Projectiles-part 1 – exam questions

Question 1: June 2006 – Q5

A football is kicked from a point O on a horizontal football ground with a velocity of $20\,\text{m s}^{-1}$ at an angle of elevation of $30°$. During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

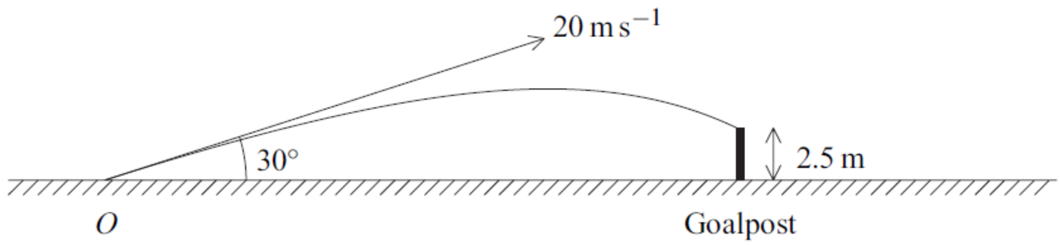
(a) Show that x and y satisfy the equation

$$y = x\tan 30° - \frac{gx^2}{800\cos^2 30°}$$

(6 marks)

(b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal.

(4 marks)



(c) State **two** modelling assumptions that you have made. *(2 marks)*

Question 2: June 2007

A ball is projected with speed $u\,\text{m s}^{-1}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$

(6 marks)

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P.

(i) By taking $g = 10\,\text{m s}^{-2}$, show that R satisfies the equation

$$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$$

(2 marks)

(ii) Hence, given that u and R are constants, show that, for $\tan\alpha$ to have real values, R must satisfy the inequality

$$R^2 \leqslant \frac{u^2(u^2 - 20)}{100}$$

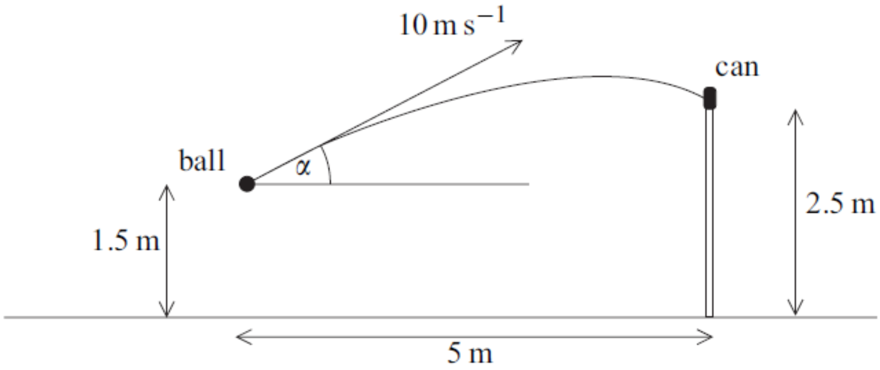
(2 marks)

(iii) Given that $R = 5$, determine the minimum possible speed of projection.

(3 marks)

Question 3: June 2008 – Q5

A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity $10\,\text{m s}^{-1}$ at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



(a) Show that α satisfies the equation

$$49\tan^2\alpha - 200\tan\alpha + 89 = 0$$ *(7 marks)*

(b) Find the **two** possible values of α, giving your answers to the nearest 0.1°. *(3 marks)*

(c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than $8\,\text{m s}^{-1}$.

Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. *(3 marks)*

(ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. *(4 marks)*

Question 4: June 2009 – Q2

A particle is projected from a point O on a horizontal plane and has initial velocity components of $2\,\text{m s}^{-1}$ and $10\,\text{m s}^{-1}$ parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x$$ *(4 marks)*

(b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. *(4 marks)*

(c) Hence find the time for which the particle is more than 1 metre above the plane. *(2 marks)*

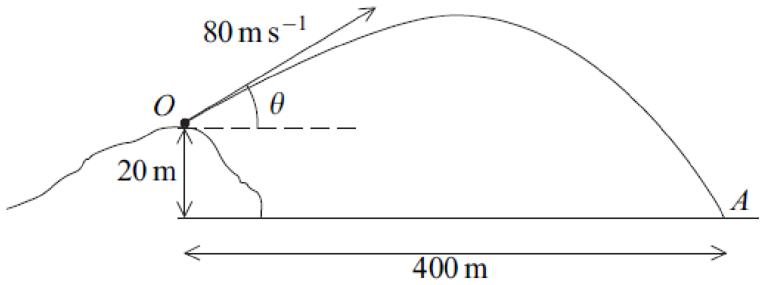
Question 5: June 2010 – Q2

2 A projectile is fired from a point O on top of a hill with initial velocity $80\,\text{m s}^{-1}$ at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{gx^2}{12\,800}(1 + \tan^2\theta)$$ *(5 marks)*

(ii) The projectile hits a target A, which is 20 m vertically below O and 400 m horizontally from O.



Show that

$$49\tan^2\theta - 160\tan\theta + 41 = 0$$ *(2 marks)*

(b) (i) Find the two possible values of θ. Give your answers to the nearest 0.1°. *(3 marks)*

(ii) Hence find the shortest possible time of the flight of the projectile from O to A. *(2 marks)*

(c) State a necessary modelling assumption for answering part **(a)(i)**. *(1 mark)*

Question 6: June 2011 – Q3

(In this question, use $g = 10\,\text{m s}^{-2}$.)

A golf ball is hit from a point O on a horizontal golf course with a velocity of $40\,\text{m s}^{-1}$ at an angle of elevation θ. The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical distances of the golf ball from O are x and y metres respectively.

(a) Show that the equation of the trajectory of the golf ball during its flight is given by

$$x^2\tan^2\theta - 320x\tan\theta + (x^2 + 320y) = 0$$ *(6 marks)*

(b) (i) The golf ball hits the top of a tree, which has a vertical height of 8 m and is at a horizontal distance of 150 m from O.

Find the two possible values of θ. *(5 marks)*

(ii) Which value of θ gives the shortest possible time for the golf ball to travel from O to the top of the tree? Give a reason for your choice of θ. *(2 marks)*

Question 7: June 2012 – Q3

(In this question, take $g = 10\ \mathrm{m\,s^{-2}}$.)

A projectile is fired from a point O with speed u at an angle of elevation α above the horizontal so as to pass through a point P. The projectile travels in a vertical plane through O and P. The point P is at a horizontal distance $2k$ from O and at a vertical distance k above O.

(a) Show that α satisfies the equation

$$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0$$ *(7 marks)*

(b) Deduce that

$$u^4 - 20ku^2 - 400k^2 \geqslant 0$$ *(3 marks)*

Projectiles-part 1 – exam questions

Question 1: June 2006 – Q5

5(a)	$y=-\frac{1}{2}gt^2+20\sin 30.t$	M1A1	
	$x=20\cos 30.t$	M1	
	$t=\frac{x}{20\cos 30}$	A1	
	$y=-\frac{1}{2}g\frac{x^2}{400\cos^2 30}+20\sin 30\frac{x}{20\cos 30}$	M1	
	$y=x\tan 30-\frac{gx^2}{800\cos^2 30°}$	A1	6
(b)	$2.5=x\tan 30-\frac{9.8x^2}{800\cos^2 30}$		
	$9.8x^2-346x+1500=0$	M1A1	
	$x=\frac{346\pm\sqrt{119716-58800}}{19.6}$	M1	
	=30.3 (or 30.2) & 5.06 (or 5.05)		
	answer: 30.3m (or 30.2m)	A1F	4
(c)	no air resistance,	B1	
	the ball is a particle	B1	2
	etc.		
	Total		**12**

