Particles on a slope (with Friction)

Here, as in leaflet 2.7, particles on a slope are considered, but this time including friction.

Worked Example 1.

If a particle, of mass M kg, is on the point of slipping down a rough plane that is inclined at an angle θ to the horizontal, what is the coefficient of friction?

Solution

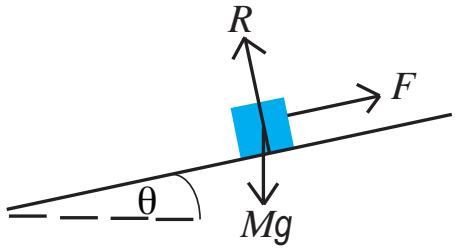

Figure 1

Resolving perpendicular to the plane:

$$R = Mg\cos\theta$$

Resolving parallel to the plane:

$$F = Mg\sin\theta$$

As the particle is on the point of slipping, friction is limiting, $F = F_{MAX}$, so:

$$\begin{aligned} F &= \mu R \\ Mg\sin\theta &= \mu \times Mg\cos\theta \\ \frac{Mg\sin\theta}{Mg\cos\theta} &= \mu \\ \tan\theta &= \mu \end{aligned}$$

Note: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

This can be written as $\theta = \arctan\mu$; θ is referred to as the **Angle of Friction**.

Worked Example 2.

A box of mass 6 kg is on the point of slipping down a rough slope, which is inclined at an angle 30° to the horizontal. A force S is applied to the box and acts up the plane. Given that the coefficient of friction is 0.45, what is the magnitude of S? (Figure 2 shows the forces acting on the box, when modelled as a particle.)

Solution

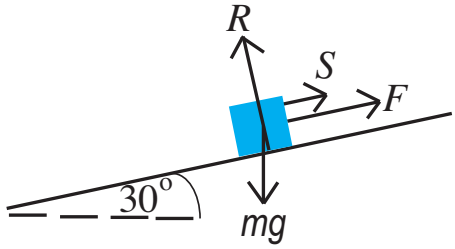

Figure 2

Resolving perpendicular to the plane:

$$R = 6g\cos 30°$$

Resolving parallel to the plane:

$$F = 6g\sin 30° - S$$

As the box is on the point of slipping, friction is limiting, $F = F_{MAX}$, so:

$$\begin{aligned} F &= \mu R \\ 6g\sin 30° - S &= 0.45 \times 6g\cos 30° \\ S &= 6g\sin 30° - 0.45 \times 6g\cos 30° = 6.5 \text{ N (2 s.f.)} \end{aligned}$$

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Worked Example 3.

A ski is dropped by a skier ascending a ski slope. The ski begins to slide down the slope, which is inclined at an angle 25° to the horizontal. Given the ski has a mass m kg and the coefficient of sliding friction between the ski and the slope is 0.21, what is the acceleration of the ski? (Figure 3 shows the forces acting on the ski, when modelled as a particle.)

Solution

As the motion is down the slope, the sum of the perpendicular forces equals zero.

Resolving perpendicular to the plane:

$$R = mg\cos 25°$$

As the ski is sliding, the frictional force is given by:

$$F = \mu R = 0.21 \times mg\cos 25°$$

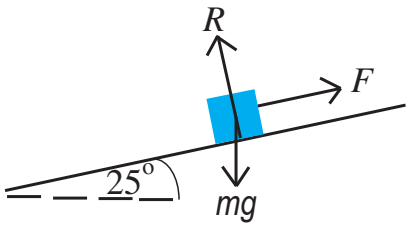

Figure 3

Use Newton's Second Law, parallel to the plane. The resultant force is $mg\sin 25° - F$:

$$\begin{aligned} mg\sin 25° - 0.21 \times mg\cos 25° &= ma \\ a = \quad g\sin 25° - 0.21 \times g\cos 25° &= 2.3 \text{ m s}^{-2} \text{ (2 s.f.)} \end{aligned}$$

Exercises

1. If a particle, of mass 11 kg, is on the point of slipping down a rough plane that is inclined at an angle 16° to the horizontal, what is the coefficient of friction?

2. A box of mass 5 kg is on the point of slipping down a rough slope, which is inclined at an angle 12° to the horizontal. A force S is applied to the box and acts up the plane. Given friction is acting and the coefficient of friction is 0.12, what is the magnitude of S?

3. A ski is dropped by a skier ascending a ski slope. The ski begins to slide down the slope, which is inclined at an angle 22° to the horizontal. Given the ski has a mass 2 kg and the coefficient of sliding friction between the ski and the slope is 0.19, what is the acceleration of the ski?

4. If a particle, of mass m kg, is on the point of slipping down a rough inclined plane that has a coefficient of friction of 0.14, what is the angle of the incline from the horizontal?

5. A box of mass 4 kg is on the point of slipping down a rough slope, which is inclined at an angle 40° to the horizontal. A force S of magnitude $\frac{1}{4}$ R (where R is the normal reaction force) is applied to the box and acts up the plane. Given friction is acting, what is the coefficient of friction?

6. A ski is dropped by a skier ascending a ski slope. The ski begins to slide down the slope, which is inclined at an angle 15° to the horizontal. Given the ski has a mass m kg and has an acceleration of 1.2 m s $^{-2}$, what is the coefficient of sliding friction between the ski and the slope?

Answers (all to 2 s.f.)

1. 0.29 2. 4.4 N 3. 1.9 m s $^{-2}$ 4. 8.0° 5. 0.59 6. 0.14

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