Auxiliary equations method.

Definitions

$a\dfrac{dy}{dx}+by=f(x)$ with $a,b\in\mathbb{R}$

- The REDUCED equation is $a\dfrac{dy}{dx}+by=0$.

  The general solution of the reduced equation is called

  The COMPLEMENTARY FUNCTION

- A PARTICULAR INTEGRAL satisfies the equation $a\dfrac{dy}{dx}+by=f(x)$

- The general solution of $a\dfrac{dy}{dx}+by=f(x)$ is

  the sum of the complementary function and the particular integral

  $y_G=y_P+y_C$

Solving first order linear differential equations

$a\dfrac{dy}{dx}+by=f(x)$ is a differential equation where a and b are real numbers

The reduced eqaution is $a\dfrac{dy}{dx}+by=0$

- The AUXILIARY equation associated with this equation is $a\lambda+b=0$
- The complementary function is : $y=Ce^{\lambda x}$ where λ is solution to $a\lambda+b=0$.
- Finding the particular integral:
  - ⊗if $f(x)$ is a polynomial then y_P is also a polynomial of the same degree
  - ⊗if $f(x)=ACos(kx)+BSin(kx)$ then $y_P=aCos(kx)+bSin(kx)$

    a and b to be worked out.
  - ⊗if $f(x)=Ae^{kx}$ then $y_P=ae^{kx}$ if $k\neq\lambda$

    $y_P=axe^{kx}$ if $k=\lambda$ where a is to be worked out
- The general solution is $y_G=y_P+y_C$