Question 2: June 2007 – Q5

5(a)	$y=ut\sin\alpha-\frac{1}{2}gt^2$	M1 A1	
	$x=ut\cos\alpha$	M1	
	$t=\frac{x}{u\cos\alpha}$	A1	
	$y=u\left(\frac{x}{u\cos\alpha}\right)\sin\alpha-\frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	M1	
	$y=x\tan\alpha-\frac{gx^2}{u^2\cos^2\alpha}$		
	$y=x\tan\alpha-\frac{gx^2}{2u^2}(1+\tan^2\alpha)$	A1	6
b)(i)	$1=R\tan\alpha-\frac{10R^2}{2u^2}(1+\tan^2\alpha)$	M1	
	$5R^2\tan^2\alpha-u^2R\tan\alpha+5R^2+u^2=0$	A1	2
(ii)	For real solutions of the quadratic :		
	$u^4R^2-20R^2(5R^2+u^2)\geq 0$	M1	
	$R^2\leq\frac{u^4-20u^2}{100}$		
	$R^2\leq\frac{u^2(u^2-20)}{100}$	A1	2
(iii)	$5^2\leq\frac{u^2(u^2-20)}{100}$		
	$u^4-20u^2-2500\geq 0$	M1	
	${u_{min}}^2=61.0$ (or $10+\sqrt{2600}$)	A1	
	$u_{min}=7.81$	A1F	3
	Total		**13**

Question 3: June 2008 – Q5

5(a)	$5=10\cos\alpha t$	M1	
	$t=\frac{5}{10\cos\alpha}$	A1	
	$1=-\frac{1}{2}(9.8)t^2+10\sin\alpha t$	M1A1	
	$1=-\frac{1}{2}(9.8)\frac{25}{100\cos^2\alpha}+10\sin\alpha\frac{5}{10\cos\alpha}$	m1	
	$1=-\frac{1}{2}(9.8)\frac{25}{100}(1+\tan^2\alpha)+10\sin\alpha\frac{5}{10\cos\alpha}$	A1	
	$49\tan^2\alpha-200\tan\alpha+89=0$	A1	7
(b)	$\tan\alpha=\frac{200\pm\sqrt{40000-4(49)(89)}}{2\times 49}$	M1	
	$=3.57, 0.508$	A1	
	$\alpha=74.4°, 26.9°$	A1F	3
(c)(i)	$10\cos 26.9°=8.92$ (or 8.91) >8		
	⇒ The can will be knocked off the wall	M1	
	$10\cos 74.4°=2.69<8$	A1F	
	⇒ The can will not be knocked off the wall	E1	3
		ALTERNATI The can will be $10\cos\alpha>8$ $\cos\alpha>0.8$ $\alpha<36.9°$ So, for $\alpha=26$ and for $\alpha=74.$	
5(c)(ii)	$x=ut$		
	$t=\frac{5}{10\cos 26.9°}$		
	$v=10\sin 26.9°-9.8(\frac{5}{10\cos 26.9°})$	M1	
	$v=-0.970$	A1F	
	$\tan\theta=\frac{-0.970}{8.92}$	M1	
	$\theta=-6.2°$		
	At an angle of depression of 6.2°	A1F	4
	Total		**17**

Question 4: June 2009 – Q2

2(a)	$x=2t$	M1	
	$y=-\frac{1}{2}gt^2+10t$	M1	
	$t=\frac{x}{2}$		
	$y=-\frac{1}{2}g\left(\frac{x}{2}\right)^2+10\left(\frac{x}{2}\right)$	m1	
	$y=-\frac{g}{8}x^2+5x$	A1	4
(b)	$1=-\frac{g}{8}x^2+5x$	M1	
	$gx^2-40x+8=0$		
	$x=\frac{40\pm\sqrt{(-40)^2-4\times 8g}}{2g}$	M1	
	$x=3.871,\ 0.211$	A1	
	Distance $=3.66$m	A1	4
(c)	$t=\frac{3.66}{2}$	M1	
	$t=1.83$ sec	A1	2
	Total		**10**

