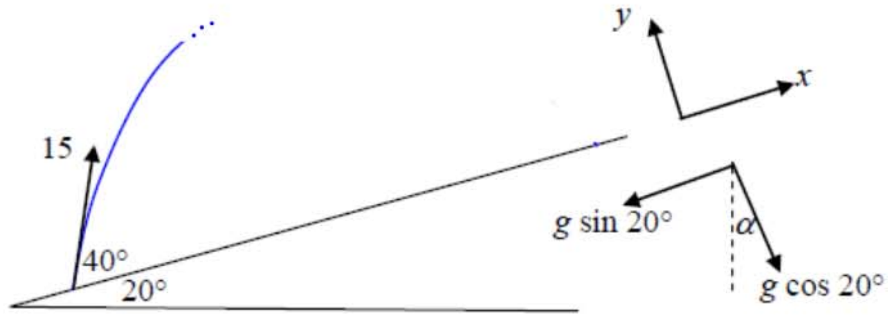


a) Work out the time of the flight.
b) work out the range on the slope.

Numerical examples

Example 1

A stone is thrown with initial speed 15 ms^{-1} onto a plane which is inclined at 20° above the horizontal. The stone is thrown at an angle of 40° above the plane.
How far from the point of projection does the stone hit the plane?



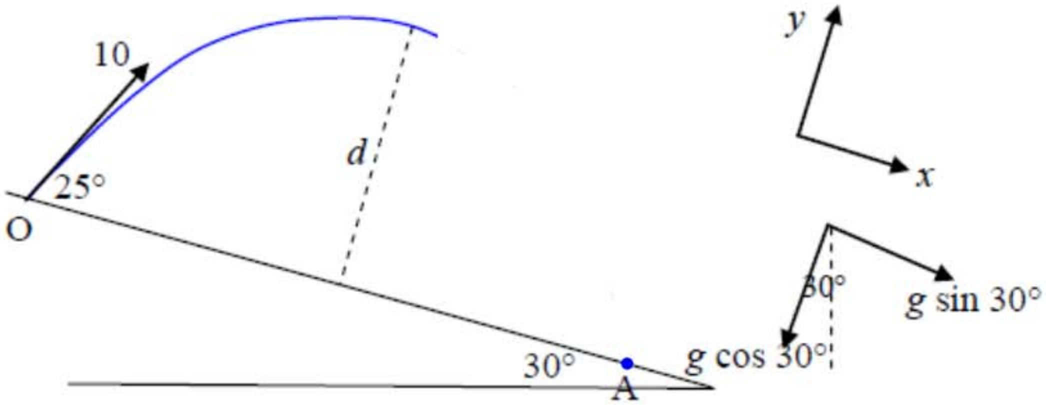
The stone hits the plane 16.7 m from its point of projection.

Example 2

A particle is thrown from a point O down a plane inclined at 30° to the horizontal. The particle has an initial speed of 10 ms^{-1} at 25° above the plane. It lands at a point A on the plane.

(i) Find the greatest perpendicular distance of the particle from the plane.

(ii) Find the distance OA.



i) d=1.05m ii)OA = 11.5m

Exam question:

A slope is inclined at an angle of 20° below the horizontal. A ball is projected at a speed of 30 m s^{-1} from the slope at an angle of 40° above the slope. The ball moves in a plane that contains the line of greatest slope of the plane.

(a) Find the time of flight of the ball, given that it moves down the slope. *(5 marks)*

(b) Find the range of the ball. *(4 marks)*

(c) Find the speed of the ball when it hits the slope, giving your answer correct to 2 significant figures. *(4 marks)*

a) $t = 4.188s$

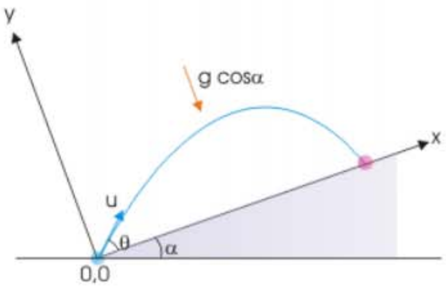
b) $x = 126m$

c) $v = 42ms^{-1}$

Multiple choice questions

EXERCISE 1

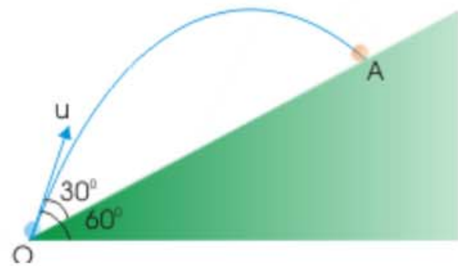
A projectile is thrown from the base of an incline of angle 30° as shown in the figure. It is thrown at an angle of 30° from the slope direction at a speed of 10 m/s.(Consider $g = 10\ m/s^2$) :



The total time of flight is : $(a)2 \quad (b)\sqrt{3} \quad (c)\frac{\sqrt{3}}{2} \quad (d)\frac{2}{\sqrt{3}}$

EXERCISE 2

A ball is projected on an incline of 30° from its base with a speed 20 m/s, making an angle 60° from the horizontal. The magnitude of the component of velocity, perpendicular to the incline, at the time ball hits the incline is :



$(a)10 \quad m/s \quad (b)10\sqrt{3} \quad m/s \quad (c)20\sqrt{3} \quad m/s \quad (d)20\sqrt{3} \quad m/s$

1)d) 2) a)