Differential equations – Numerical methods – Exam questions

Question: June 2007 Q2

The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \sqrt{x^2 + y^2 + 3}$$

and $$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(6 marks)*

Question: Jan 2006 Q5

(a) The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x\ln x + \frac{y}{x}$$

and $$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{x}y = x\ln x$$ *(3 marks)*

(ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

(iii) Calculate the value of y when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

Question: Jan 2007 Q1

The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \ln(1 + x^2 + y)$$

and $$y(1) = 0.6$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,f(x_r, y_r)$ and $k_2 = h\,f(x_r + h, y_r + k_1)$ and $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(6 marks)*

Question: Jun 2009 Q1

The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \sqrt{x^2 + y + 1}$$

and $$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,f(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to $y(3.2)$, giving your answer to three decimal places. *(3 marks)*

Question: June 2006 Q2

The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{x^2 + y^2}{xy}$$

and $$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(6 marks)*

Question: Jan 2009 Q1

The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{x^2 + y^2}{x + y}$$

and $$y(1) = 3$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.2$, to obtain an approximation to $y(1.2)$. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.2$, to obtain an approximation to $y(1.2)$, giving your answer to four decimal places. *(5 marks)*

Question: June 2008 Q1

The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \ln(x + y)$$

and $$y(2) = 3$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(2.1)$, giving your answer to four decimal places. *(6 marks)*

Question: Jan 2010 Q1

The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x\ln(2x + y)$$

and $$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,\mathrm{f}(x_r, y_r)$ and $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(5 marks)*

Differential equations – Numerical methods – Exam questions'answers

Question: June 2007 Q2

(a)	$y_1 = 2 + 0.1\times\sqrt{1^2+2^2+3}$	M1	
	$y(1.1) = 2 + 0.1\times\sqrt{8}$	A1	
	$y(1.1) = 2.28284\ldots = 2.2828$ to 4dp	A1	3
(b)	$k_1 = 0.1\times\sqrt{8} = 0.2828$	M1 A1ft	
	$k_2 = 0.1\times \mathrm{f}(1.1,\ 2.2828\ldots)$	M1	
	$= 0.1\times\sqrt{9.42137\ldots} = 0.3069(425\ldots)$	A1	
	$y(1.1) = y(1) + \frac{1}{2}[0.28284\ldots + 0.30694\ldots]$	m1	
	$2.29489\ldots = 2.2949$ to 4dp	A1	6
	Total		**9**

Question: Jan 2006 Q5

(a)(i)	$y(1.1) = y(1) + 0.1[1\ln 1 + 1/1]$	M1A1	
	$= 1 + 0.1 = 1.1$	A1	3
(ii)	$y(1.2) = y(1) + 2(0.1)[\mathrm{f}(1.1, y(1.1))]$	M1A1	
	$\ldots. = 1 + 2(0.1)[1.1\ln 1.1 + (1.1)/1.1]$	A1√	
	$\ldots. = 1 + 0.2\times 1.104841198\ldots.$		
	$\ldots. = 1.22096824 = 1.221$ to 3dp	A1	4
(b)(i)	IF is $e^{\int -\frac{1}{x}\,dx}$	M1	
	$= e^{-\ln x}$	A1	
	$= e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$	A1	3
(ii)	$\frac{d}{dx}\left(\frac{y}{x}\right) = \ln x$	M1A1	
	$\frac{y}{x} = \int \ln x\,dx = x\ln x - \int x\left(\frac{1}{x}\right)dx$	M1	
	$\frac{y}{x} = x\ln x - x + c$	A1	
	$y(1) = 1 \Rightarrow 1 = \ln 1 - 1 + c$	m1	
	$\Rightarrow c = 2 \Rightarrow y = x^2\ln x - x^2 + 2x$	A1	6
(iii)	$y(1.2) = 1.222543\ldots = 1.223$ to 3dp	B1	1
	Total		**17**