Question 5: June 2010 – Q2

2(a)(i)	$x = 80\cos\theta.\,t$	B1	
	$t = \dfrac{x}{80\cos\theta}$	B1	
	$y = 80\sin\theta.t - \dfrac{1}{2}gt^2$	B1	
	$y = 80\sin\theta\dfrac{x}{80\cos\theta} - \dfrac{1}{2}g(\dfrac{x}{80\cos\theta})^2$	M1	
	$y = x\tan\theta - \dfrac{gx^2}{12800}(1+\tan^2\theta)$	A1	5
(ii)	$-20 = 400\tan\theta - \dfrac{9.8\times400^2}{12800}(1+\tan^2\theta)$	M1	
	$122.5\tan^2\theta - 400\tan\theta + 102.5 = 0$ $49\tan^2\theta - 160\tan\theta + 41 = 0$	A1	2
(b)(i)	$\tan\theta = \dfrac{160\pm\sqrt{25600-4(49)(41)}}{2\times49}$	M1	
	$= 2.9850,\ 0.2803$	A1	
	$\theta = 71.5°,\ 15.7°$	A1F	3
(ii)	For the shortest time $400 = 80\cos15.7°.t$	M1	
	$t = 5.19$	AIF	2
(c)	• The projectile is a particle • The air resistance is negligible	E1	1
	Total		**13**

Question 6: June 2011 – Q3

3 (a)	$x = 40\cos\theta.t$	M1	
	$y = -\dfrac{1}{2}(10)t^2 + 40\sin\theta.t$	M1 A1	
	$y = -\dfrac{1}{2}(10)(\dfrac{x}{40\cos\theta})^2 + 40\sin\theta.(\dfrac{x}{40\cos\theta})$	m1	
	$y = -\dfrac{x^2}{320\cos^2\theta} + x\tan\theta$		
	$320y = -x^2(1+\tan^2\theta) + 320x\tan\theta$	m1	
	$x^2\tan^2\theta - 320x\tan\theta + (x^2+320y) = 0$	A1	6
(b)(i)	$150^2\tan^2\theta - 320(150)\tan\theta + (150^2 + 320\times8) = 0$	M1	
	$1125\tan^2\theta - 2400\tan\theta + 1253 = 0$	A1	
	$\tan\theta = \dfrac{2400\pm\sqrt{2400^2-4(1125)(1253)}}{2(1125)}$	m1	
	$\tan\theta = 1.22\ ,\ 0.912$	A1F	
	$\theta = 50.7°\ ,\ 42.4°$	A1F	5
(b)(ii)	$\theta = 42.4°$	B1F	
	$t = \dfrac{150}{40\cos\theta}$ and $\cos42.4 > \cos50.7$	E1	2
			13

Question 7: June 2012 – Q3

3(a)	$x = ut\cos\alpha$	M1	
	$t = \dfrac{x}{u\cos\alpha}$	A1	
	$y = -\dfrac{1}{2}gt^2 + ut\sin\alpha$	M1	
	$y = -\dfrac{1}{2}g(\dfrac{x}{u\cos\alpha})^2 + u(\dfrac{x}{u\cos\alpha})\sin\alpha$	M1	
	$y = -\dfrac{gx^2}{2u^2\cos^2\alpha} + \dfrac{x\sin\alpha}{\cos\alpha}$		
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\alpha) + x\tan\alpha$	A1	
	$k = -\dfrac{10(2k)^2}{2u^2}(1+\tan^2\alpha) + 2k\tan\alpha$	M1	
	$u^2 = -20k(1+\tan^2\alpha) + 2u^2\tan\alpha$		
	$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0$	A1	7
(b)	Pass through $P \Longrightarrow$ Discriminant ≥ 0		
	$(-2u^2)^2 - 4(20k)(u^2+20k) \geq 0$	M1A1	
	$4u^4 - 80ku^2 - 1600k^2 \geq 0$		
	$u^4 - 20ku^2 - 400k^2 \geq 0$	A1	3
	Total		**10**