Forces acting at an angle (with Friction)

Here as in leaflet 2.6, we consider forces that act at an angle, but this time including friction.

Worked Example 1.

A force of 18 N acts on a particle, of mass 7.5 kg, at an angle of 30° above the horizontal. The particle is on a rough horizontal plane. Given that the particle is on the point of slipping, what is the coefficient of friction, between the particle and the plane?

Solution

Figure 1 shows the forces acting on the particle.

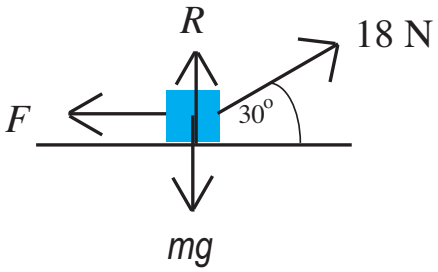

Figure 1

Resolving vertically: $\mathrm{R} = 7.5g - 18\sin 30°$

Resolving horizontally: $\mathrm{F} = 18\cos 30°$

As the particle is on the point of slipping, friction is limiting ($F = F_{MAX}$), so $F = \mu R$:

$$\begin{aligned} 18\cos 30° &= \mu(7.5g - 18\sin 30°) \\ \mu &= \frac{18\cos 30°}{7.5g - 18\sin 30°} = 0.24 \end{aligned}$$

Worked Example 2.

A particle, of mass m kg, is in equilibrium under a force of magnitude T N, which acts at an angle α above the horizontal. Given the coefficient of friction between the particle and the rough horizontal plane is μ, show that $T \leq \dfrac{\mu mg}{\cos\alpha + \mu\sin\alpha}$

Solution

Resolving vertically:

$$R + T\sin\alpha - mg = 0 \Rightarrow R = mg - T\sin\alpha$$

Resolving horizontally:

$$T\cos\alpha - F = 0 \Rightarrow F = T\cos\alpha$$

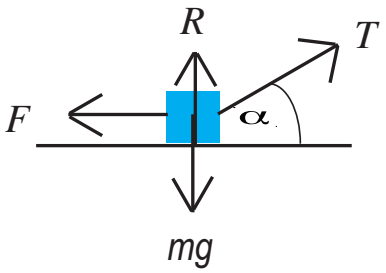

Figure 2

As the particle is in Equilibrium:

$$\begin{aligned} F &\leq \mu R \\ T\cos\alpha &\leq \mu(mg - T\sin\alpha) \\ T\cos\alpha &\leq \mu mg - \mu T\sin\alpha \\ T\cos\alpha + \mu T\sin\alpha &\leq \mu mg \\ T(\cos\alpha + \mu\sin\alpha) &\leq \mu mg \\ T &\leq \frac{\mu mg}{\cos\alpha + \mu\sin\alpha} \end{aligned}$$

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Worked Example 3.

A light inextensible rope is used to pull a particle of mass 2.5 kg along a rough horizontal plane. If the tension in the rope is 25 N and acts at an angle of 25° above the horizontal and the coefficient of sliding friction between the particle and the surface is 0.55, what is the acceleration of the particle?

Solution

As the motion is horizontal, the sum of vertical components of the force equals zero.

Resolving vertically: $R = 2.5g - 25\sin 25°$

As the particle is moving the frictional force satisfies $F = \mu R$, where μ is the coefficient of sliding friction:

$F = \mu R = 0.55(2.5g - 25\sin 25°)$

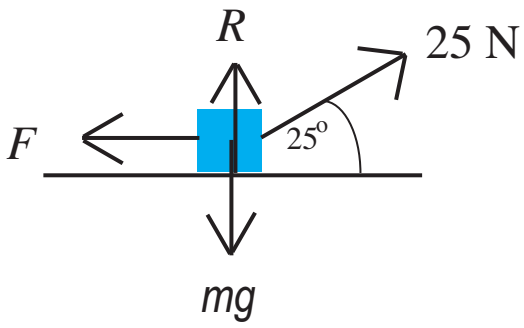

Figure 3

The resultant force is $25\cos 25° - F$.
Use Newton's Second Law parallel to the plane:

$$ma = 25\cos 25° - 0.55(2.5g - 25\sin 25°)$$

$$a = \frac{25\cos 25° - 0.55(2.5g - 25\sin 25°)}{2.5} = 6.0 \text{ m s}^{-2} \text{ (2 s.f.)}$$

Exercises

1. A force of 16 N acts on a particle, of mass 11 kg, at an angle of 18° above the horizontal. The particle is on a rough horizontal plane. Given the particle is on the point of slipping, what is the coefficient of friction, between the particle and the plane?

2. A particle, of mass 6 kg, is in equilibrium on a rough horizontal plane under a force of magnitude T N, which acts at an angle 15° above the horizontal. Given the coefficient of friction between the particle and the rough horizontal plane is 0.35, what values could T take?

3. A light inextensible rope is used to pull a particle of mass 3 kg along a rough horizontal plane. The tension in the rope is 15 N and acts at an angle of 40° above the horizontal. If the coefficient of sliding friction between the particle and the surface is 0.45, what is the acceleration of the particle?

4. A force of 48 N acts on a particle on a rough horizontal plane at an angle of 20° above the horizontal. Given the particle is on the point of slipping, and the coefficient of friction between the particle and the plane is 0.6, what is the mass of the particle?

5. The coefficient of friction between a particle, of mass 8.5 kg, and a rough horizontal plane is 0.8. Given a force of 50 N acts on the particle at an angle of 40° above the horizontal, does slipping occur?

6. A light inextensible rope is used to pull a particle of mass 2 kg along a rough horizontal plane. Given the tension in the rope is 27 N and acts at an angle of 30° above the horizontal and that the particle is moving with an acceleration of 10 m s $^{-2}$, what is the coefficient of sliding friction between the particle and the surface?

Answers (all to 2 s.f.)
1. 0.15 2. $T \leq 19$ N 3. 0.86 m s $^{-2}$ 4. 9.3 kg 5. No, F = 38 N< 41 N (F_{MAX}) 6. 0.55

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