Numerical methods to solve first order differential equation

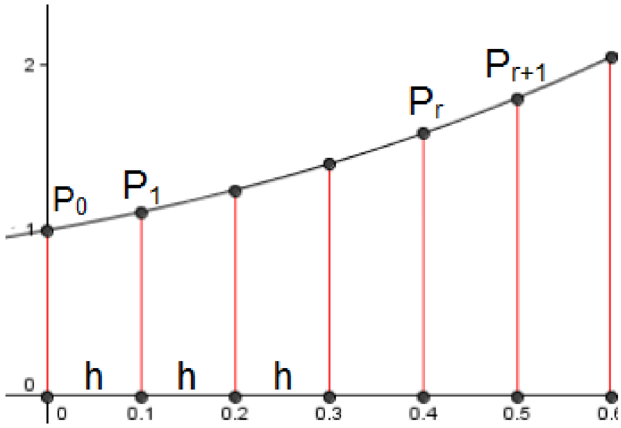
In this chapter, we want to solve equations which can be written

$$\frac{dy}{dx} = f(x,y) \quad and \quad y(x_0) = y_0$$

There are three methods to solve numerically this equation.

Formulae to be used will be stated explicitly in questions.

Knowing $P_0(x_0, y_0)$, we work out P_1 *then* P_2 *then* P_3 *etc.*



Euler's formula

To work out P_{r+1}, we consider that

the gradient of the line P_rP_{r+1} is (approx.) equal to the gradient at P_r.

This gives: $y_{r+1} = y_r + hf(x_r, y_r)$

The mid-point formula

We consider that the gradient of the line $P_{r-1}P_{r+1}$ is (approx.) equal to the gradient at P_r :

This gives $\quad y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$

The improved Euler's formula

We consider that the gradient of the line P_rP_{r+1} is (approx.) the mean of

the gradient at P_r *and* the gradient at P_{r+1}.

This gives :

$$y_{r+1} = y_r + \frac{h}{2}\left[f(x_r, y_r) + f(x_{r+1}, y^*_{r+1}) \right]$$

$$with \ \ y^*_{r+1} = y_r + hf(x_r, y_r)$$

Or as it is given in the exam question:

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

$$where \ \ k_1 = hf(x_r, y_r) \ \ and \ \ k_2 = hf(x_r + h, y_r + k_1)$$

Possible layout for your workings out:

r	x_r	y_r	k_1	$x_r + h$	$y_r + k_1$	k_2	y_{r+1}
0							
1							
2							