Question: Jan 2007 Q1

(a)	$y(1.05) = 0.6 + 0.05\times[\ln(1+1+0.6)]$ $= 0.6477(7557..) = 0.6478$ to 4dp	M1A1 A1	3
(b)	$k_1 = 0.05\times\ln(1+1+0.6) = 0.0477(75\ldots)$	M1 A1F	
	$k_2 = 0.05\times \mathrm{f}(1.05,\ 0.6477\ldots)$ $\ldots = 0.05\times\ln\left(1+1.05^2+0.6477\ldots\right)]$	M1	
	$\ldots = 0.0505(85\ldots)$	A1F	
	$y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$ $= 0.6 + 0.5\times 0.09836\ldots$	m1	
	$= 0.6492$ to 4dp	A1F	6
	Total		**9**

Question: Jun 2009 Q1

(a)	$y(3.1) = y(3) + 0.1\sqrt{3^2+2+1}$	M1A1	
	$= 2 + 0.1\times\sqrt{12} = 2.3464(10..)$ $= 2.3464$	A1	3
(b)	$y(3.2) = y(3) + 2(0.1)[\mathrm{f}(3.1, y(3.1))]$	M1	
	$\ldots. = 2 + 2(0.1)[\sqrt{(3.1^2+2.3464+1)}\ ]$	A1F	
	$\ldots. = 2 + 0.2\times 3.599499.. = 2.719(89..)$ $= 2.720$	A1	3
	Total		**6**

Question: June 2006 Q2

(a)	$y_1 = 2 + 0.1\times\left[\frac{1^2+2^2}{1\times 2}\right]$	M1 A1	
	$= 2 + 0.1\times 2.5 = 2.25$	A1	3
(b)	$k_1 = 0.1\times 2.5 = 0.25$	M1 A1√	
	$k_2 = 0.1\times \mathrm{f}(1.1,\ 2.25)$	M1	
	$\ldots = 0.1\times 2.53434\ldots = 0.2534(34\ldots)$	A1√	
	$y(1.1) = y(1) + \frac{1}{2}[0.25 + 0.253434\ldots]$	m1	
	$= 2.2517$ to 4dp	A1√	6
	Total		**9**

Question: Jan 2009 Q1

(a)	$y_1 = 3 + 0.2\times\left[\frac{1^2+3^2}{1+3}\right]$	M1A1	
	$= 3.5$	A1	3
(b)	$k_1 = 0.2\times 2.5 = 0.5$	B1ft	
	$k_2 = 0.2\times \mathrm{f}(1.2,\ 3.5)$	M1	
	$\ldots = 0.2\times\frac{1.2^2+3.5^2}{1.2+3.5} = 0.5825(53\ldots)$	A1ft	
	$y(1.2) = y(1) + \frac{1}{2}[0.5 + 0.5825(53\ldots)]$	m1	
	$= 3.54127\ldots = 3.5413$ to 4dp	A1ft	5
	Total		**8**

Question: June 2008 Q1

$k_1 = 0.1 \times \ln(2+3)$	M1	
$= 0.1609(4379\ldots)$ $(= *)$	A1	
$k_2 = 0.1 \times \mathrm{f}(2.1,\ 3 + *\ldots)$ $\ldots = 0.1 \times \ln(2.1 + 3.16094\ldots)]$	M1	
$\ldots. = 0.1660(31\ldots)$	A1	
$y(2.1) = y(2) + \frac{1}{2}[k_1 + k_2]$ $= 3 + 0.5 \times 0.3269748\ldots.$	m1	
$= 3.163487\ldots = 3.1635$ to 4dp	A1	6
Total		**6**

Question: Jan 2010 Q1

(a)	$y_1 = 2 + 0.1 \times [3\ln(2 \times 3 + 2)] = 2 + 0.3\ln 8$ $= 2.6238(3\ldots)$	M1A1	
	$y(3.1) = 2.6238$ (to 4dp)	A1	3
(b)	$k_1 = 0.1 \times 3\ln 8 = 0.6238(32\ldots)$	B1F	
	$k_2 = 0.1 \times \mathrm{f}(3.1,\ 2.6238(32\ldots))$	M1	
	$\ldots = 0.1 \times 3.1 \times \ln 8.8238(32..)$ $[= 0.6750(1\ldots)$	A1F	
	$y(3.1) = 2 + \frac{1}{2}[0.6238(3..) + 0.6750(1..)]$	m1	
	$= 2.6494(2\ldots) = 2.6494$ to 4dp	A1	5
	Total		**8**