Differential equations - Numerical methods - Exam questions

Question: June 2007 Q2

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \sqrt{x^2 + y^2 + 3}$$

and

$$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(1.1), giving your answer to four decimal places. (3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and h = 0.1, to obtain an approximation to y(1.1), giving your answer to four decimal places. (6 marks)

Question: Jan 2006 Q5

(a) The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x,y) = x \ln x + \frac{y}{x}$$

and

$$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(1.1).

(3 marks)

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h f(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to y(1.2), giving your answer to three decimal places. (4 marks)

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{x}y = x \ln x \tag{3 marks}$$

- (ii) Solve this differential equation, given that y = 1 when x = 1. (6 marks)
- (iii) Calculate the value of y when x = 1.2, giving your answer to three decimal places. (1 mark)

Question: Jan 2007 Q1

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = ln(1 + x^2 + y)$$

and

$$v(1) = 0.6$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with h = 0.05, to obtain an approximation to y(1.05), giving your answer to four decimal places. (3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h f(x_r, y_r)$ and $k_2 = h f(x_r + h, y_r + k_1)$ and h = 0.05, to obtain an approximation to y(1.05), giving your answer to four decimal places. (6 marks)

Question: Jun 2009 Q1

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \sqrt{x^2 + y + 1}$$

and

$$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(3.1), giving your answer to four decimal places. (3 marks)

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h f(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to y(3.2), giving your answer to three decimal places. (3 marks)

Question: June 2006 Q2

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \frac{x^2 + y^2}{xy}$$

and

$$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(1.1).

(3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and h = 0.1, to obtain an approximation to y(1.1), giving your answer to four decimal places. (6 marks)

Question: Jan 2009 Q1

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \frac{x^2 + y^2}{x + y}$$

and

$$v(1) = 3$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with h = 0.2, to obtain an approximation to y(1.2).

(3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h f(x_r, y_r)$ and $k_2 = h f(x_r + h, y_r + k_1)$ and h = 0.2, to obtain an approximation to y(1.2), giving your answer to four decimal places. (5 marks)

Question: June 2008 Q1

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \ln(x + y)$$

and

$$v(2) = 3$$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and h = 0.1, to obtain an approximation to y(2.1), giving your answer to four decimal places. (6 marks)

Question: Jan 2010 Q1

The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = x \ln(2x + y)$$

and

$$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(3.1), giving your answer to four decimal places. (3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h f(x_r, y_r)$ and $k_2 = h f(x_r + h, y_r + k_1)$ and h = 0.1, to obtain an approximation to y(3.1), giving your answer to four decimal places. (5 marks)

