Algebra

Indices and surds

KEY POINTS

Definition : *for* n integer and $a \neq 0$, $a^n = a \times a \times a ... \times a$ (n *factors*)

$$a^1 = a \quad and \quad a^0 = 1$$

(a is called the base)

Multiplication : $a^n \times a^p = a^{n+p}$

Division : $a^n \div a^p = \dfrac{a^n}{a^p} = a^{n-p}$

Negative index : $a^{-n} = \dfrac{1}{a^n}$ *and* $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Power of a power : $\left(a^n\right)^p = a^{n \times p}$

Fractional index : $a^{\frac{1}{n}} = \sqrt[n]{a}$ *and* $a^{\frac{n}{p}} = \sqrt[p]{a^n} = \left(\sqrt[p]{a}\right)^n$

Different bases : $a^n \times b^n = (ab)^n$ *and* $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$

KEY POINTS

Definition : *for* $a \geq 0$, $\sqrt{a}$ is the positive number so that $\left(\sqrt{a}\right)^2 = a$

Multiplication : $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

   in particular $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = \left(\sqrt{a}\right)^2 = a$ (*for* $a \geq 0$)

Division : $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ (*for* $a \geq 0, b > 0$)

Difference of squares : $(x+y)(x-y) = x^2 - y^2$

   hence $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$

$\sqrt{a} \pm \sqrt{b}$ is called the conjugate (expression) to $\sqrt{a} \mp \sqrt{b}$

Rationalise the denominator:

- $\dfrac{a}{\sqrt{b}} = \dfrac{a}{\sqrt{b}} \times \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{a\sqrt{b}}{b}$

- $\dfrac{1}{\sqrt{a} \pm \sqrt{b}} = \dfrac{1}{\sqrt{a} \pm \sqrt{b}} \times \dfrac{\sqrt{a} \mp \sqrt{b}}{\sqrt{a} \mp \sqrt{b}} = \dfrac{\sqrt{a} \mp \sqrt{b}}{a - b}$

(Multiple numerator and denominator by the conjugate expression)