Calculus

Differentiation

Notation

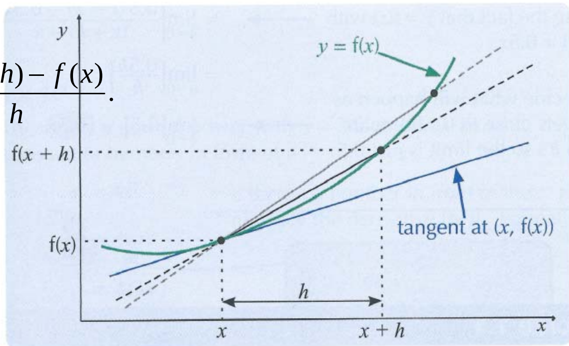
- The function you get from differentiating y with respect to x is called the DERIVATIVE of y and it's written $\frac{dy}{dx}$.
- $\frac{dy}{dx}$ is the rate of change of y with respect to x.

  It is the gradient of the curve/the tangent to the curve.
- The notation $f\,'(x)\,(f \text{ prime of } x)$ is sometimes used instead of $\frac{dy}{dx}$.

Differentiation from first principle.

Consider two points on a curve $A(x, f(x))$ *and* a point B close to A

$B(x+h, f(x+h))$ *where H is "small".*



The chord AB has gradient $\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$.

When B get closer and closer to A, h tends to 0.

If $\frac{f(x+h)-f(x)}{h}$ has a value when h tends to 0,

this value is the gradient of the curve at A: $f\,'(x)$.

Example: $f(x) = x^2$

Let's work out the gradient of the curve at $x = 3$.

- $A(3, 3^2)$ *and* $B(3+h, (3+h)^2)$

  the gradient of AB: $m = \frac{(3+h)^2 - 3^2}{3+h-3} = \frac{9+6h+h^2-9}{h} = 6+h$

When h *tends to* 0, m *tends to* 6:

Conclusion: $\frac{dy}{dx}(x=3) = f\,'(3) = 6$

Differentiating polynomials

- $\text{if } y = x^n \quad \text{then} \quad \frac{dy}{dx} = nx^{n-1}$
- $\text{if } y = x^n + x^p \quad \text{then} \quad \frac{dy}{dx} = nx^{n-1} + px^{p-1}$
- $\text{if } y = k \times x^n \quad \text{then} \quad \frac{dy}{dx} = k \times nx^{n-1} \quad \text{where } k \in \mathbb{R}.$

Example: $y = x^4 \qquad \frac{dy}{dx} = 4x^3$

$y = 5x^6 \qquad \frac{dy}{dx} = 5 \times 6x^5 = 30x^5$

$y = 3x^4 + 5x^3 + x \qquad \frac{dy}{dx} = 12x^3 + 15x^2 + 1$