Differential equations – Numerical methods – Exam questions'answers

	Differential equation	ns – N	lumer	rical methods – Exam question
Que	stion: June 2007 Q2			Question: Jan 2007 Q1
1	$y_1 = 2 + 0.1 \times \sqrt{1^2 + 2^2 + 3}$	M1		(a) $y(1.05) = 0.6 + 0.05 \times [\ln(1 + 0.6477(7557) = 0.64781]$
)	$v(1.1) = 2 + 0.1 \times \sqrt{8}$	A1		= 0.0477 (7337) = 0.0478
3	y(1.1) = 2.28284 = 2.2828 to 4dp	A1	3	(b) $k_1 = 0.05 \times \ln(1+1+0.6) = 0.0$
(b)	$k_1 = 0.1 \times \sqrt{8} = 0.2828$	M1 A1ft		$k_2 = 0.05 \times f (1.05, 0.6477)$
	$k_2 = 0.1 \times f (1.1, 2.2828)$	M1		$\dots = 0.05 \times \ln(1 + 1.05^2 + 0.647)$
	$= 0.1 \times \sqrt{9.42137} = 0.3069(425)$	A1		= 0.0505(85)
)	$y(1.1) = y(1) + \frac{1}{2} [0.28284 + 0.30694]$	ml		$y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$
2	2.29489 = 2.2949 to 4dp	A1	6	$= 0.6 + 0.5 \times 0.09836$
Que	Total stion: Jan 2006 Q5		9	= 0.6492 to 4dp
(a)(i)	$y(1.1) = y(1) + 0.1[1\ln 1 + 1/1]$	MIAI		Question: Jun 2009 Q1
	= 1+0.1 = 1.1	A1	3	(a) $y(3.1) = y(3) + 0.1\sqrt{3^2 + 2 + 1}$
				$= 2 + 0.1 \times \sqrt{12} = 2.34$
(ii)	y(1.2) = y(1) + 2(0.1)[f(1.1, y(1.1))]	MIAI		= 2.340
				(b) $y(3.2) = y(3) + 2(0.1)[f(3.1, y($
	= 1+2(0.1)[1.1ln1.1+(1.1)/1.1]	Al√		$\dots = 2 + 2(0.1)[\sqrt{(3.1^2 + 2.340)}]$
	= 1+0.2×1.104841198			
	= 1.22096824 = 1.221 to 3dp	Αl	4	$\dots = 2 + 0.2 \times 3.599499 = 2.7$ = 2.7
(ISA/II)				
(b)(i)	IF is $e^{\int -\frac{1}{x} dx}$	M1		Question: June 2006 Q2 (a)
	$=e^{-\ln x}$	A1		(a) $y_1 = 2 + 0.1 \times \left[\frac{1^2 + 2^2}{1 \times 2} \right]$
	$= e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$	A1	3	$= 2 + 0.1 \times 2.5 = 2.25$
				(b) $k_1 = 0.1 \times 2.5 = 0.25$
				$k_2 = 0.1 \times f(1.1, 2.25)$
(ii)	$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{y}{x} \right) = \ln x$			$\dots = 0.1 \times 2.53434 = 0.25$
	$\frac{dx}{dx} \left(\frac{x}{x} \right)^{-mx}$	MIAI		$y(1.1) = y(1) + \frac{1}{2} [0.25 + 0.25]$
	$\frac{y}{x} = \int \ln x dx = x \ln x - \int x \left(\frac{1}{x}\right) dx$	M1		= 2.2517 to 4dp
	x y y			Question: Jan 2009 Q1
	$y_{-x \ln x - x + c}$			(a) $y_1 = 3 + 0.2 \times \left[\frac{1^2 + 3^2}{1 + 3} \right]$
	$\frac{y}{x} = x \ln x - x + c$	A1		= 3.5
	$y(1) = 1 \Rightarrow 1 = \ln 1 - 1 + c$	m1		(b) $k_1 = 0.2 \times 2.5 = 0.5$
	$\Rightarrow c = 2 \Rightarrow y = x^2 \ln x - x^2 + 2x$	Al	6	$k_2 = 0.2 \times f(1.2, 3.5)$
	- v 2-j-x ma-x 12x	13.1	Ÿ	$\dots = 0.2 \times \frac{1.2^2 + 3.5^2}{1.2 + 3.5} = 0.55$
(iii)	y(1.2) = 1.222543 = 1.223 to 3dp	Bl	1	1.2 5.5
	Total			$y(1.2) = y(1) + \frac{1}{2} [0.5 + 0.5825]$

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Qu	estion: Jan 2007 Q1		
(a)	$y(1.05) = 0.6 + 0.05 \times [\ln(1+1+0.6)]$	M1A1	
()	= 0.6477 (7557) = 0.6478 to 4dp	A1	3
	= 0.0477 (7557) = 0.0470 to 4ap		
(b)	$k_1 = 0.05 \times \ln(1 + 1 + 0.6) = 0.0477(75)$	M1	
. ,		AlF	
	$k_2 = 0.05 \times f (1.05, 0.6477)$	AII	
	_	M1	
	$\dots = 0.05 \times \ln \left(1 + 1.05^2 + 0.6477 \dots \right)$	1411	
	= 0.0505(85)	A1F	
	$y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$,	
	$= 0.6 + 0.5 \times 0.09836$	ml	
	0.0 1 0.0 × 0.0 > 0.0 1		
	= 0.6492 to 4dp	A1F	6
-	Total		9
Qu	estion: Jun 2009 Q1		
(a)	$y(3.1) = y(3) + 0.1\sqrt{3^2 + 2 + 1}$	3.61.4.1	
()	$y(3.1) - y(3) + 0.1 \sqrt{3} + 2 + 1$	M1A1	
	$= 2 + 0.1 \times \sqrt{12} = 2.3464(10)$		
	= 2.3464	A1	3
(b)	y(3.2) = y(3) + 2(0.1)[f(3.1, y(3.1))]	M1	
	$\dots = 2 + 2(0.1) \left[\sqrt{(3.1^2 + 2.3464 + 1)} \right]$	A1F	
	2 · 2(012)[\(\frac{1}{2}\)(012 · 218 · 01 · 12)]	7111	
	$\dots = 2 + 0.2 \times 3.599499 = 2.719(89)$		
	= 2.720	A1	3
	2.720	211	
	Total		6
Qu	estion: June 2006 Q2		
(a)	1		
()	$y_1 = 2 + 0.1 \times \left[\frac{1^2 + 2^2}{1 \times 2} \right]$	M1 A1	
	$= 2 + 0.1 \times 2.5 = 2.25$	A1	3
(b)	$k_1 = 0.1 \times 2.5 = 0.25$	M1	
		A1√	
	$k_2 = 0.1 \times f(1.1, 2.25)$	M1	
	$\dots = 0.1 \times 2.53434 = 0.2534(34)$	A1√	
	$y(1.1) = y(1) + \frac{1}{2} [0.25 + 0.253434]$	m1	
	<u>~</u>		
	= 2.2517 to 4dp	A1√	- 6
0	Total	I	9
	estion: Jan 2009 Q1	I	I
(a)	$y_1 = 3 + 0.2 \times \left[\frac{1^2 + 3^2}{1 + 3} \right]$	M1A1	
	[1+3]		
	= 3 .5	A1	3
(b)	$k_1 = 0.2 \times 2.5 = 0.5$	B1ft	
(6)			
	$k_2 = 0.2 \times f(1.2, 3.5)$	M1	
	$\dots = 0.2 \times \frac{1.2^2 + 3.5^2}{1.2 + 3.5} = 0.5825(53)$		
	1.2 + 3.5	Alft	
	1 -		
	$y(1.2) = y(1) + \frac{1}{2} [0.5 + 0.5825(53)]$	m1	
	2	1111	
	2.54125		_
	= 3.54127 = 3.5413 to 4dp	Alft	5
	T-4-1		0
	Total	1	8

Question: June 2008 Q1

-	Question: Jan 2010 Q1		T	
	Nucetions Ion 2010 O1	Total		6
	= 3.163487 = 3.1635 to 4dp		A1	6
	$y(2.1) = y(2) + \frac{1}{2} [k_1 + k_2]$ = 3 + 0.5 × 0.3269748		m1	
	= 0.1660(31)		Al	
	$k_2 = 0.1 \times f(2.1, 3 + *)$ = 0.1 × ln(2.1 + 3.16094)]		M1	
	= 0.1609(4379) (= *)		Al	
	$k_1 = 0.1 \times \ln(2+3)$		M1	

(a)	$y_1 = 2 + 0.1 \times [3\ln(2 \times 3 + 2)] = 2 + 0.3\ln 8$		
	= 2.6238(3)	M1A1	
	y(3.1) = 2.6238 (to 4dp)	A1	3
(b)	$k_1 = 0.1 \times 3 \ln 8 = 0.6238(32)$	B1F	
	$k_2 = 0.1 \times f(3.1, 2.6238(32))$	M1	
	$\dots = 0.1 \times 3.1 \times \ln 8.8238(32)$	A1F	
	[= 0.6750(1)		
	$y(3.1) = 2 + \frac{1}{2} [0.6238(3) + 0.6750(1)]$	m1	
	= 2.6494(2) = 2.6494 to 4dp	A1	5
	Total		8